

1. **Claim:** For all natural numbers n , if n is even then $n^2 + 2011$ is odd.

Proof: *by Induction*

Let

$$P(n) = n^2 + 2011 \forall n \in \mathbb{N}$$

Base Case:

$$P(0) = 0^2 + 2011 = 2011$$

which is odd. So the base case is true.

Inductive Step:

Assume $P(n)$ is true. Taking $P(n + 2)$

$$P(n + 2) = (n + 2)^2 + 2011$$

$$P(n + 2) = n^2 + 4n + 2015$$

Take $K(q) = q^2 + 4q + 2015$. There is only one possibility

- (a) q is even. If q is even, q^2 is also even. Moreover $4q$ becomes even.
So we have

$$K(q) = \text{Even} + \text{Even} + \text{Odd}$$

$$K(q) = \text{Even} + \text{Odd}$$

$$K(q) = \text{Odd}$$

So, $P(n + 2) = n^2 + 4n + 2015$ is odd. So, $P(n) \rightarrow P(n + 2)$ is true.
The claim is correct.