

Claim: For all natural numbers k , if the pile starts with $4k+1$ tokens, then Thuc has a winning strategy. **Proof:** *by Induction* Let $P(k) = 4k+1 \forall k \in \mathbb{N}$ **Base Case:**

$$P(0) = 4(0) + 1 = 1$$

If there's only 1 token, then Tamara will end up picking it firstly, and thus Thuc wins.

Inductive Step: Assume $P(k)$ is true. Taking $P(k+1)$

$$P(k+1) = 4(k+1) + 1$$

$$P(k+1) = (4k+1) + 4$$

$$P(k+1) = P(k) + 4$$

Tamara has to go first, and can pick up either 1, 2 or 3 tiles. Thus, Thuc can afterwards pick

$$4 - \text{No of tiles Tamara picked}$$

to get the number of tiles in the form of $P(k) = 4k+1$, under which condition Thuc knows he will win.