

1. **Claim:** For all $n \in \mathbb{N}$ prove that $\sum_1^n a \cdot r^{n-1} = a \cdot \left(\frac{r^n - 1}{r - 1}\right)$

Let $P(n)$ be the proposition $\sum_1^n a \cdot r^{n-1} = a \cdot \left(\frac{r^n - 1}{r - 1}\right)$

Base Case: for $n = 1$, we have

LHS

$$a \cdot r^0 = a$$

RHS

$$a \cdot \left(\frac{r^1 - 1}{r - 1}\right) = a$$

So the LHS = RHS, and we conclude that the Base Case holds.

Inductive Step: Suppose that $P(n)$ is true. So $P(n+1)$. The LHS becomes

$$\begin{aligned} & \sum_1^{n+1} a \cdot r^{n-1} \\ &= a + a \cdot r + a \cdot r^2 + \dots + a \cdot r^{n-1} + a \cdot r^n \\ &= a \cdot \left(\frac{r^n - 1}{r - 1}\right) + a \cdot r^n \end{aligned}$$

Further Manipulation gives,

$$a \cdot \left(\frac{r^{n+1} - 1}{r - 1}\right)$$

which is RHS. Proved.