

①

a) $y = v_0 t - 5t^2$

$y' = v_0 - 10t = 0$

$v_0 = 10t$

$t = \frac{v_0}{10}$

b) $f(x, y) = e^{\frac{-(x-x_0)^2 - (y-y_0)^2}{2\sigma^2}}$

$f'_x = e^{\frac{-(x-x_0)^2 - (y-y_0)^2}{2\sigma^2}} \left(-\frac{x-x_0}{\sigma^2} \right) = 0$

~~and~~ $x = x_0$

~~$f'_y = e^{\frac{0 - (y-y_0)^2}{2\sigma^2}} \left(-\frac{y-y_0}{\sigma^2} \right) = 0$~~

$y = y_0$

2.

% code:

```
dark = double(rgb2gray(imread('u2dark.png')));
```

% a

```
avg=mean2(dark);
```

```
min=min(min(dark));
```

```
max=max(max(dark));
```

%b

```
fixedimg = (dark-min)*256/(max-min+1);
```

```
imshow(uint8(fixedimg));
```

%c

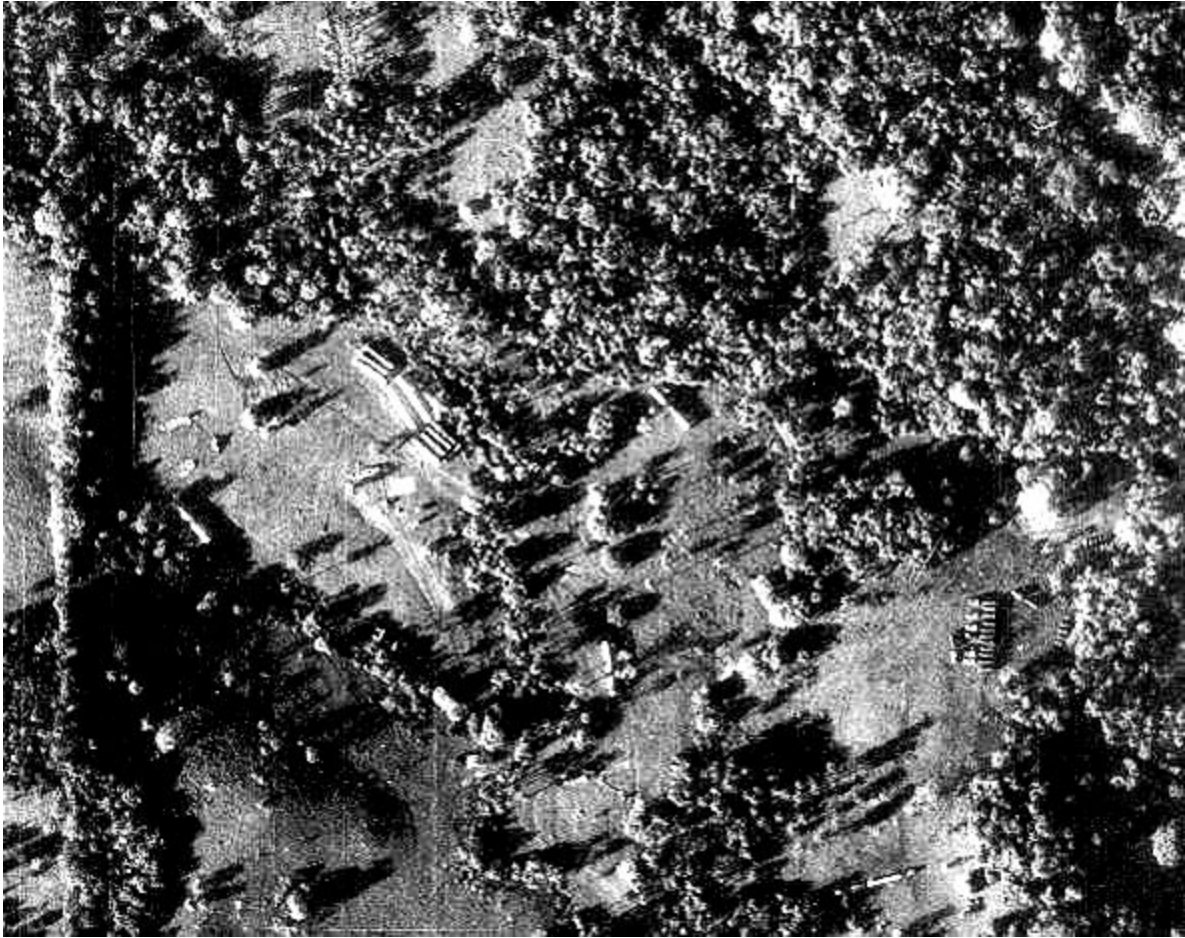
```
contrasted = 2*(fixedimg-128) + 128;
```

```
contrasted = uint8(contrasted);
```

```
imshow(contrasted);
```

Images: fixed and contrasted (in this order).





3.code:

```
function edgedetector()
    img = double(rgb2gray(imread('buoys.jpg')));

    edges = DetectVerticalEdges(img);
    blurred_edges = BoxBlur(edges);

    figure('Name','Original Image')
    imshow(img, []);

    figure('Name','Edges')
    imshow(edges, []);

    figure('Name','Blurred Edges')
    imshow(blurred_edges, []);

end
% a. Tiny edges are waves.
function edges = DetectVerticalEdges(img)

    left=circshift(img,[0,1]);
    edges =uint8(img-left); %zeros(height, width-1);

end
% b
function blurred = BoxBlur(img)
    img = double(img);

    height = size(img, 1);
    width = size(img, 2);
    n=5; % width of the blur
    blurred = zeros(height-(n-1),width-(n-1));
    for y=1:height-(n-1)
        for x=1:width-(n-1)
            bla=img(y:y+n-1,x:x+n-1);
            blurred(y,x) = sum(sum(bla));
        end
    end

    blurred = blurred / n^2;
end
```

Images: edges and blurred (in this order).



4.

a) Rotation by 45° and then rotation by 45° is the same as rotation by 90° .

by 90°

$$X_2 = A \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

b)

$$T = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/4 \\ \sqrt{2}/2 & \sqrt{2}/4 \end{bmatrix}$$

c)

d)

$$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then \neq then

$p_1 \neq p_2$

e)

$$\begin{aligned} RYA &= B \\ YA &= R^T B \\ Y &= R^T B A^{-1} \end{aligned}$$

5.

```
flower = double(rgb2gray(imread('flower.bmp')));  
[u,s,v] = svd(flower);  
diagonal = diag(s);  
ten = diagonal(1:10);  
x = 1:10;  
graph = plot(x,ten);  
% first values are the largest
```

```
k = 10;  
a = u(:, 1:k);  
b = s(1:k, 1:k);  
vt = transpose(v);  
c = vt(1:k, :);  
res = a*b*c;  
figure;  
imshow(uint8(res));
```

```
k = 50;  
a = u(:, 1:k);  
b = s(1:k, 1:k);  
vt = transpose(v);  
c = vt(1:k, :);  
res = a*b*c;  
figure;  
imshow(uint8(res));
```

```
k = 100;  
a = u(:, 1:k);  
b = s(1:k, 1:k);  
vt = transpose(v);  
c = vt(1:k, :);  
res = a*b*c;  
figure;  
imshow(uint8(res));
```

300p x 300p is 90 000p. Now we have $300 \times k + k \times k + k \times 300$ so $300 \times 200 + 200 \times 200 + 200 \times 300 = 160000$

We used 1:k range so k+1:r is error

6.

$$\sigma) \|A\| = \max \sqrt{(Ax)^T Ax} = \max \sqrt{x^T A^T A x}$$

$$= \max \sqrt{(U \Sigma V^T)^T (U \Sigma V^T) x} =$$

$$= \max \sqrt{x^T V \Sigma^T U^T U \Sigma V^T x} =$$

$$= \max \sqrt{w^T \Sigma^T \Sigma w}$$

$$\Sigma^T \Sigma = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ \vdots & \vdots & \ddots \end{bmatrix} \text{ for } w = I \text{ (2-norm)}$$

2-norm ist σ_1 , also 2-norm von A ist σ_1
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$$b) \|A\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^m |a_{ij}|^2}$$

$$A^T A = (U \Sigma V^T)^T (U \Sigma V^T) = V \Sigma^T U^T U \Sigma V^T =$$

$$= V \Sigma^T \Sigma V^T \text{ also } \text{trace}(A^T A) = \text{trace}(\Sigma^T \Sigma) =$$

$$= \sigma_1^2 + \sigma_2^2 + \dots + \sigma_p^2$$