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Cálculo II

Integrais Triplas

a - $\iiint_E y\,dV$ $\quad E = \{(x,y,z) \mid 0 \le x \le 3,\ 0 \le y \le x,\ x-y \le z \le x+y\}$

$$\int_0^3 \int_0^x \int_{z-y}^{x+y} y \; dz\,dx\,dy$$

$$\int_{x-y}^{x+y} y\,dz = yz \Big|_{x-y}^{x+y} = y(x+y) - y(x-y)$$

$$xy + y^2 - xy + y^2 = 2y^2$$

$$\int_0^x 2y^2\,dy = \frac{2y^3}{3}\Big|_0^x = \frac{2}{3}x^3$$

$$\int_0^3 \frac{2}{3}x^3 \quad \frac{2}{3}\cdot\frac{x^4}{4} = \frac{x^4}{6}\Big|_0^3$$

$$\frac{81}{6} - \frac{27}{2}$$

b. $\iiint_E e^{z/y}$ $\quad E = \{(x,y,z) \mid 0 \le y \le 1,\ y \le x \le 1,\ x-y \le z \le x+y\}$

$$\int_0^1 \int_y^1 \int_{x-y}^{x+y} e^{z/y}\, dz\, dx\, dy$$

$$\int_{x-y}^{x+y} e^{z/y}\, dz = y\, e^{z/y} \Big|_{x-y}^{x+y} = y \cdot e^{\frac{x+y}{y}} - y\, e^{\frac{x-y}{y}}$$

$$\int_y^1 y \cdot e^{\frac{x+y}{y}} - y \cdot \frac{1}{y} \cdot e^{\frac{x-y}{y}} = e^{\frac{x+y}{y}} - e^{\frac{x-y}{y}} \Big|_y^1$$

$$\int_0^1 e^{\frac{y-1}{y}} - e^{\frac{1-y}{y}} + 1 - e^2\, dy$$

$$= e^{\frac{y-1}{y}} - e^{\frac{1-y}{y}} + 1 - e^2 =$$