Cálculo

a. $\iint_D \frac{y}{x^2+1}$

$$\int_0^4 \int_0^{\sqrt{x}} \frac{y}{x^2+1} \, dy \, dx$$

$$\frac{y^2}{2} \cdot \frac{1}{x^2+1} \Big|_0^{\sqrt{x}} = \frac{x}{2x^2+2}$$

$$\frac{1}{2} \int_0^4 \frac{x}{x^2+1} \Big| = \frac{1}{4} \int \frac{du}{u} = 1/4 \ln u = 1/4 \ln(x^2+1) \Big|_0^4$$

$$1/4 \left[\ln 17 - \ln(1) \right] = 1/4 \cdot \ln 17$$

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b- $\iint_D (2x+y)$

$$\int_1^2 \int_{y-1}^1 2x+y \; dx\,dy$$

$$\int_{y-1}^1 2x+y \; dx = x^2 + xy \Big|_{y-1}^{1}$$

$$(1+y) - [(y-1)^2 + (y-1)\cdot y]$$
$$(1+y) - [y^2 - 2y + 1 + y^2 - y]$$
$$(1+y) - [2y^2 - 3y + 1]$$

$$1+y-2y^2+3y-1 = -2y^2+4y$$

$$\int_1^2 4y - 2y^2 \; dy$$

$$\frac{\cancel{4}^{2}y^2}{\cancel{2}} - \frac{2y^3}{3} = 2y^2 - \frac{2y^3}{3} \Big|_1^2$$

$$8 - \frac{16}{3} - 2 + 2/3 = 6 - 14/3 = \frac{18}{3} - \frac{14}{3} = \frac{4}{3} \;//$$

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c- $\iint_D (x \cos y)$

$$\int_0^1 \int_0^{x^2} x \cdot \cos y \; dy\, dx$$

$$\int_0^{x^2} x \cos y = x \int_0^{x^2} \cos y = x \cdot \operatorname{sen} y \Big|_0^{x^2}$$

$$x \cdot \operatorname{sen} x^2$$

$$\int_0^1 x \operatorname{sen} x^2 \, dx = \frac{1}{2} \int \operatorname{sen} u \, du$$

$$\frac{1}{2} \int (-\cos u) = -\frac{1}{2} \cos u$$

$$-\frac{1}{2} \cos x^2 \Big|_0^1$$

$$-\frac{1}{2} \cos 1 - \left(-\frac{1}{2} \cos 0\right)$$

$$\boxed{1 - \frac{1}{2} \cos 1}$$

D- $\iint_D (xy^2)$

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} xy^2$$

$$\int_{-1}^{1} \frac{1-y^4}{2}\, dy = 1/2 \int_{-1}^{1} 1-y^4$$

$$1/2 \left. y - y^5/5 \right|_{-1}^{1} = 1/2 \cdot 8/5 = 4/5$$

Refreshing