Day- 2080/01/03, Sunday (1)

Exercise

Nppahari

Eigenvalue/ Eigen vector/ Eigen decomposition

1. Find the eigenvalues and eigen vectors of the following matrices. Are the eigen vectors orthogonal?

(a) $A=\begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}$ (b) $B=\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ (c) $C=\begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$

Ans; Yes in all cases.

2. Find the eigen vectors of the matrix $A=\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$. Are the eigen vectors orthogonal? Ans No.

3. Show that the matrix $B=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ has complex eigen values. Find the corresponding eigen vectors.

Ans; $\lambda = \pm i$, $v_1=\begin{pmatrix} 1 \\ i \end{pmatrix}$, $v_2=\begin{pmatrix} i \\ 1 \end{pmatrix}$.

4. Consider the following matrices

(a) $A=\begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix}$ (b) $C=\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$ (c) $L=\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$

(d) $E=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, reflection matrix

Find the eigen value and eigen vectors in each case. Express the matrices as the

(i) product of three matrices in the form $A=PDP^{-1}$, where the symbol as usual.

(ii) Decompose the matrices A as the eigen value decomposition of the form $A=\lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T$. where the symbol as usual.

(5) Consider the following matrice

(a) $A=\begin{pmatrix} 1 & 4 & 3 \\ 4 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ (b) $B=\begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$

Find the eigen value and eigen vectors in each case.
Express the matrics as the

(i) Product of the three matrics of the form
$A = PDP^{-1}$ (Diagonalize)

(ii) Decompose the matrics as the spectral value decomposition of the form $A = \lambda_1 u_1 u_1^T + \lambda_2 u_2 u_2^T + \lambda_3 u_3 u_3^T$

Hint: (a) $\lambda_1 = -4$, $\lambda_2 = 1$, $\lambda_3 = 6$, $u_1 = \frac{1}{\sqrt{50}}\begin{pmatrix}5\\4\\3\end{pmatrix}$,

$u_2 = \frac{1}{\sqrt{25}}\begin{pmatrix}0\\-3\\4\end{pmatrix}$, $u_3 = \frac{1}{\sqrt{50}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$

Find P and D.

(b) $\lambda_1 = 8$, $u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\\0\end{pmatrix}$, $\lambda_2 = 6$, $u_2 = \frac{1}{\sqrt{6}}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$,

$\lambda_3 = 3$, $u_3 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}$.

6. Prove the following theorem:
If the matrix A is orthogonally diagonalizable, then A is symmetric.

Note: If λ is repeated or $|P| = 0$. Then the matrix A is not diagonalizable.

$P^{-1}AP = D$ not hold.