

Exercise (Singular value decomposition) (5)
 Find the singular value decomposition of the matrix A

where (a) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{pmatrix}$ (b) $B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

(c) $C = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{pmatrix}$

Ans (a) $A = USV^T$, $U = \begin{pmatrix} \frac{1}{\sqrt{30}} & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{30}} & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{6}} \\ \frac{5}{\sqrt{30}} & 0 & -\frac{1}{\sqrt{6}} \end{pmatrix}$

$S = \begin{pmatrix} \sqrt{6} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$, $V^T = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

(b) $B = USV^T$, $U = \begin{pmatrix} \frac{6}{\sqrt{180}} & -\frac{2}{\sqrt{5}} \\ \frac{12}{\sqrt{180}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $S = \begin{pmatrix} \sqrt{6} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$V^T = \begin{pmatrix} \frac{1}{\sqrt{30}} & \frac{2}{\sqrt{30}} & \frac{5}{\sqrt{30}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \end{pmatrix}$

(c) $C = USV^T$, $U = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix}$, $S = \begin{pmatrix} \sqrt{30} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$V^T = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \end{pmatrix}$