Decentralized Optimal Control for Connected Automated Vehicles at Intersections Including Left and Right Turns

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Abstract— In prior work, we addressed the problem of optimally controlling on line connected and automated vehicles crossing two adjacent intersections in an urban area to minimize fuel consumption while achieving maximal throughput without any explicit traffic signaling and without considering left and right turns. In this paper, we extend the solution of this problem to account for left and right turns under hard safety constraints. Furthermore, we formulate and solve another optimization problem to minimize a measure of passenger discomfort while the vehicle turns at the intersection and we investigate the associated tradeoff between minimizing fuel consumption and passenger discomfort.

I. INTRODUCTION

Traffic light signaling is the prevailing method used to control the traffic flow through an intersection. Aside from the infrastructure cost and the need for dynamically controlling green/red cycles, traffic light systems can increase the number of rear-end collisions at the intersection [1]. Serious delays can occur during hours of heavy traffic if the light cycle is not adjusted appropriately. These challenges have motivated research efforts for new approaches capable of providing a smoother traffic flow and more fuel-efficient driving while also improving safety.

Dresner and Stone [2] proposed the use of a centralized reservation scheme to control a single intersection. Since then, several other efforts using reservation schemes have been reported in the literature [3]–[5] with some focused on minimizing vehicle travel time under collision-avoidance constraints [6]–[10]. A detailed discussion of the research in this area reported in the literature to date can be found in [11].

Connected and automated vehicles (CAVs) provide the most intriguing and promising opportunity to reduce fuel consumption, greenhouse gas emissions, travel delays and to improve safety. In earlier work [12], we established a decentralized optimal control framework to address the problem of optimally controlling CAVs crossing two adjacent intersections in an urban area, without any explicit traffic signaling and without considering left and right turns, with the objective of minimizing fuel consumption while achieving maximal throughput using a First-In-First-Out queue to designate the order in which the CAVs cross the intersection. We also established the conditions under which feasible solutions exist and showed that they can be enforced through an appropriately designed *Feasibility Enforcement Zone* (FEZ) that precedes the *Control Zone* (CZ) in [13].

To consider left and right turns within this framework, "comfort" becomes of fundamental importance in addition to safety. In this paper, we extend the solution of the problem addressed in [12] to account for left and right turns. Then, another optimization problem is formulated with the objective of minimizing a measure of passenger discomfort while the vehicle turns. Furthermore, we investigate the associated tradeoff between minimizing fuel consumption and passenger discomfort inside the *Merging Zone* (MZ).

The problem of coordinating CAVs at intersections including left and right turns has been addressed before. Kim and Kumar in [14] proposed an approach based on Model Predictive Control towards developing an effective intersection management algorithm to achieve system-wide safety and liveness of intersection-crossing traffic. However, in our approach, the objective is to jointly minimize fuel consumption and passenger discomfort for CAVs crossing an intersection, while safety is a hard constraint.

The paper is organized as follows. In Section II, we review the model in [12] and its generalization in [15]. In Section III, we present the model and the analytical solution of the decentralized optimal control problem with the new collision-avoidance terminal conditions. In Section IV, the new optimization problem is formulated and solved for each CAV to address passenger discomfort inside the MZ. Finally, we present simulation results in Section V, and concluding remarks in Section VI.

II. THE MODEL

We briefly review the model introduced in [12] and [15] where there are two intersections, 1 and 2, located within a distance D (Fig. 1). The region at the center of each intersection, called *Merging Zone* (MZ), is the area of potential lateral CAV collision. Although it is not restrictive, this is taken to be a square of side S. Each intersection has a *Control Zone* (CZ) and a coordinator that can communicate with the CAVs traveling within it. The distance between the entry of the CZ and the entry of the MZ is $L > S$, and it is assumed to be the same for all entry points to a given CZ.

Let $N_z(t) \in \mathbb{N}$ be the cumulative number of CAVs which have entered the CZ and formed a queue by time t, $z =$
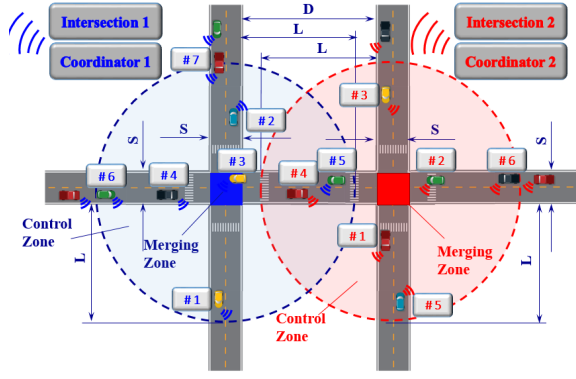
Fig. 1. Connected and automated crossing two adjacent intersections.

1, 2. The way the queue is formed is not restrictive to our analysis in the rest of the paper. When a CAV reaches the CZ of intersection z, the coordinator assigns it an integer value $i = N_z(t) + 1$. If two or more CAVs enter a CZ at the same time, then the corresponding coordinator selects randomly the first one to be assigned the value $N_z(t) + 1$.

For simplicity, we assume that each CAV is governed by second order dynamics

$$\dot{p}_i = v_i(t), \; p_i(t_i^0) = 0; \; \dot{v}_i = u_i(t), \, v_i(t_i^0) \text{ given} \quad (1)$$

where $p_i(t) \in \mathcal{P}_i$, $v_i(t) \in \mathcal{V}_i$, and $u_i(t) \in \mathcal{U}_i$ denote the position of the CAV i starting from the entry of the CZ, speed and acceleration/deceleration (control input) of each CAV i. These dynamics are in force over an interval $[t_i^0, t_i^f]$, where t_i^0 and t_i^f are the times that the vehicle i enters the CZ and exits the MZ of intersection z respectively.

To ensure that the control input and vehicle speed are within a given admissible range, the following constraints are imposed:

$$u_{i,min} \leq u_i(t) \leq u_{i,max}, \quad \text{and} \\ 0 \leq v_{min} \leq v_i(t) \leq v_{max}, \quad \forall t \in [t_i^0, t_i^m], \quad (2)$$

where t_i^m is the time that the vehicle i enters the MZ. To ensure the absence of any rear-end collision throughout the CZ, we impose the *rear-end safety* constraint

$$s_i(t) = p_k(t) - p_i(t) \geq \delta, \quad \forall t \in [t_i^0, t_i^m] \quad (3)$$

where δ is the *minimal safe distance* allowable and k is the CAV physically ahead of i.

The objective of each CAV is to derive an optimal acceleration/deceleration, in terms of fuel consumption, inside the CZ, i.e., over the time interval $[t_i^0, t_i^m]$. In addition, we impose safety constraints to avoid both rear-end and lateral collisions inside the MZ. The conditions under which the rear-end collision avoidance constraint does not become active inside the CZ are provided in [13].

III. VEHICLE COORDINATION AND CONTROL

A. Modeling Left and Right Turns

In order to include left and right turns, we impose the following assumptions:

Assumption 3.1: Each vehicle i has proximity sensors and can observe and/or estimate local information that can be shared with other vehicles.

Assumption 3.2: The decision of each vehicle i on whether a turn is to be made at the MZ is known upon its entry in the CZ.

Let d_i denote the decision of vehicle i on whether a turn is to be made at the MZ, where $d_i = 0$ indicates left turn, $d_i = 1$ indicates going straight and $d_i = 2$ indicates right turn. Left and right turns need special attention in the context of safety while ensuring passenger comfort. We impose the following three *Maximal Allowable Speed* limits inside the MZ: (1) v_L^a for CAVs planning to make left turns, (2) v_R^a for CAVs making right turns, and (3) v^a for CAVs going straight.

The objective for each CAV is to make a safe turn while minimizing a measure of passenger discomfort. Discomfort due to a turn originates in the tangential and the centripetal forces. As the latter is perpendicular to the former, it cannot change the speed of the vehicle and it is the magnitude of the tangential force which must be maintained fixed so that passengers are minimally affected by the turn. Thus, we use *jerk*, i.e., the rate of change of acceleration which results in speed vibrations, as a measure of passenger discomfort.

B. Vehicle Communication Structure

When a CAV enters a CZ, $z = 1, 2$, it is assigned a set $\mathcal{Q}_i^z(t)$ from the coordinator, where $\mathcal{Q}_i^z(t) = \{\mathcal{E}_i^z(t), \mathcal{S}_i^z(t), \mathcal{L}_i^z(t), \mathcal{O}_i^z(t)\}$, defined next, indicates the positional relationship between CAV i and all other CAVs j, $0 < j < i$. With respect to CAV i, CAV j, $0 < j < i$ belongs to one and only one of these subsets defined as follows:

(i) $\mathcal{E}_i^z(t)$ contains all vehicles that can cause rear-end collision at the *end* of the MZ with i, e.g., $\mathcal{E}_3^z(t)$ contains CAV #2 (Fig. 2(a)). Note that this subset does not contain the indices corresponding to vehicles cruising on the same lane and towards the same direction.

To ensure the absence of rear-end collision, the following constraint is applied:

$$t_i^f > t_j^f, \; j \in \mathcal{E}_i^z(t). \quad (4)$$

(ii) $\mathcal{S}_i^z(t)$ contains all vehicles traveling on the same lane that can cause rear-end collision at the *beginning* of the MZ as i, e.g., $\mathcal{S}_3^z(t)$ contains CAV #2 (Fig. 2(b)). Note that this subset contains the indices corresponding to vehicles cruising on the same lane and towards the same direction.

To ensure the absence of rear-end collision at the beginning of the MZ, the following condition is applied:

$$t_i^m > t_k^m, \; k \in \mathcal{S}_i^z(t). \quad (5)$$

(iii) $\mathcal{L}_i^z(t)$ contains all vehicles traveling on different lanes and towards different lanes that can cause lateral collision inside the MZ as i, e.g., $\mathcal{L}_3^z(t)$ contains CAV #2 (Fig. 2(c)).

(iv) $\mathcal{O}^z(t)$ contains all vehicles that cannot cause lateral or rear-end collisions at the MZ as i, e.g., $\mathcal{O}_3^z(t)$ contains CAV #2 (Fig. 2(d)).
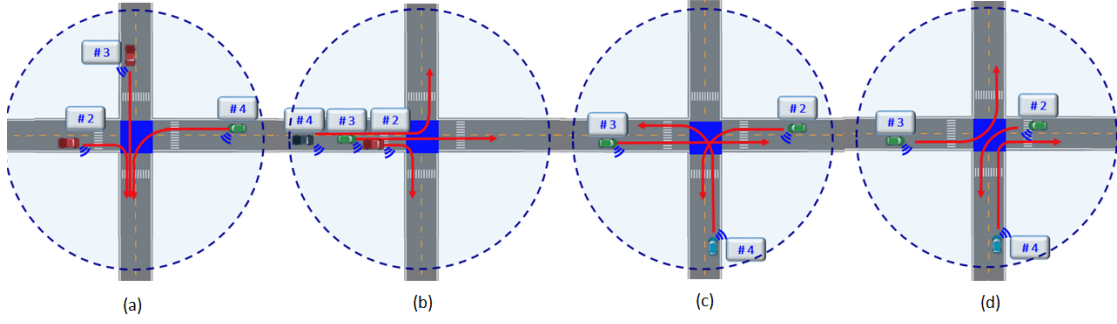
Fig. 2. Illustration of different subsets of $\mathcal{Q}^z(t)$: (a) subset $\mathcal{E}^z(t)$; (b) subset $\mathcal{S}^z(t)$; (c) subset $\mathcal{L}^z(t)$; (d) subset $\mathcal{O}^z(t)$

Note that $\mathcal{Q}_i^z(t)$ defined above is different from the definition in [12] where we had no turns. With left and right turns being considered, the collision scenarios are more complicated. As the positional relationship between CAV i and j, $j < i$, $j \neq i-1$ cannot be explicitly determined through CAV $i-1$ with turns being involved, the terminal conditions of CAV i cannot solely depend on CAV $i-1$.

Recalling that t_i^m is the assigned time for CAV i to enter the MZ, we require the following condition:

$$t_i^f \geq t_{i-1}^f, \ i > 1. \tag{6}$$

Note that in [12] the speed in the MZ was considered to be constant, hence $t_i^m > t_k^m$ could be ensured by $t_i^f > t_k^f$. However, with turns being considered, the speed and the trajectories in the MZ may be different for CAV $i-1$ and i, therefore, $t_i^f \geq t_{i-1}^f$ does not imply $t_i^m \geq t_{i-1}^m$. This becomes an issue only when $i-1 \in \mathcal{S}_i^z(t)$. In that case, we have to ensure (5) holds. Also, for $i-1 \in \mathcal{E}_i^z(t)$, (4) must be satisfied.

There is a number of ways to satisfy (6). For example, we may impose a strict First-In-First-Out (FIFO) queueing structure, where each vehicle must enter the MZ in the same order it entered the CZ. The crossing sequence may also be determined in a priority-based fashion. More generally, t_i^f (and t_i^m) may be determined for each vehicle i at time t_i^0 when the vehicle enters the CZ. If (6) is satisfied and (4) and (5) both hold, then the order in the queue is preserved. Otherwise, (6) is violated, then the order may need to be updated so that CAV i is placed in the jth, $j < i$, queue position such that (6) is satisfied, and both (4) and (5) still hold. The policy through which the order ("schedule") is specified may be the result of a higher level optimization problem. In what follows, we will adopt a specific scheme for determining t_i^m and t_i^f (upon arrival of CAV i) based on our problem formulation, without affecting the terminal conditions of $1, \cdots, i-1$, but we emphasize that our analysis is not restricted by the policy designating the order of the vehicles within the queue.

For each CAV i, we define its *information set* $Y_i(t)$, $t \in [t_i^0, t_i^f]$, as

$$Y_i(t) \triangleq \left\{ p_i(t), v_i(t), \mathcal{Q}_i^z(t), z=1,2, s_i(t), t_i^m, t_i^f, d_i \right\} \tag{7}$$

where $p_i(t), v_i(t)$ are the traveling distance and speed of CAV i inside the CZ it belongs to, and $\mathcal{Q}_i^z(t) = \{\mathcal{E}_i^z(t),$ $\mathcal{S}_i^z(t)$, $\mathcal{L}_i^z(t)$, $\mathcal{O}_i^z(t)\}$, $z = 1, 2$. indicates the positional relationship with respect to CAV j, $j < i$. The fourth element in $Y_i(t)$ is $s_i(t) = p_k(t) - p_i(t)$, the distance between CAV i and CAV k which is immediately ahead of i in the same lane (the index k is made available to i by the coordinator under Assumption 3.1). t_i^m and t_i^f are the times targeted for CAV i to enter and exit the MZ respectively, whose evaluation is discussed next. The last element d_i, indicates whether i is making a left or right turn, or going straight at the MZ, which becomes known once the vehicle enters the CZ (Assumption 3.2). Note that once CAV i enters the CZ, then all information in $Y_i(t)$ becomes available to i.

For safety, comfort and fuel efficiency, it is appropriate for vehicles to make turns at an intersection at low speeds. The speed for which an intersection curve is designed depends on speed limit, the type of intersection, and the traffic volume [16]. Generally, the "desirable time" Δ_i that a vehicle needs to make a turn at an intersection [16] is

$$\Delta_i = \begin{cases} \dfrac{R_i}{\sqrt{15R_i(0.01E+F)}}, & \text{if } d_i = 0, 2, \\ \dfrac{S}{v^a}, & \text{if } d_i = 1, \end{cases} \tag{8}$$

where R_i is the centerline turning radius, E is the super-elevation, which is zero in urban conditions, and F is the side friction factor. Therefore, the time t_i^m that the vehicle i enters the MZ is directly related to the time t_i^f that the vehicle exits the MZ through Δ_i:

$$t_i^f = t_i^m + \Delta_i. \tag{9}$$

Note that Δ_i is different for left and right turns since the corresponding turning radii R_i are different.

C. Terminal Conditions

We now turn our attention to the terminal conditions, i.e., CAV i's time, speed, and position to enter/exit the MZ.

(i) Let $e = \max_j \{j \in \mathcal{E}_i^z(t)\}$. In this case, CAV e is immediately ahead of CAV i in the FIFO queue that may cause rear-end collision at the end of the MZ. To avoid such rear-end collision, e and i should maintain a minimal safety distance δ, by the time vehicle i exits the MZ. For simplicity, we make the following assumption:

Assumption 3.3: For each vehicle i, the speed remains constant after the MZ exit for at least a length δ.

Note that this assumption is simply made for math calculation purpose.

Given the assumption above, we set $t_i^f = t_e^f + \frac{\delta}{v_e^f}$, where t_e^f and t_i^f is the time that vehicle e and i exits the MZ, and v_e^f is the speed of the vehicle e at the exit of the MZ. The terminal speeds at t_i^m and t_i^f are set as follows:

$$v_i^f = v_i^m = \begin{cases} v_L^a, & \text{if } d_i = 0, \\ v^a, & \text{if } d_i = 1, \\ v_R^a, & \text{if } d_i = 2, \end{cases} \quad (10)$$

where v_i^m is the speed of the vehicle i at the entry of the MZ. Note that v_e^f can also be determined through (10). In our earlier work [12], we considered a constant speed for the CAVs inside the MZ. To consider left and right turns, the speed can no longer be constant. Therefore, we formulate a new optimization problem to address a measure of passenger discomfort within the MZ. For this problem, (10) are the initial and terminal speeds for each CAV, and t_i^m is the initial time which can be evaluated according to (9) given t_i^f. Note that v_i^m in (10) and t_i^m are the terminal conditions for the decentralized optimal control problem in the CZ.

(ii) Let $s = \max_j \{j \in \mathcal{S}_i^z(t)\}$. In this case, CAV s is immediately ahead of CAV i in the FIFO queue that may cause rear-end collision at the beginning of the MZ. To guarantee the rear-end collision constraint does not become active we set $t_i^m = t_s^m + \Delta_s^\delta$, where

$$\Delta_s^\delta = \begin{cases} \frac{\delta}{\sqrt{15R_i(0.01E+F)}}, & \text{if } d_s = 0, 2, \\ \frac{\delta}{v^a}, & \text{if } d_s = 1 \end{cases} \quad (11)$$

is the time vehicle s needs to travel a distance δ inside the MZ. The time t_i^f that the vehicle i will be exiting the MZ can be evaluated from (9) while v_i^f is defined in (10).

Let S_L and S_R denote the length of the left and right turn trajectories, respectively. If vehicle s makes a left turn and vehicle i makes a right turn, $S_L > S_R$, implies that $t_i^f < t_s^f$, thus (6) does not hold. In that case, we set $t_i^f = t_s^f$, and t_i^m is evaluated according to (9). The re-evaluation of t_i^m can only make t_i^m larger, thus, (5) still holds. Hence, we have

$$t_i^f = \max\{t_s^m + \Delta_s^\delta + \Delta_i, t_s^f\} \quad (12)$$

(iii) Let $l = \max_j \{j \in \mathcal{L}_i^z(t)\}$. In this case, CAV l is immediately ahead of CAV i in the FIFO queue that may cause lateral collision with i inside the MZ. We constrain the MZ to contain only l or i so as to avoid lateral collision, that is, $t_i^m = t_l^f$. The time t_i^f is evaluated through (9) and v_i^f is defined in (10).

(iv) Let $o = \max_j \{j \in \mathcal{O}_i^z(t)\}$. In this case, CAV o is immediately ahead of CAV i in the FIFO queue that will not generate collision with i in the MZ, so we set $t_i^f = t_o^f$. The time t_i^m is evaluated through (9) and v_i^f is defined in (10).

In order to ensure the absence of any collision type, we set t_i^f as follows:

$$t_i^f = \max\{t_e^f + \delta/v_e^f, t_s^m + \Delta_s^\delta + \Delta_i, t_s^f, t_l^f + \Delta_i, t_o^f\} \quad (13)$$

Recall that in [12], t_i^f and t_i^m can be recursively determined through CAV $i-1$ and k. However, with left and right turns being considered, the positional relationship between i and j, $j < i, j \neq i-1$ becomes more complicated. CAV i now depends on four CAVs e, s, l, and o. However, the essence of the recursive structure stays the same. It follows from (9) and (13) that t_i^f and t_i^m can always be recursively determined from CAVs e, s, l, and o, which preserves simplicity in the solution and enables decentralization.

Although (9) through (13) provide a simple recursive structure for determining t_i^m, the presence of the control and state constraints (2) may prevent these values from being admissible. This may happen by (2) becoming active at some internal point during an optimal trajectory (see [15] for details). In addition, however, there is a global lower bound to t_i^m, which depends on t_i^0 and on whether CAV i can reach v_{max} prior to t_{i-1}^m or not: (i) If CAV i enters the CZ at t_i^0, accelerates with $u_{i,max}$ until it reaches v_{max} and then cruises at this speed until it leaves the MZ at time t_i^1, it was shown in [12] that $t_i^1 = t_i^0 + \frac{L}{v_{max}} + \frac{(v_{max}-v_i^0)^2}{2u_{i,max}v_{max}}$. (ii) If CAV i accelerates with $u_{i,max}$ but reaches the MZ at t_i^m with speed $v_i^m < v_{max}$, it was shown in [12] that $t_i^2 = t_i^0 + \frac{v_i(t_i^m)-v_i^0}{u_{i,max}}$, where $v_i(t_i^m) = \sqrt{2Lu_{i,max} + (v_i^0)^2}$. Thus, $t_i^c = t_i^1 \mathbb{1}_{v_i^m=v_{max}} + t_i^2(1 - \mathbb{1}_{v_i^m=v_{max}})$ is a lower bound of t_i^m regardless of the solution of the problem.

D. Decentralized Control Problem Formulation and Analytical Solution

Recall that at time t, the values of t_{i-1}^f, t_{i-1}^m, v_{i-1}^f, $\mathcal{Q}_i^z(t)$, $z = 1, 2$, d_{i-1} are available to CAV i through its information set in (7). This is necessary for i to compute t_i^f and t_i^m appropriately and satisfy (6) and (5).

The decentralized optimal control problem for each CAV is formulated so as to minimize the L^2-norm of its control input (acceleration/deceleration). There is a monotonic relationship between fuel consumption for each CAV i, and its control input u_i [17]. Therefore, we formulate the following problem for each i:

$$\min_{u_i \in U_i} \frac{1}{2} \int_{t_i^0}^{t_i^m} K_i \cdot u_i^2 \, dt$$

$$\text{subject to} : (1), (2), t_i^m, \; p_i(t_i^0) = 0, \, p_i(t_i^m) = L, \quad (14)$$
$$z = 1, 2, \text{and given } t_i^0, v_i(t_i^0),$$

where K_i is a factor to capture CAV diversity (for simplicity we set $K_i = 1$ for the rest of this paper). Note that this formulation does not include the safety constraint (3).

An analytical solution of problem (14) may be obtained through a Hamiltonian analysis. The presence of constraints (2) and (3) complicates this analysis. The complete solution including the constraints in (2) is given in [15]. Assuming that the constraints are not active upon entering the CZ and that they remain inactive throughout $[t_i^0, t_i^m]$, a complete solution was derived in [17] and [18] for highway on-ramps, and in [12] for two adjacent intersections. The solution to (14) differs from what we have derived in [12] in the values of the coefficients instead of the structure, as the terminal

conditions determined in Sec. III-C consider left and right turns. The optimal control input (acceleration/deceleration) over $t \in [t_i^0, t_i^m]$ is given by

$$u_i^*(t) = a_i t + b_i \qquad (15)$$

where a_i and b_i are constants. Using (15) in the CAV dynamics (1) we also obtain the optimal speed and position:

$$v_i^*(t) = \frac{1}{2} a_i t^2 + b_i t + c_i \qquad (16)$$

$$p_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \qquad (17)$$

where c_i and d_i are constants of integration. The constants a_i, b_i, c_i, d_i can be computed by using the given initial and final conditions. The analytical solution (15) is only valid as long as all initial conditions satisfy (2) and (3) and none of these constraints becomes active in $[t_i^0, t_i^m]$. Otherwise, the solution needs to be modified as described in [15]. Recall that the constraint (3) is not included in (14) and it is a much more challenging matter. To address this, we derive the conditions under which the CAV's state maintains feasibility in terms of satisfying (3) over $[t_i^0, t_i^m]$ in [13].

IV. JOINT MINIMIZATION OF PASSENGER DISCOMFORT AND FUEL CONSUMPTION IN THE MZ

Passenger Discomfort It is reported in [19] that the comfort of the passengers in transportation can be quantified as a function of jerk, which is the time derivative of acceleration, i.e., $J_i(t) = \dot{u}_i(t)$. Hence, the following optimization problem is formulated with the objective of minimizing the L^2-norm of jerk for each vehicle i, where the acceleration/deceleration $u_i(t)$ is the control input:

$$\min_{u_i} \frac{1}{2} \int_{t_i^m}^{t_f^m} J_i^2 \; dt \qquad (18)$$

$$\text{subject to} : (1), \; J_i(t) = \dot{u}_i(t),$$
$$u_i(t_i^m), u_i(t_i^f), v_i^m, v_i^f, p_i(t_i^m), p_i(t_i^f), \text{given } t_i^m, t_i^f,$$

The analytical solution of problem (18) has been obtained in [20] using Hamiltonian analysis and considering the jerk as the control input. However, here we control jerk indirectly through the acceleration/deceleration, so the analytical closed-form solution is

$$u_i^*(t) = \frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + d_i, \qquad (19)$$

$$v_i^*(t) = \frac{1}{24} a_i t^4 + \frac{1}{6} b_i t^3 + \frac{1}{2} c_i t^2 + d_i t + e_i, \qquad (20)$$

$$p_i^*(t) = \frac{1}{120} a_i t^5 + \frac{1}{24} b_i t^4 + \frac{1}{6} c_i t^3 + \frac{1}{2} d_i t^2 + e_i t + f_i, \qquad (21)$$

where a_i, b_i, c_i, d_i, e_i and f_i are constants of integration, which can be computed by using the given initial and final conditions at t_i^m and t_i^f.

Tradeoff between Fuel Consumption and Passenger Discomfort To investigate this tradeoff between fuel consumption and passenger discomfort, we consider a convex
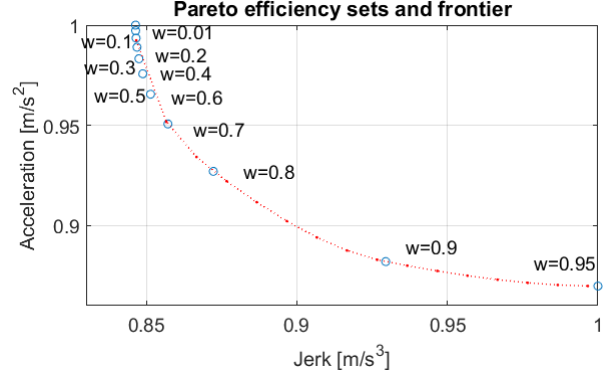

Fig. 3. Pareto efficiency sets and frontier corresponding to different combinations of fuel consumption and passenger discomfort in the MZ.

combination of acceleration/deceleration and jerk to formulate the following optimization problem:

$$\min_{u_i} \frac{1}{2} \int_{t_i^m}^{t_f^m} \left(w \cdot q_1 \cdot u_i^2 + (1-w) \cdot q_2 \cdot J_i^2 \right) dt \qquad (22)$$

$$\text{subject to} : (1), \; J_i(t) = \dot{u}_i(t), \; 0 \le w \le 1,$$
$$u_i(t_i^m), u_i(t_i^f), v_i^m, v_i^f, p_i(t_i^m) \text{ and } p_i(t_i^f), \text{given } t_i^m, t_i^f.$$

where q_1, q_2 are normalization factors which are selected so that $q_1 \cdot u_i^2 \in [0, 1]$ and $q_2 \cdot J_i^2 \in [0, 1]$. The optimal solution can be derived as

$$J_i^*(t) = \frac{a_i}{w q_1} + e_i A_1^3 e^{A_1 t} + f_i A_2^3 e^{A_2 t} \qquad (23)$$

$$u_i^*(t) = \frac{1}{w q_1}(a_i t + b_i) + e_i A_1^2 e^{A_1 t} + f_i A_2^2 e^{A_2 t} \qquad (24)$$

$$v_i^*(t) = \frac{1}{w q_1}\left(\frac{1}{2} a_i t^2 + b_i t + c_i + \frac{a_i(1-w)q_2}{w q_1}\right)$$
$$+ e_i A_1 e^{A_1 t} + f_i A_2 e^{A_2 t} \qquad (25)$$

$$p_i^*(t) = \frac{1}{w q_1}\left(\frac{1}{6} a_i t^3 + \frac{1}{2} b_i t^2 + c_i t + \frac{a_i(1-w)q_2}{w q_1} t + d_i\right)$$
$$+ e_i e^{A_1 t} + f_i e^{A_2 t} \qquad (26)$$

where $A_1 = \sqrt{\frac{w q_1}{(1-w)q_2}}$, $A_2 = -\sqrt{\frac{w q_1}{(1-w)q_2}}$. The constants a_i, b_i, c_i, d_i, e_i and f_i can be computed using the initial and final conditions at t_i^m and t_i^f. Note that since $0 \le w \le 1$, the optimal solution is only valid when $w \ne 1$ and $w \ne 0$. When $w = 0$, the problem reduces to (18). When $w = 1$, the problem becomes the same as the one formulated for the CZ in (14), which now minimizes the fuel consumption in the MZ.

To illustrate the tradeoff between passenger discomfort and fuel consumption, we examine a range of cases with different weights and produce the Pareto sets. The parameters used are listed in Sec. V except for the initial and terminal acceleration which are set to 0. By yielding all of the optimal solutions to (22) while varying the weight w, we can derive the Pareto sets and the Pareto frontier corresponding to different combinations of fuel consumption and passenger discomfort as shown in Fig. 3. Note that as $w \to 0$, the solution of (22) becomes that of (18).

V. SIMULATION EXAMPLES

The proposed decentralized optimal control framework incorporating turns is illustrated through simulation in MAT-LAB. For each direction, only one lane is considered. The parameters used are: $L = 400$ m, $S = 30$ m, $S_L = \frac{3}{8}\pi S$, $S_R = \frac{1}{8}\pi S$, $\delta = 10$ m, $v_L^a = 8$ m/s, $v_R^a = 6$ m/s, $v^a = 10$ m/s and $\Delta_i = 5, 3, 3$s for left turn, going straight and right turn respectively. CAVs arrive rate is assumed to be given by a Poisson process with rate $\lambda = 1$ and the speeds are uniformly distributed over $[10, 12]$.
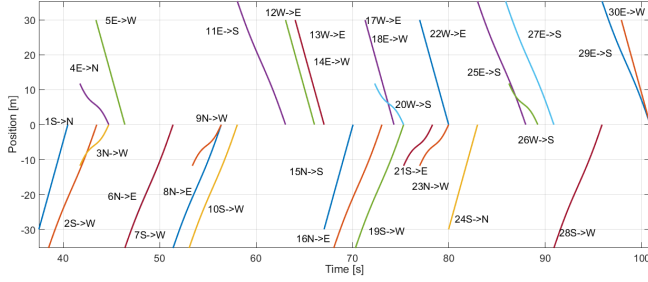

Fig. 4.   Distance to the end of MZ of the first 30 CAVs in the MZ.

We first consider the case where only the L^2-norm of jerk is optimized over the MZ. The initial and terminal conditions of time and speed are defined in Sec. III-C. Observe that $p_i(t_i^m) = L$ and $p_i(t_i^f) = L + S$ if i is going straight $(p_i(t_i^f) = L + S_L$ for left turn and $p_i(t_i^f) = L + S_R$ for right turn). The two additional conditions needed for acceleration/deceleration are set as follows: (a) the initial acceleration for (18) is set to the terminal value derived from the optimal control problem in the CZ (14), under which the acceleration/deceleration is continuous at t_i^m; (b) the terminal acceleration is set as zero. The position trajectories of the first 30 CAVs in the MZ are shown in Fig. 4. CAVs are separated into two groups: CAVs shown above zero are driving from east or west, and those below zero are driving from north or south, with labels indicating the position of the vehicles in the FIFO queue and the driving direction. These figures include different instances from each of Cases 1), 2), 3) or 4) in Sec. III-B regarding the value of t_i^f. For example, CAV #11 is assigned $t_{11}^m = t_{10}^f$, which corresponds to Case 3).

VI. CONCLUSIONS

In this paper, we have extended the decentralized optimal control framework in [12] to account for left and right turns. In addition, we formulated and solved another optimization problem to minimize a measure of passenger discomfort while the vehicle turns at the intersection, and investigated the associated tradeoff between minimizing fuel consumption and passenger discomfort. The optimal solution including turns do not require any additional computational time than what is required by the solution in [12] since the terminal conditions are determined based on another set of collision-avoidance constraints, which can still enable online implementation. Future research should investigate the implications of having information with errors and/or delays to the system behavior.

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