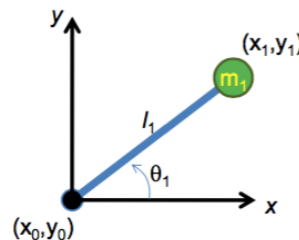




### Problem#1 (1-Link Arm Robot)

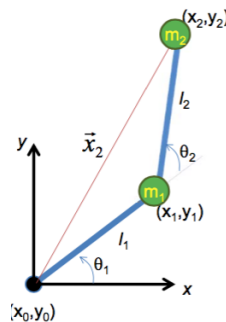
For the 1-link arm robot shown in the figure below, develop a trajectory where  $q(t = 0) = 20^\circ$ ,  $q(t = 1) = 70^\circ$ ,  $q(t = 2) = 100^\circ$ , and  $q(t = 3) = 120^\circ$  with zero velocities and accelerations at 0, 1, 2, and 3 seconds. Choose appropriate polynomials and write a MATLAB code which plots the time-position, time-velocity, and time-acceleration trajectory graphs in the joint space from  $t = 0$  to  $t = 3$  seconds in the joint space.



### Problem#2 (Two-Link Arm Robot)

For the two-link arm robot shown in the figure below with  $l_1 = l_2 = 300 \text{ mm}$  and  $m_1 = m_2 = 0.05 \text{ Kg}$ ,

- Develop a trajectory where the robot starts at  $t = 0$  with the initial position of  $\vec{x}_2 = (300, 450) \text{ mm}$  and zero velocity and ends at  $t = 5 \text{ sec}$  with the final position of  $\vec{x}_2 = (-300, 450) \text{ mm}$  and zero velocity. Choose appropriate polynomials and write a MATLAB code which plots the time-position, time-velocity, and time-acceleration trajectory graphs for both joints from  $t = 0$  to  $t = 5$  seconds. Finally, plot the resultant end-effector path/trajectory i.e. track the position of the end-effector (task space) within the motion.
- Write out the full computed torque controller to execute this motion. The detailed expanded matrix form is desired. Draw the corresponding control block diagram.



**Problem#3 (Mobile Wheeled Robot)**

In the following figures, a Neobotix mobile wheeled robot is shown. The robot's mobile base has two independently driven wheels with a wheelbase of  $\mathbf{a}$ , and with angular velocities  $\omega_R$  and  $\omega_L$  for the right and left wheels, respectively. The direction of the angular velocities is such that if  $\omega_R$  and  $\omega_L$  are both the same with positive values, the robot will then drive in the forward  $x_R$  direction. Treat the castor wheel as a single omniwheel centered with distance  $\mathbf{c}$  to the axis passing through the driven wheels' centers as shown in the figure. All wheels are of radius  $\mathbf{r}$ .

- Constraint Equations: Determine the constraint equations with respect to the robot-fixed frame,  $\mathbf{F}_R$ , for each of the three wheels on the mobile base (6 equations total).
- Constraint Matrix: *Generate* the constraint matrices  $\mathbf{J}$  &  $\mathbf{C}$ . Then *compile* the constraints into a single matrix constraint equation. Reduce the equation to *eliminate* any redundant constraints.
- Mobile Kinematics: Determine the velocity kinematics of the mobile robot base (defined by the robot-fixed frame,  $\mathbf{F}_R$ , on the robot between the wheels) with respect to the world coordinate frame  $\mathbf{F}_W$  in terms of wheel angular velocities as defined in the figure. Be sure to define any assumptions or unlabeled information.

For the following questions, as shown in the last figure in the last page, assume that a 3-DOF arm manipulator (arm robot with three joints) is attached on top of the base of the mobile robot right above the omniwheel. Assume that, from your previous knowledge of D-H parameters, you have already calculated the 4x4 homogenous transformation matrix from the mobile robot fixed frame,  $\mathbf{F}_R$ , to the arm tip frame,  $\mathbf{F}_T$ , of the elbow manipulator, that is, you already have the transformation of the tip frame  $\mathbf{F}_T$  with respect to the mobile robot fixed frame  $\mathbf{F}_R$  i.e.  $\mathbf{H} = \mathbf{T}_T^R$ . Also, assume that you have already calculated the full 6x3 velocity Jacobian from  $\mathbf{T}_T^R$ .

- Combined Position Kinematics: For the complete combined system, solve for the position forward kinematics that symbolically represent the tip frame  $\mathbf{F}_T$  with respect to the world coordinate frame  $\mathbf{F}_W$ . Assume that  $x_r$  and  $y_r$  are the x and y coordinates of the robot, respectively. Define any assumptions or unlabeled information.
- Combined Velocity Kinematics: Solve for the full velocity kinematics that represent the 6-DOF velocity (linear and angular velocities) of the robot tip frame  $\mathbf{F}_T$  with respect to the world coordinate  $\mathbf{F}_W$  as a function of all of the drive wheel and joint angular velocities (5 joint variables).
- Combined Jacobian: Write out the corresponding Jacobian and matrix- vector equation that gives the linear and angular velocities of the tip as a function of the two wheel angular velocities and the three joint angular velocities.
- Force Propagation: Symbolically, represent the required torque at each wheel to keep the position of the mobile base stable as it is pushing against the wall with a force/moment vector equal to  $\mathbf{F} = [f_x \ f_y \ f_z \ n_x \ n_y \ n_z]^T$  (assuming no slip).

HW#4, 11/15/17

Robot Dynamics

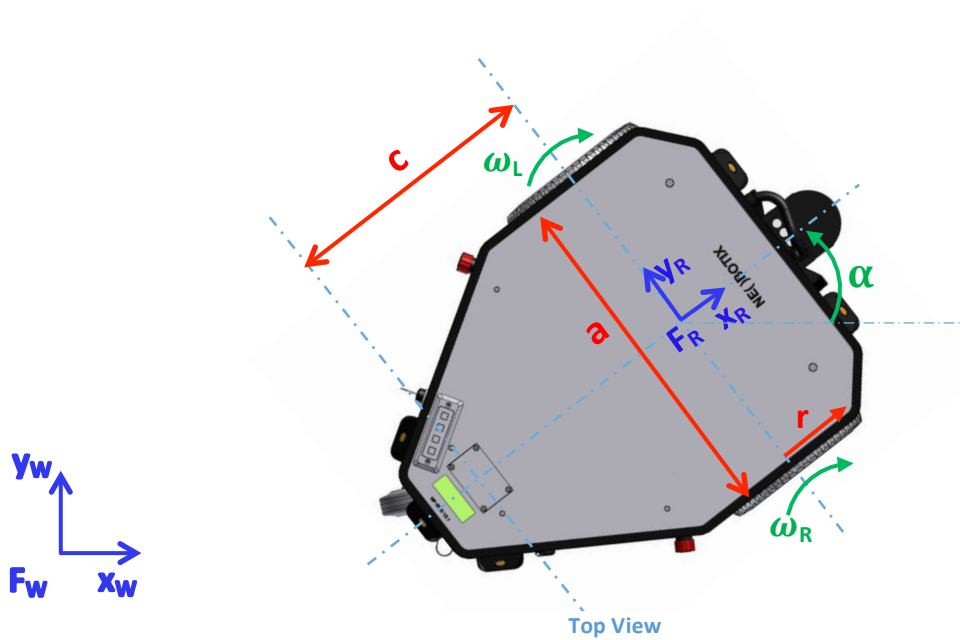
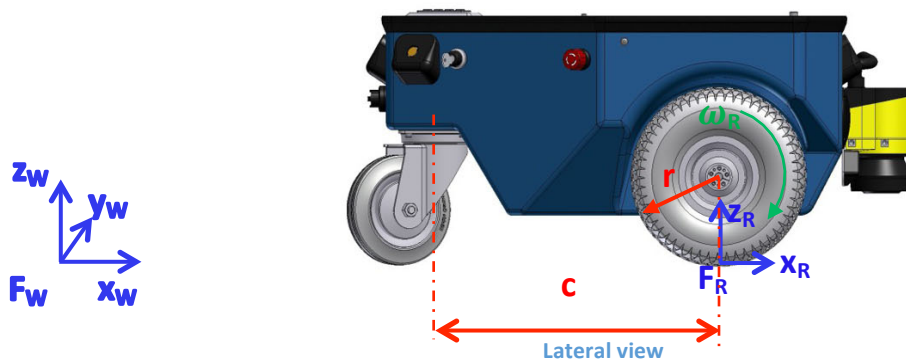
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# WPI



Neobotix Mobile Wheeled Robot MM-500



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HW#4, 11/15/17

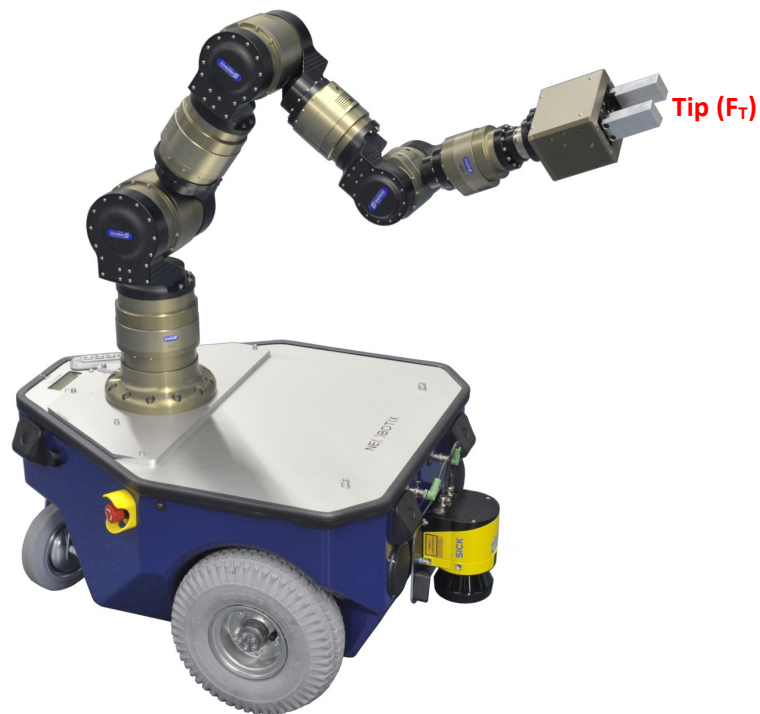
Robot Dynamics

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# WPI



Mobile Robot with Attached Arm

Good Luck!