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RBE 502 Robot Control

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HOMEWORK 01

$= [K_1 \;\; K_2]$

(A) $\dot{x} = Ax + Bu$ $\qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\dot{x}_1 = x_2$

$\dot{x}_2 = u$

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$\therefore$ rank of $[B \;\; AB]$

$= \text{rank} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 2 = n \Rightarrow$ full rank

$\therefore$ The system is reachable.

H.W:- 2.

(b) $\dot{x} = Ax + Bu$, $u = -Kx$

$\therefore \dot{x} = Ax + B(-Kx)$

$= Ax - BKx = (A-BK)x$

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $K = [k_1 \; k_2]$

$M = A - BK = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [k_1 \; k_2]$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$

$\therefore (M - \lambda I) = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$(M - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ -k_1 & -\lambda - k_2 \end{bmatrix}$

$\det |M - \lambda I| = 0$

$\therefore \begin{vmatrix} -\lambda & 1 \\ -k_1 & -k_2 - \lambda \end{vmatrix} = 0$

$\therefore \lambda(\lambda + k) + k_1 = 0$

$\Rightarrow \lambda^2 + \lambda k_2 + k_1 = 0$

$\Rightarrow$ Given $s^2 + 1.4s + 1 = 0$

$\therefore k_2 = 1.4$ & $k_1 = 1$ //

$\therefore$ Given $s^2 + 1.4s + 1 = 0$

$\therefore k_2 = 1.4$ & $k_1 = 1$

(c) $x_f = (1, 1)$, $x_0 = (-5, 0)$

$\dot{x}_d = Ax_d + Bu_d$.

$\therefore x_{1d}(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$\therefore x_{2d}(t) = a_1 + 2a_2 t + 3a_3 t^2$.

$\therefore x_{1d}(0) = -5$ $\quad x_{2d}(0) = 0$

$\therefore x_{1d}(5) = 1$ $\quad x_{2d}(5) = 1$

$\therefore$ for $t = 0$

$\therefore x_{1d}(0) = a_0 = -5$

$\therefore x_{2d}(0) \Rightarrow a_1 \cancel{+2a_2} = 0$

$\Rightarrow$ for $t = 5$

$\therefore x_{1d}(5) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$

$\therefore 1 = -5 + (0)(5) + 25a_2 + 125a_3$

$\therefore 25a_2 + 125a_3 = 6$ — (1)

$\therefore x_{2d}(5) = a_1 + 2a_2 t + 3a_3 t^2$

$\therefore 1 = 0 + 10a_2 + 75a_3$

$\therefore 10a_2 + 75a_3 = 1$ — (2)

∴ from eq^n (1) & (2) we get.

$$a_3 = \frac{-7}{125} = -0.0560$$

$$a_2 = \frac{26}{50} = 0.5200$$

Ans
$a_0 = -5$
$a_1 = 0$
$a_2 = 0.5200$
$a_3 = -0.0560$

$$\therefore \dot{x}_2 = u.$$

$$\therefore u_d(t) = \dot{x}_{2d}(t)$$

$$\ddot{u}_d(t) = \cancel{a_1} + 2a_2 t + 2\times 3a_3 t^2$$

$$\cancel{= 0 + 0.5200\times 2t + 3\times -0.0560 t^2}$$

$$\boxed{\cancel{u_d(t) = 1.0400t - 0.1680t^2}}$$

$$u_d(t) = 1.0400 - 0.3360t$$

(d) $e(t) = x(t) - x_d(t)$

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t)$$

$$= Ax + Bu - Ax_d - Bu_d$$

$$\dot{e}(t) = A(x - x_d) + B(u - u_d)$$

$e = x - x_d$ & $v = u - u_d = Ke$

$$\therefore \dot{e}(t) = Ae + Bv = Ae + BKe$$

$$\underline{\dot{e}(t) = (A + BK)e}$$

(e) $u(t) = Ke(t) + u^d(t)$

$\therefore K = [K_1 \quad K_2]$

$e(t) = x - x_d$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad x_d = \begin{bmatrix} x_{1d} \\ x_{2d} \end{bmatrix}$

$$x_d = \begin{bmatrix} -5 + 0.52t^2 - 0.056t^3 \\ 1.04t - 0.1680^2 \end{bmatrix}$$

$u^d(t) = 1.04 - 0.3360t$

$$\therefore u(t) = [K_1 \quad K_2]\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} -5 + 0.52t^2 - 0.056t^3 \\ 1.04t - 0.1680^2 \end{bmatrix}\right)$$

$$+ 1.04 - 0.3360t.$$

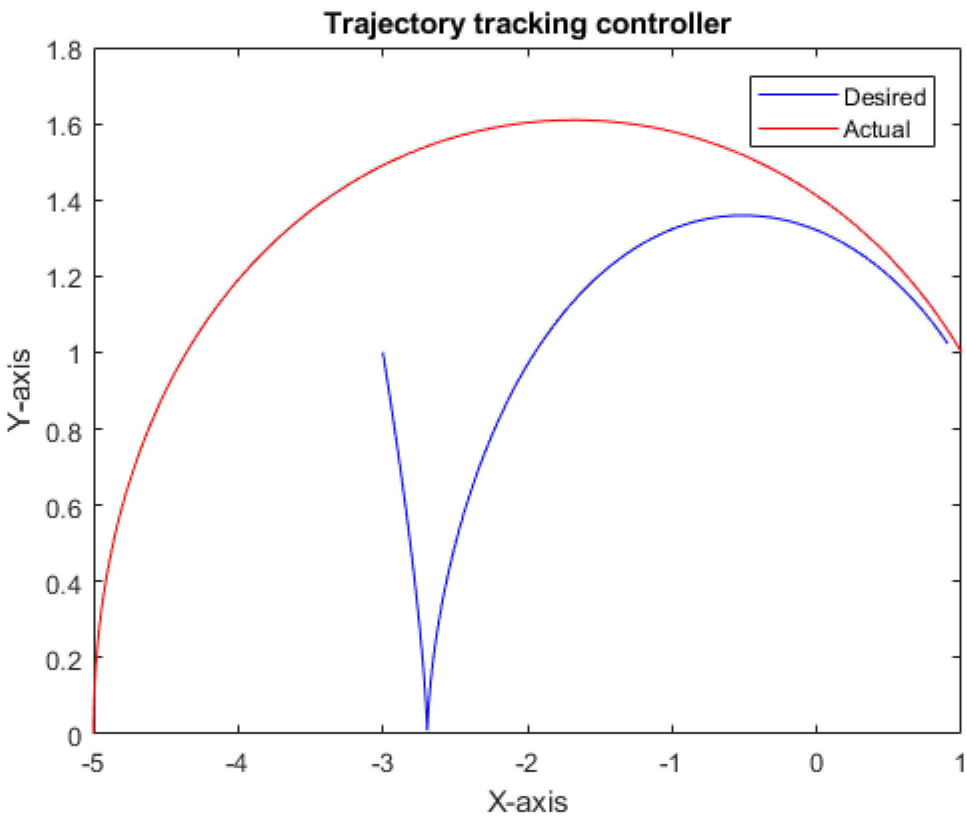
(f)

```
clc

clear all
close all
A = [ 0 1 ; 0 0 ];
B = [ 0 ; 1 ];
K = [1 1.4];
temp_x=[];
temp_x_d=[];
x_0=[-3;1];
dt = 0.01;
for t=0:0.01:5
x1_d = - (0.0560*t^3) + (0.52*t^2) - 5;
x2_d = (1.04*t) - (0.1680*t^2);
x_d  = [x1_d; x2_d];
u_d = 1.04 - (0.3360*t);
u_t=-K*(x_0-x_d)+u_d;
x_dot=A*x_0+B*u_t;
temp_x=[temp_x,x_0];
temp_x_d=[temp_x_d,x_d];
x_0=x_dot*dt+x_0;
```

```
end
figure

plot(temp_x(1,:),temp_x(2,:),'b-');
hold on
plot(temp_x_d(1,:), temp_x_d(2,:), 'r-');
legend('Desired','Actual')
title('Trajectory tracking controller');
xlabel('X-axis')
ylabel('Y-axis')
hold off
```



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