ECE36800 Data structures

Insertion sort and Shell sort
Chapter 6 (pp. 253-300)

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Overview

- Sorting background
- Insertion sort
- Shell sort

- Reference pages: pp. 253-300

Sorting background

- Why sorting at all?
  - Necessary if need to process items in sorted order (e.g., priority queue)
  - If set of items is sorted, one can
    - Find items faster
    - Find particular items in constant time (e.g., min, max, median)
    - Find identical items faster
- Trade off:
  - Faster searching after slow sorting
  - Slow searching without sorting

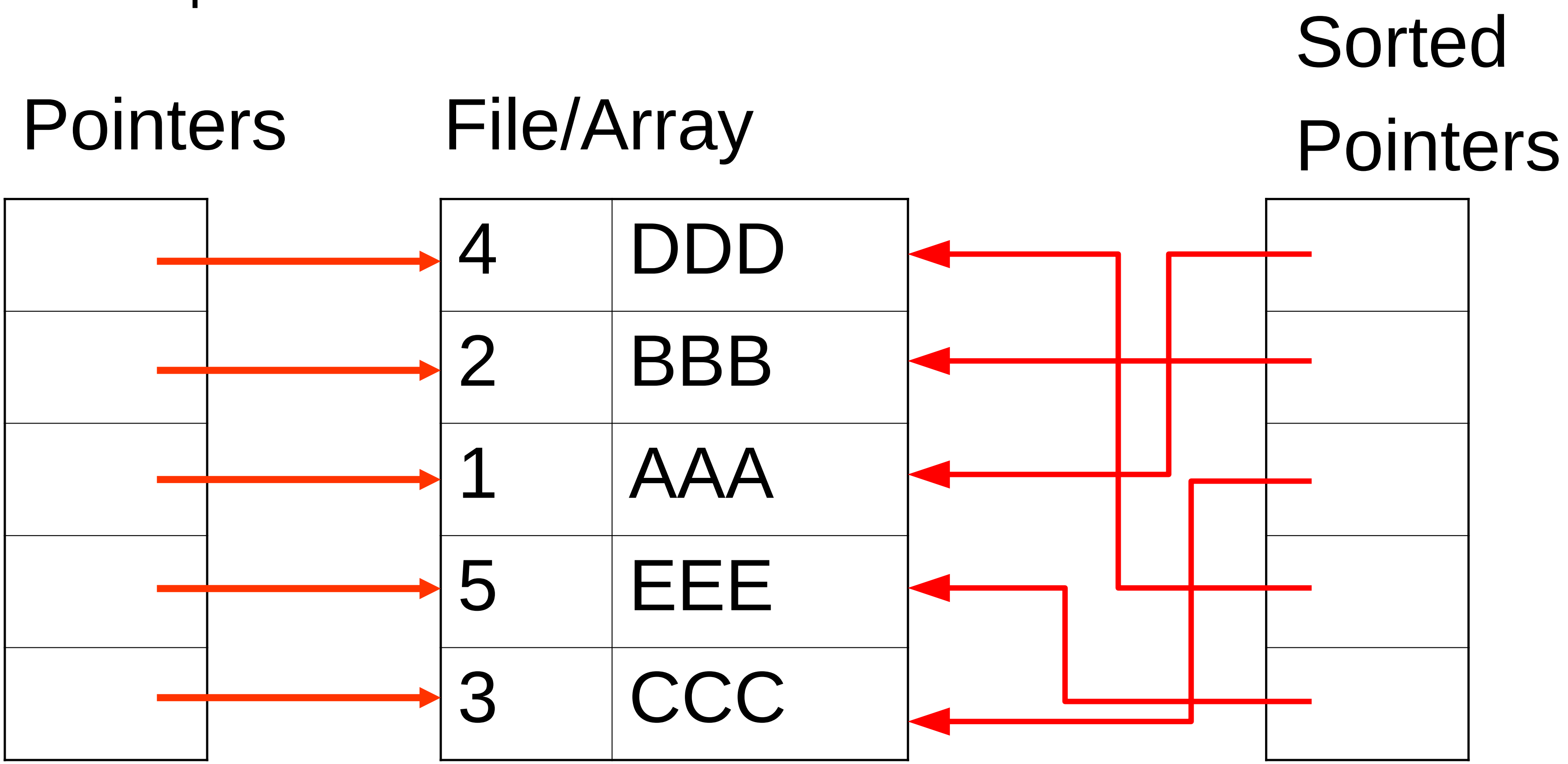
Sorting records

k[i]	r[i]
4	DDD
2	BBB
1	AAA
5	EEE
3	CCC

k[i]	r[i]
1	AAA
2	BBB
3	CCC
4	DDD
5	EEE

Sorting pointers

- Instead of records, pointers (or indices) to them are sorted (based on keys of records)
  - More efficient than copying records, but at higher memory requirement



Simple insertion sort

- Insert an item into an already sorted array
- Compare item with items in the sorted array from largest to smallest
  - If reverse order, swap items
- Continue until all items are sorted

Insertion sort: algorithm

Sort *n* integers r[0] to r[*n*-1] in ascending order

```
for j ← 1 to n-1
    for i ← j downto 1
        if r[i-1] > r[i]
            r[i-1] ↔ r[i]
        else
            break
```

Insertion sort: example

								j	i	swap
3	1	6	3	2	5	4	5	1	1	yes
1	3	6	3	2	5	4	5	2	2	no
1	3	6	3	2	5	4	5	3	3	yes
1	3	3	6	2	5	4	5	3	2	no
1	3	3	6	2	5	4	5	4	4	yes

Insertion sort: example

								j	i	swap
1	3	3	2	6	5	4	5	4	3	yes
1	3	2	3	6	5	4	5	4	2	yes
1	2	3	3	6	5	4	5	4	1	no
1	2	3	3	6	5	4	5	5	5	yes
1	2	3	3	5	6	4	5	5	4	no

Insertion sort: example

								j	i	swap
1	2	3	3	5	6	4	5	6	6	yes
1	2	3	3	5	4	6	5	6	5	yes
1	2	3	3	4	5	6	5	6	4	no
1	2	3	3	4	5	6	5	7	7	yes
1	2	3	3	4	5	5	6	7	6	no

→ exit

Insertion sort comments

- Only neighbors are swapped and identical items are never exchanged
  - Thus, insertion sort is stable, i.e.,the relative positions of identical items do not change
- The algorithm builds up a sorted subarray at the start and successively inserts items into it
- Move up array to find proper position to insert

Insertion sort: move vs. swap

Move larger items down the array to make room, insert new record into correct position

```
for j ← 1 to n-1
    temp_r ← r[j]
    for i ← j downto 1
        if r[i-1] > temp_r
            r[i] ← r[i-1]
        else
            break
    r[i] ← temp_r
```

Making room

Insertion sort: while-loop vs. for-loop

Use `while` instead of `for` in inner loop and combine the checking of sortedness

```
for j ← 1 to n-1
    temp_r ← r[j]
    i ← j
    while i > 0 and r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

i > 0 often tested but seldom false

Insertion sort: using sentinel

```
for j ← n-1 downto 1
    if r[j] < r[j-1]
        r[j] ↔ r[j-1]
for j ← 2 to n-1
    temp_r ← r[j]
    i ← j
    while r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

Find min item

r[0] ≤ temp_r guaranteed

Insertion sort: efficiency

- Memory:
  - In-place sort, $O(1)$
- Time: examine critical operations
  - Swaps or moves
  - Comparisons
  - O(n) for finding the sentinel

Insertion sort: time – best case

Best case: array already sorted

```
for j ← 1 to n-1
    temp_r ← r[j]
    i ← j
    while i > 0 and r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

Insertion sort: time – best case

Best case: array already sorted

- For the items 1 to *n*-1
  - O(1) copy operation or assignment operation
  - O(1) comparison
- Result: $O(n)$ (both copy and compare)

Insertion sort: time – worst case

Worst case: array sorted in descending order

```
for j ← 1 to n-1
    temp_r ←  r[j]
    i ←  j
    while i > 0 and r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

Insertion sort: time – worst case

Worst case: array sorted in descending order

- For the i-th item (i from 1 to n-1), dominated by i comparisons and i moves or copy operations

$(n-1) + (n-2) + \ldots + 1 = (n-1)*((n-1) + 1) / 2$

$= n^2/2 - n/2$ comparisons or copy operations

- Result: $O(n^2)$ (both copy and compare)

Insertion sort: time – average case

Average case: array in random order

```
for j ← 1 to n-1
    temp_r ← r[j]
    i ← j
    while i > 0 and r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

Insertion sort: time – average case

Average case: array in random order

- For the i-th record (i from 1 to n-1)
  - On average i / 2 positions from the correct one
  - On average $(1+i)/2$ comparisons and copy operations

$$((n) + (n-1) + \ldots + 3 + 2) / 2 = (n+1)*(n)/4 - 1/2$$

$$= n^2/4 - n/4 - 1/2 \text{ comparisons/copies}$$

- Result: $O(n^2)$ (both copy and compare)

Insertion sort: summary

- Memory complexity: $O(1)$, in-place
- Time complexity:
  - Perfectly sorted: $O(n)$
  - Average input: very poor $O(n^2)$
  - On average $n^2/4$ comparisons and moves
- Advantages:
  - Easy to implement
  - One of the best sorts for sorted input and/or small input sizes

Shortcomings of insertion sort

- Perform comparisons between adjacent neighbors
  - $O(n^2)$ comparisons (worst-case and average-case)
- Perform swaps/moves between adjacent items
  - Take several swaps/moves to correct one inversion (pair of integers out of order)
  - $O(n^2)$ swaps/moves

Shell sort: improving insertion sorts

- Allow comparisons and swaps between non-adjacent neighbors
- Perform insertion sort on *k* subarrays of original array
  - Subarray 0: r[0], r[k], r[2k], r[3k], …
  - Subarray 1: r[1], r[1+k], r[1+2k], r[1+3k], …
  - …
  - Subarray *k*-1: r[k-1], r[2k-1], r[3k-1], r[4k-1], …
- The adjacent item in a subarray is *k* positions away in the original away, allowing items to move far
- Iterate the process, use a smaller *k* in each pass
- In the last pass, use *k* = 1

Example

- Given array
  - 3 7 9 0 5 1 6 8 4 2 0 6 1 5 7 3 4 9 8 2
- Arrange into a 2-D array with k = 7 rows (filling in one column at a time)
  - Subarray 0: 3 8 7 $\rightarrow$ 3 7 8
  - Subarray 1: 7 4 3 $\rightarrow$ 3 4 7
  - Subarray 2: 9 2 4 $\rightarrow$ 2 4 9
  - Subarray 3: 0 0 9 $\rightarrow$ 0 0 9
  - Subarray 4: 5 6 8 $\rightarrow$ 5 6 8
  - Subarray 5: 1 1 2 $\rightarrow$ 1 1 2
  - Subarray 6: 6 5 $\rightarrow$ 5 6
- Put it back: 3 3 2 0 5 1 5 7 4 4 0 6 1 6 8 7 9 9 8 2

Example

- Arrange [3 3 2 0 5 1 5 7 4 4 0 6 1 6 8 7 9 9 8 2] into a 2-D array with k = 3 rows (filling in one column at a time)
  - Subarray 0: 3 0 5 4 1 7 8 → 0 1 3 4 5 7 8
  - Subarray 1: 3 5 7 0 6 9 2 → 0 2 3 5 6 7 9
  - Subarray 2: 2 1 4 6 8 9 → 1 2 4 6 8 9
- Put it back: 0 0 1 1 2 2 3 3 4 4 5 6 5 6 8 7 7 9 8 9
- Apply insertion sort (k = 1) in the last pass

Modify Insertion sort to Shell sort

```
for j ← 1 to n-1
    temp_r ← r[j]
    i ← j
    while i > 0 and r[i-1] > temp_r
        r[i] ← r[i-1]
        i ← i-1
    r[i] ← temp_r
```

Shell sort: example

Pass 1, $k = 5$

3	5	6	3	4	5	1	2

j = 5 no

3	5	6	3	4	5	1	2

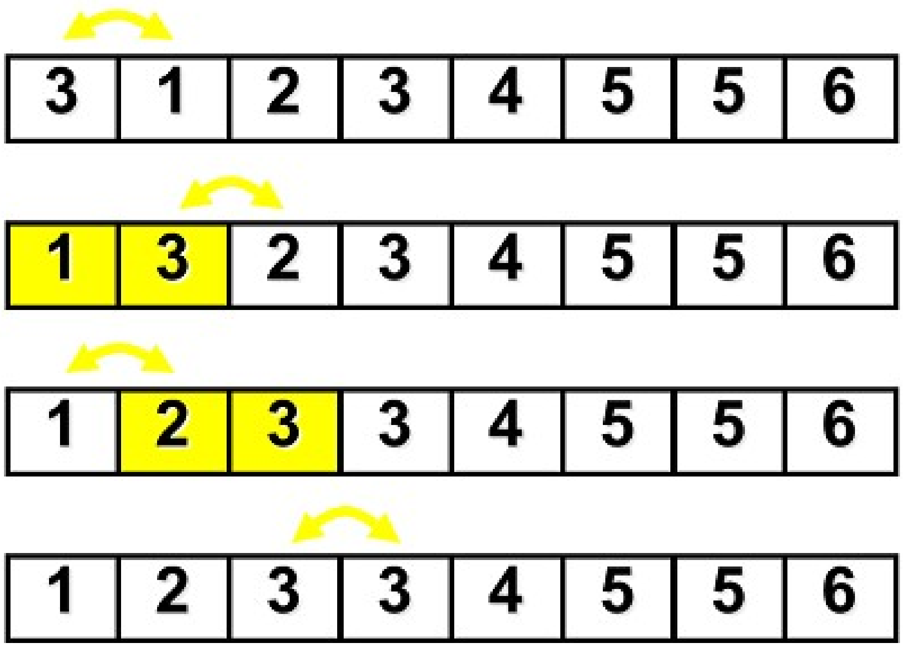
j = 6 yes

3	1	6	3	4	5	5	2

j = 7 yes

3	1	2	3	4	5	5	6

Shell sort: example



Pass 2, *k* = 1

j = 1, yes

j = 2, yes

no

j = 3, no

No more swaps in the remainder of insertion sort

Shell sort: algorithm

```
for each k (in descending order)
   for j ← k to n-1
      temp_r ← r[j]
      i ← j
      while i ≥ k and r[i-k] > temp_r
         r[i] ← r[i-k]
         i ← i-k
      r[i] ← temp_r
```

Shell sort: choosing k

- Sequence {1, 4, 13, …, $3h(k\text{-}1) + 1$, …}
- Define recursively
  - $h(1) = 1$
  - $h(i + 1) = 3 * h(i) + 1$
- Let x be the smallest integer such that $h(x) \geq n$
- Set the number of passes to be $x – 1$
- For pass j, use $k = h(x – j)$

Shell sort: complexity

- Analysis is quite sophisticated, will skip that for this course
- Sequence of integers of the form $2^p 3^q$ (< n):

1
2 3
4 6 9
8 12 18 27

Bottom to top
Right to left

  - $O(n(\log n)^2)$
- Sequence {1, 3, 7, 15, …, $2^k - 1$, …}: $O(n^{1.5})$
- Sequence {1, 4, 13, …, $3h(k\text{-}1) + 1$, …}: $O(n^{1.5})$