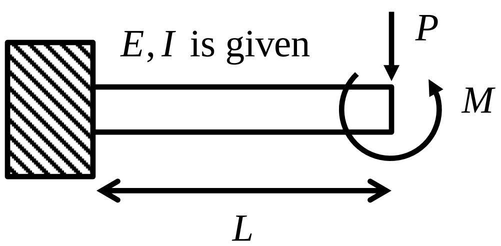
Homework

Find approximate v(x) of given beam using Galerkin method



$$q(x) = -EI\frac{d^4V(x)}{dx^4} = 0 \qquad 0 \le x \le L$$

$$V(0) = 0$$

$$\theta(0) = \left.\frac{dV(x)}{dx}\right|_{x=0} = 0$$

$$\frac{M(L)}{EI} = \left.\frac{d^2V(x)}{dx^2}\right|_{x=L} = \frac{M}{EI}$$

$$\frac{V(L)}{EI} = \left.\frac{d^3V(x)}{dx^3}\right|_{x=L} = -\frac{P}{EI}$$

Use approximate solution:

$$\tilde{v}(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 \qquad V(0) = 0 = a_0$$

$$\frac{d\tilde{v}(x)}{dx} = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 \qquad \left.\frac{dV(x)}{dx}\right|_{x=0} = 0 = a_1$$

$$\frac{d^2\tilde{v}(x)}{dx^2} = 2a_2 + 6a_3x + 12a_4x^2 \qquad \left.\frac{d^2V(x)}{dx^2}\right|_{x=L} = \frac{M}{EI} = 2a_2 + 6a_3L + 12a_4L^2 \qquad a_2 = \frac{PL}{2EI} + 6L^2a_4 + \frac{M}{2EI}$$

$$\frac{d^3\tilde{v}(x)}{dx^3} = 6a_3 + 24a_4x. \qquad \left.\frac{d^3V(x)}{dx^3}\right|_{x=L} = -\frac{P}{EI} = 6a_3 + 24a_4L. \qquad a_3 = -\frac{P}{6EI} - 4a_4L$$

$$\tilde{v}(x) = \frac{P}{2EI}\left((L + \frac{M}{P})x^2 - \frac{x^3}{3}\right) + a_4(x^4 - 4Lx^3 + 6Lx^2) = \sum_{i=0}^{N} W_iC_i$$

$$C_0 = 1 \qquad N_0 = \frac{P}{2EI}\left((L + \frac{M}{P})x^2 - \frac{x^3}{3}\right)$$

$$C_1 = a_4 \qquad N_1 = x^4 - 4Lx^3 + 6Lx^2$$

$$R(x) = 0 - \left(-EI\frac{d^4\tilde{v}(x)}{dx^4}\right) = 24EIa_4$$

$$\int_0^L R(x)N_1(x)\,dx = 0$$

$$\int_0^L (24EIC_1)(x^4 - 4Lx^3 + 6Lx^2)\,dx$$

$$\therefore C_1 = 0$$

$$\tilde{v}(x) = \frac{P}{2EI}\left[(L + \frac{M}{P})x^2 - \frac{x^3}{3}\right]$$