

# STAT 511: HW #7

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4/19/2021

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# Multiple Regression & Brand Preference Dataset

## Setting up workspace

```
library(nortest)
library(olsrr)
library(car)
library(lmtest)
library(MASS)
library(tidyverse)

setwd("C:/Users/RUMIL/Desktop/APU/STAT 511 - Millie Mao (Applied Regression Analysis)/Week 10/Week 10/")

brand_data = read.table(file = "Brand.txt", header = FALSE, sep = "")

View(brand_data)

# Adding headers
names(brand_data) <- c("Rating", "Moisture", "Sweetness")

# names(bank_data) <- c("", "")

# Defining dependent and independent vars
Rating = brand_data$Rating #Y
Moisture = brand_data$Moisture #X1
Sweetness = brand_data$Sweetness #X2
```

1a. Regress degree of brand liking on sweetness only. Write down the estimated regression model.

```
# Coding Sweetness as a dummy variable
cat_sweetness <- as.factor(Sweetness)
cat_sweetness

## [1] 2 4 2 4 2 4 2 4 2 4 2 4 2 4 2 4
## Levels: 2 4

# Regressing Rating on new Sweetness dummy variable
Sweetness_only <- lm(Rating ~ cat_sweetness, data = brand_data)

summary(Sweetness_only)

##
## Call:
## lm(formula = Rating ~ cat_sweetness, data = brand_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -16.375 -7.312 -0.125 8.688 16.625
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)    77.375      3.851  20.094 1.01e-11 ***
## cat_sweetness4    8.750      5.446   1.607    0.13
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.89 on 14 degrees of freedom
## Multiple R-squared:  0.1557, Adjusted R-squared:  0.09539
## F-statistic: 2.582 on 1 and 14 DF,  p-value: 0.1304
```

Estimated Regression model:

$$\hat{Y} = 77.375 + 8.750X$$

```
#Checking how many levels there are in sweetness as a dummy
levels(cat_sweetness)
```

```
## [1] "2" "4"
```

```
typeof(cat_sweetness)
```

```
## [1] "integer"
```

```
attributes(cat_sweetness)
```

```
## $levels
## [1] "2" "4"
##
## $class
## [1] "factor"
```

So in this case since sweetness is a category with only 2 levels (sweetness level 2 and sweetness level 4) we can think of our regression model like so:

We will use sweetness level 2 as our reference group

```
#the "2" indicates the level marked as "2" given from our level() function and now setting sweetness le
cat_sweetness_new <- relevel(cat_sweetness, ref = "2")
```

```
#Regressing
Sweetness_only2 <- lm(Rating ~ cat_sweetness_new, data = brand_data)

summary(Sweetness_only2)
```

```
##
## Call:
## lm(formula = Rating ~ cat_sweetness_new, data = brand_data)
##
## Residuals:
```

```
##      Min      1Q  Median      3Q      Max
## -16.375  -7.312  -0.125   8.688  16.625
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      77.375      3.851  20.094 1.01e-11 ***
## cat_sweetness_new4   8.750      5.446   1.607   0.13
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.89 on 14 degrees of freedom
## Multiple R-squared:  0.1557, Adjusted R-squared:  0.09539
## F-statistic: 2.582 on 1 and 14 DF,  p-value: 0.1304
```

**1b. Compute the estimated mean degree of brand liking at each level of sweetness, i.e., what is**

the estimated mean degree of brand liking at sweetness level 2? At sweetness level 4?

After reading the summary we can now say the estimated means when Sweetness level is 2 and 4 to be.

Table 1: Mean Rating at Level 2 or Level 4 Sweetness

| $\hat{Y} = B_0 + B_1X_1$    | Sweetness = Level 2           | Sweetness = Level 4           |
|-----------------------------|-------------------------------|-------------------------------|
| $\hat{Y} = 77.375 + 8.750X$ | $\hat{Y} = 77.375 + 8.750(0)$ | $\hat{Y} = 77.375 + 8.750(1)$ |
|                             | $= 77.375$                    | $= 77.375 + 8.750$            |
|                             | 77.375                        | 86.125                        |

**1c. Interpret the intercept coefficient.**

$$B_0 = 77.375$$

The estimated mean Y-value when  $X = 0$  (our reference/baseline group) is 77.375. When put in context, this represents the mean Rating when the brand's sweetness level is 2.

**1d. Interpret the slope coefficient.**

The change in mean Rating for Sweetness level 4 relative to sweetness level 2 is 86.125 ( $77.375 + 8.750(1)$ ).

**1e. Is the slope coefficient significant? State the null, alternative, decision rule and conclusion.**

**Null Hypothesis:**  $H_0: \beta_j = 0$  (slopes are showing no change),  $X_j$  is **not** linearly associated with Y, therefore the partial slope is not significant.

**Alternative Hypothesis:**  $H_1: \beta_j \neq 0$  (slopes are showing change),  $X_j$  is linearly associated with Y, therefore the partial slope is significant.

Testing the significance of Sweetness Level 4 ( $\hat{\beta}_1 = 8.750$ ) p-value:

Because the p-value for sweetness level 4 is [1] 0.13 and is greater than our alpha (accepted error/significance level) of 0.05 we **fail to reject** our NULL hypothesis and conclude that our partial slope, **Sweetness Level 4** in reference to Sweetness Level of 2, does not show significance in our model.

## 2. Refer to the “Brand Preference” dataset. Code sweetness ( $X_2$ ) as a dummy variable.

2a. Fit a multiple regression model with moisture content, sweetness, and their interaction.

```
#Regressing Rating on Moisture and with Sweetness still as a dummy variable
full_lm <- lm(Rating ~ cat_sweetness + Moisture + cat_sweetness*Moisture, data = brand_data)

summary(full_lm)
```

```
##
## Call:
## lm(formula = Rating ~ cat_sweetness + Moisture + cat_sweetness *
##      Moisture, data = brand_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.150 -1.488  0.125  1.700  3.700
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      42.9000     2.8912  14.838 4.40e-09 ***
## cat_sweetness4     15.7500     4.0887   3.852  0.0023 **
## Moisture           4.9250     0.3934  12.518 3.02e-08 ***
## cat_sweetness4:Moisture -1.0000     0.5564  -1.797  0.0975 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.488 on 12 degrees of freedom
## Multiple R-squared:  0.9622, Adjusted R-squared:  0.9528
## F-statistic: 101.9 on 3 and 12 DF, p-value: 8.379e-09
```

2b. Write down the estimated regression equation at each sweetness level

S = Sweetness

M = Moisture

$$\hat{Y} = 42.90 + 15.75S + 4.925M - 1.0(S * M)$$

$$\text{Sweetness level 2: } \hat{Y} = 42.90 + 4.925M$$

$$\text{Sweetness level 4: } \hat{Y} = 58.68 + 3.925M$$

**2c. Interpret the slope coefficient in each estimated regression equation in Part (b).**

$M = 4.925$ : The mean Rating when sweetness is level 2 for a 1 unit change in Moisture content increases by 4.925.

$M = 3.925$ : The increase in mean Rating when sweetness is 4 for a 1 unit increase in Moisture content is 3.925.

**2d. Is the interaction coefficient significant at  $\alpha = 5\%$ ? State the null, alternative, decision rule, conclusion.**

\*\*\*0.0975 . not significant fail to reject.

Because the p-value for sweetness level 4 is [1] 0.0975 and is greater than our alpha (accepted error/significance level) of 0.05 we **fail to reject** our NULL hypothesis and conclude that our partial slope, **Sweetness Level 4** in reference to Sweetness Level of 2, does not show significance in our model.

**2e. your answer is NO in Part (d), drop the interaction term and rerun the model. Write down the new estimated regression equation at each sweetness level.**

```
no_inter_lm <- lm(Rating ~ cat_sweetness + Moisture, data = brand_data)
summary(no_inter_lm)
```

```
##
## Call:
## lm(formula = Rating ~ cat_sweetness + Moisture, data = brand_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.400 -1.762  0.025  1.587  4.200
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    46.4000     2.3129  20.061 3.66e-11 ***
## cat_sweetness4  8.7500     1.3466   6.498 2.01e-05 ***
## Moisture       4.4250     0.3011  14.695 1.78e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.693 on 13 degrees of freedom
## Multiple R-squared:  0.9521, Adjusted R-squared:  0.9447
## F-statistic: 129.1 on 2 and 13 DF,  p-value: 2.658e-09
```

$y = 46.4 + 8.75S + 4.425M$

lvl 2:  $y = 46.4 + 4.425M$

lvl 4:  $y = 55.15 + 4.425M$

### 3. Refer to the “Assessed Valuations” dataset (Value.txt)

```
value_data = read.table(file = "C:/Users/RUMIL/Desktop/APU/STAT 511 - Millie Mao (Applied Regression Analysis)
View(value_data)

# Adding headers
names(value_data) <- c("Price", "Valuation", "Corner_lot")

# names(bank_data) <- c("", "")

# Defining dependent and independent vars
Price = value_data$Price #Y
Valuation = value_data$Valuation #X1
Corner_lot = value_data$Corner_lot #X2
```

3a. Regress selling price on lot location only. Write down the estimated regression equation.

```
# Coding Corner Location as a dummy variable
cat_corner_lot <- as.factor(Corner_lot)

# For good measure I set 0 as base reference group
cat_corner_lot <- relevel(cat_corner_lot, ref = "0")

location_onlylm <- lm(Price ~ Corner_lot, data = value_data)
summary(location_onlylm)

##
## Call:
## lm(formula = Price ~ Corner_lot, data = value_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -17.854  -5.637   1.157   6.196  16.446
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    81.154      1.198  67.715 < 2e-16 ***
## Corner_lot     -8.523      2.397  -3.556 0.000728 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.303 on 62 degrees of freedom
## Multiple R-squared:  0.1694, Adjusted R-squared:  0.156
## F-statistic: 12.64 on 1 and 62 DF, p-value: 0.0007283
```

$y = 81.154 - 8.5C$

**\*\*\*3b. Based on your regression result in Part (a), what is the estimated mean selling price for corner lots? For non-corner lots?**

R uses alpha-numeric ordering as the reference group by default so either 'a' or '0' as the base reference group.

Estimated mean selling price for non-corner lots:

**The estimated mean selling price for non-corner lots is roughly \$81K**

For corner lots:

**The estimated mean selling price for corner lots is roughly \$72.5k, which is taken by 81k - 8.5k.**

**3c. Based on your regression result in Part (a), what is the estimated difference in selling price between corner and non-corner lots? Is this difference statistically significant?**

The estimated mean selling price between corner and non-corner lots is \$8.5k. Looking at the p-value of 0.000728 we can see that the difference is statistically significant.

**3d. Regress selling price on assessed valuation, lot location, and their interaction. Write down the estimated regression equation for corner lots, and for non-corner lots respectively**

```
#regressing on valuation, lot location, plus their interaction
val_lot_interlm <- lm(Price ~ Corner_lot + Valuation + Corner_lot*Valuation, data = value_data)
summary(val_lot_interlm)
```

```
##
## Call:
## lm(formula = Price ~ Corner_lot + Valuation + Corner_lot * Valuation,
##     data = value_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -10.8470  -2.1639   0.0913   1.9348   9.9836
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -126.9052     14.7225  -8.620 4.33e-12 ***
## Corner_lot        76.0215     30.1314   2.523  0.01430 *
## Valuation         2.7759      0.1963  14.142 < 2e-16 ***
## Corner_lot:Valuation -1.1075      0.4055  -2.731  0.00828 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.893 on 60 degrees of freedom
## Multiple R-squared:  0.8233, Adjusted R-squared:  0.8145
## F-statistic: 93.21 on 3 and 60 DF,  p-value: < 2.2e-16
```



C - Corner or non corner lots

V - Valuation

Estimated regression equation for **non corner lots**:

$$\hat{Y} = -126.9052 + 2.7759V$$

and for **corner lots** respectively:

$$\hat{Y} = -50.8837 + 1.67V$$

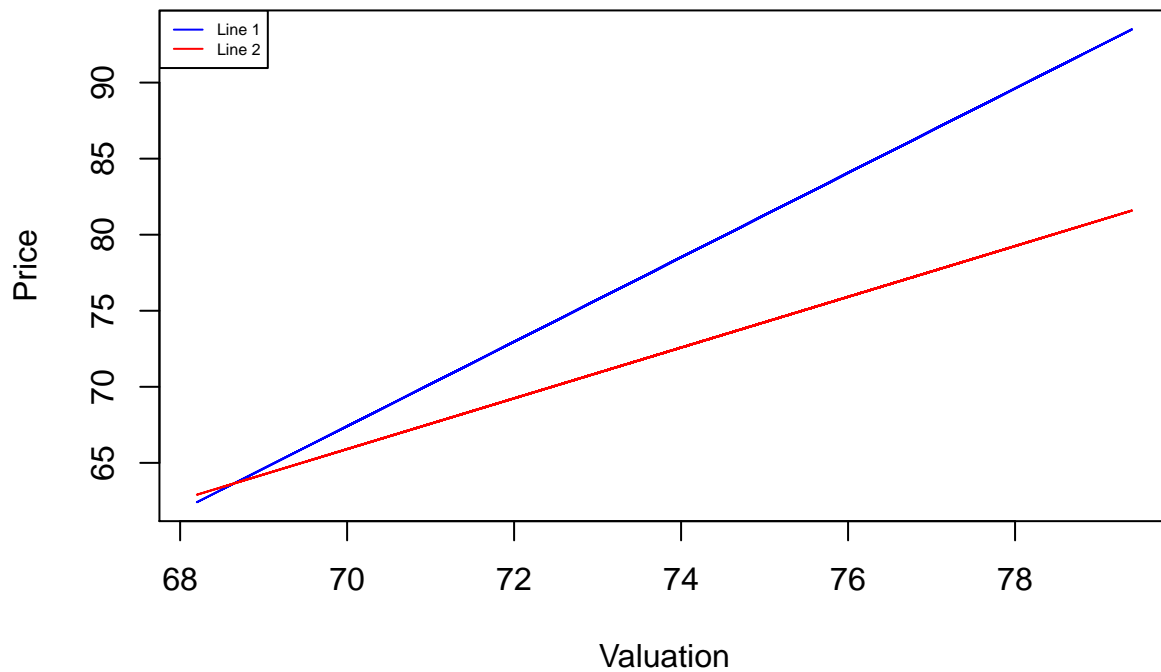
### 3e. Plot the estimated regression lines for the two groups and describe their differences

Plotting Fitted Regression Lines

```
#plot fitted regression lines  
#Save estimated coefficients  
Coef = val_lot_interlm$coefficients  
Coef
```

```
##           (Intercept)           Corner_lot           Valuation  
##          -126.905171           76.021532           2.775898  
## Corner_lot:Valuation  
##           -1.107482
```

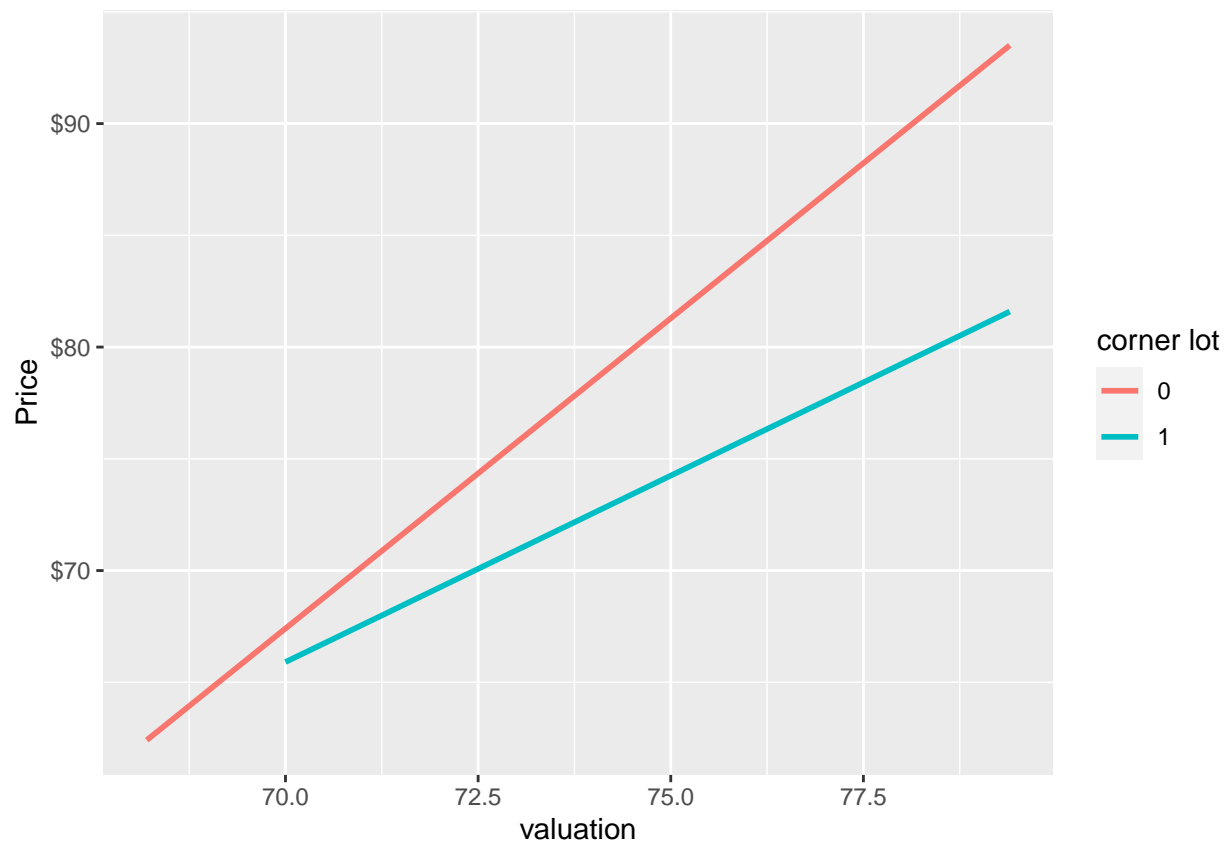
```
#Non corner lots: When Corner lots takes value 0  
price1 = Coef[1] + Coef[3] * Valuation  
  
#corner lots: When Corner lots takes value 1  
price2 = Coef[1] + Coef[3] * Valuation + Coef[2] + Coef[4] * Valuation  
  
#plotting plot()  
plot(Valuation, price1, type = 'l', col = "blue",  
      xlab = "Valuation" , ylab = "Price")  
lines(Valuation, price2, type = 'l', col = "red")  
  
#legend  
legend("topleft", legend = c("Line 1", "Line 2"),  
      col = c("blue", "red"), lty = 1, cex = 0.5)
```



```
plot_coef <- value_data %>%  
  
  ggplot(aes(x = Valuation, y = Price, color = as.factor(Corner_lot))) +  
  
  geom_smooth(method = "lm", se = FALSE) +  
  labs(x = "valuation", y = "Price", color = "corner lot")+  
  scale_y_continuous(labels = scales::dollar)
```

```
plot_coef
```

```
## `geom_smooth()` using formula 'y ~ x'
```



The two are parallel which would mean the interaction coefficient is 0?

so the interaction terms [1] 0.00828 is significant and we would keep them in our model because of the hierarchy principle.

The lines move at the same rate

insignificant means we fail to reject the null, should be parallel. considering interaction term is unnecessary.

the insignificant interaction regardless of using corner or non corner the valuation has the same effect on the price

testing with interaction terms:

we need to always include each of the single effects in ADDITION to the interaction term. ie) if we add  $X*Y$  (second order) in the model we need to always include the 1st order term such as  $3*X + 2*Y$  in our model. If the interaction is insignificant we can drop the highest order from the model.

We should look at the first step and look at the significance then either keep or remove.

WE remove in order from bottom up of the summary

if the highest order is significant, but the ones in lower order have some that are insignificant or significant we keep them.