STAT 511: Analyzing Home Sales Prices Using Multiple Regression Analysis

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Background & Objective

Given that a city tax assessor is interested in predicting residential home sales prices in a midwestern city with various characteristics, we will be conducting a **multiple linear regression analysis (MLR)** from the Real Estate Sales (APPENC07) dataset from 2002. We aim to observe and predict the relationships using the given features, ***square feet***, the absence or presence of a ***swimming pool*** and ***air conditioning***, and our response variable as ***house sales price***.

Our dataset is comprised of *522 total transactions* from midwestern home sales during the year 2002.

```
## # A tibble: 6 x 4
##   sales_price square_feet swimming_pool air_conditioning
##         <int>       <int>         <int>            <int>
## 1      360000        3032             0                1
## 2      340000        2058             0                1
## 3      250000        1780             0                1
## 4      205500        1638             0                1
## 5      275500        2196             0                1
## 6      248000        1966             1                1
```

Part 1 - Regression using a Dummy Variable

1a. Estimated regression equation from regressing sales price on swimming pool only.

```
##
## Call:
## lm(formula = sales_price ~ swimming_pool, data = house_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -188396  -94396  -46896   52604  647604
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)     272396       6195   43.97  < 2e-16 ***
## swimming_pool    79724      23589    3.38  0.00078 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 136600 on 520 degrees of freedom
## Multiple R-squared:  0.02149,    Adjusted R-squared:  0.01961
## F-statistic: 11.42 on 1 and 520 DF,  p-value: 0.0007799
```

Estimated Regression model:

$\hat{Y} = 272396 + 79724X$

1b. Interpretation of estimated intercept and slope.

Intercept: B_0 **= 272396**

The estimated mean Y-value when X = 0 (reference/baseline group) is 272396. When put in context, the mean sales price of a house when the property **does not** contain a swimming pool is estimated to be $272,396.

Slope: B_1 **= 79724**

The slope of 79724 in our model indicates the change for the sales price of a property **containing** a swimming pool, **relative** to a property **without** a swimming pool to be $352,120.

The calculations of these coefficients can be represented in this table.

Table 1: Property Sales Price With & Without Swimming Pool

$\hat{Y} = B_0 + B_1X_1$	Swimming Pool = No	Swimming Pool = Yes
$\hat{Y} = 77.375 + 8.750X$	$\hat{Y} = 272396 + 79724(0)$ = 272396	$\hat{Y} = 272396 + 79724(1)$ = 272396+ 79724
Estimated Mean Sales Price	**$272,396**	**$352,120**

1c. Hypothesis test on the significance of the slope coefficient.

Using a significance level of $\alpha = 0.05$.

Null Hypothesis: H_0: $\beta_j = 0$ (slopes are showing no change), X_j is not linearly associated with Y, therefore the partial slope is not significant.

Alternative Hypothesis: H_1: $\beta_j \neq 0$ (slopes are showing change), X_j is linearly associated with Y, therefore the partial slope is significant.

Testing the significance of a property **with** a swimming pool ($\hat{\beta}_1 = 79724$)

Conclusion and Decision Rule using p-value:

Looking at our model summary, we see that the **p-value** for owning a swimming pool is [1] 0.00078 which means we reject our NULL hypothesis and conclude with our alternative hypothesis. This means that the partial slope, or that a property **containing** a swimming pool in reference to one **without a swimming pool**, is statistically significant.

Part 2 - Fitting a MLR model With the Interaction Term of a Dummy and Continuous Variable

2a. Regressing sales price on the (1) swimming pool dummy variable, (2) area of residence, and the (3) interaction between these two variables.

```
##
## Call:
## lm(formula = sales_price ~ swimming_pool + square_feet + swimming_pool *
##     square_feet, data = house_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -247193  -40579   -7542   24476  384051
##
## Coefficients:
##                             Estimate Std. Error t value Pr(>|t|)
## (Intercept)               -88538.996  12063.237  -7.340 8.34e-13 ***
## swimming_pool             105909.972  47262.735   2.241   0.0255 *
## square_feet                  161.910      5.168  31.331  < 2e-16 ***
## swimming_pool:square_feet    -37.213     17.102  -2.176   0.0300 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 78890 on 518 degrees of freedom
## Multiple R-squared:  0.6747, Adjusted R-squared:  0.6728
## F-statistic: 358.1 on 3 and 518 DF,  p-value: < 2.2e-16
```

Estimated regression equation for each kind of property:

$\hat{Y} = -88538.996 + 105909.972X + 161.910Y - 37.213(X * Y)$

Variable Assignment:

X = Swimming pool

Y = Square feet

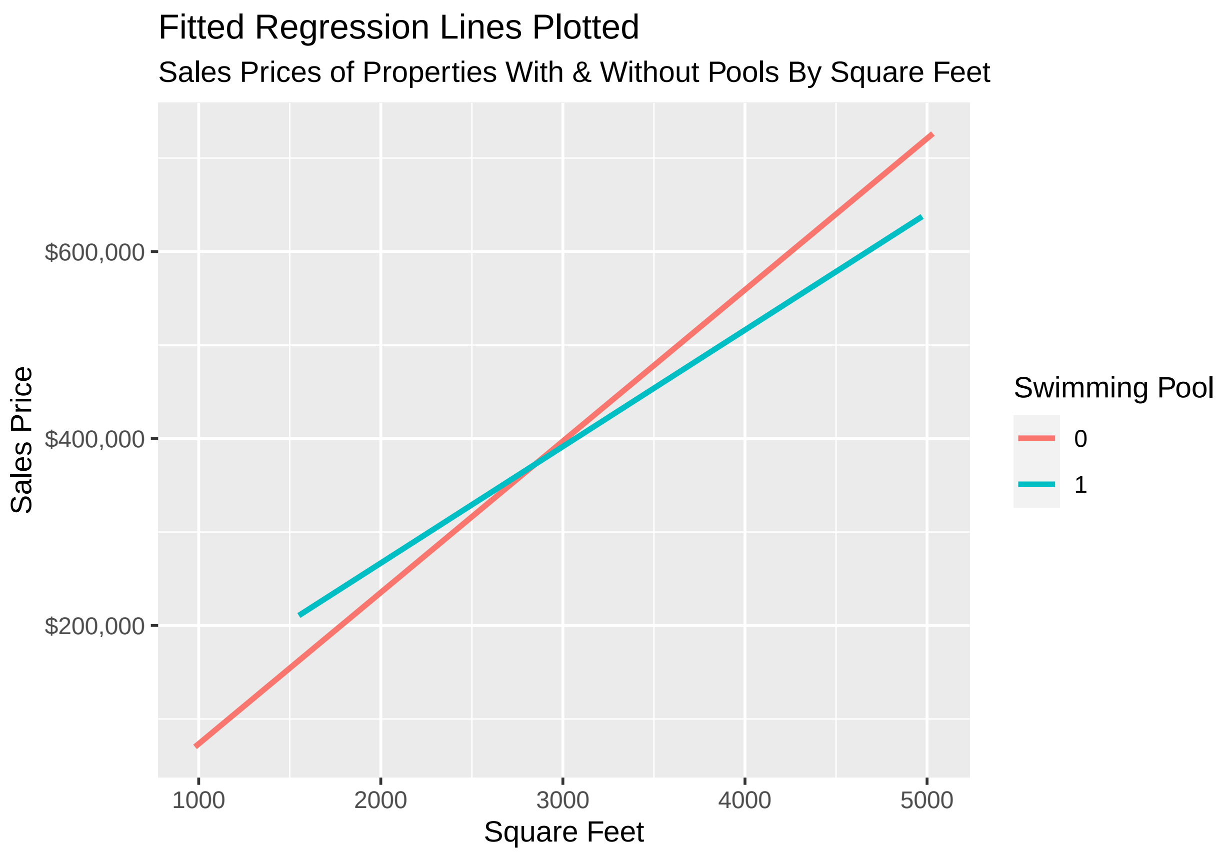
X * Y = Interaction of swimming pool and square feet

Table 2: Calculating Estimated Regression Equations for Properties With and Without Pools

$\hat{Y} = B_0 + B_1X + B_2Y + B_3(X*Y)$	Swimming Pool = No	Swimming Pool = Yes
$\hat{Y} = -88538.996 + 105909.972X + 161.910Y - 37.213(X*Y)$	$\hat{Y} = -88538.996 + 105909.972(0) + 161.910Y - 37.213(0*Y)$	$\hat{Y} = -88538.996 + 105909.972(1) + 161.910Y - 37.213(1*Y)$
	$= -88538.996 + 161.910Y$	$= -88538.996 + 105909.972 + 161.910Y - 37.213(Y)$ $= 17370.976 + 124.697Y$
Estimated Regression Equations	$= -88538.996 + 161.910Y$	$= 17370.976 + 124.697Y$

2b. Plotting fitted regression lines

```
## `geom_smooth()` using formula 'y ~ x'
```



To find the value at which these two lines intersect algebraically we can set both equations equal to one another, solving for one variable, then plugging that back into the equation to get the other variable to obtain the coordinates.

The point of intersection of these two lines are when the values of: **Square feet is 2846.04 and sales price = 372264.58**.

2c. Testing if the two regression lines are parallel.

Using a significance level of $\alpha = 0.05$.

Null Hypothesis: H_0: $\beta_c = 0$ Partial slope of the interaction is 0.

Alternative Hypothesis: H_1: $\beta_c \neq 0$ Partial slope of the interaction is not 0.

Testing the significance of a property **with** a swimming pool ($\hat{\beta}_3 = -37.213$)

Conclusion and Decision Rule using p-value:

Looking at our model summary, we see that the p-value of our interaction term is [1]0.0300 which means we **reject** NULL hypothesis and conclude with our alternative hypothesis and that our regression lines are not parallel and a relationship exists between the two lines (because the interaction coefficient is not 0).

Part 3 - MLR Only With the Interaction of Dummy Variables

3a. Fitting a MLR on both swimming pool and AC dummy variables and find the estimated regression equation.

```
##
## Call:
## lm(formula = sales_price ~ swimming_pool + (air_conditioning) +
##     swimming_pool * air_conditioning, data = house_data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -181752  -92704  -35504   44546  629546
##
## Coefficients:
##                                Estimate Std. Error t value Pr(>|t|)
## (Intercept)                    189578.2    14087.8  13.457  < 2e-16 ***
## swimming_pool                     421.8   132154.9   0.003    0.997
## air_conditioning               100875.8    15548.0   6.488 2.03e-10 ***
## swimming_pool:air_conditioning  65876.5   134169.7   0.491    0.624
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 131400 on 518 degrees of freedom
## Multiple R-squared:  0.09756,    Adjusted R-squared:  0.09233
## F-statistic: 18.67 on 3 and 518 DF,  p-value: 1.642e-11
```

Estimated regression equation for each kind of property:

$\hat{Y} = 189578.2 + 421.8X + 100875.8Z - 65876.5(X * Z)$

Variable Assignment:

X = Swimming pool

Z = Air conditioning

X * Z = Interaction of swimming pool and air conditioning

3c. Calculating estimated mean sales prices for 4 types of properties using estimated regression equation:

1. No swimming pool and no AC

```
## [1] 189578.2
```

The estimated mean sales price of a property without a swimming pool and AC is $189,578.2.

2. No swimming pool and has AC

```
## [1] 290454
```

The estimated mean sales price of a property without a swimming pool but has AC is $290,454.

3. Has swimming pool and no AC

```
## [1] 190000
```

The estimated mean sales price of a property with a swimming pool and no AC is $190,000.

4. Has swimming pool and has AC

```
## [1] 356752.3
```

The estimated mean sales price of a property with a swimming pool and AC is $356,752.3.

Conclusion

Based on our regression analysis it is clear that a property with a swimming pool and air conditioning ($356,752.3), cost significantly more than a property without ($189,578.2.).

For future reference we understand that our dataset is unbalanced with only about 7%, or 36 out of 522 observations owning swimming pools and 16%, or 88 out of 522 observations having air conditioning. Moving forward one way to correct this would be to collect more data from houses owning these features. Also, since the Interaction term between owning a swimming pool and having air conditioning is not significant we should remove this from the model.

What might these mean for the city tax assessor or a home owner?

Over time as square feet increases the price of a house with a swimming pool increases at a faster rate then a house without swimming pool. ***summarize interaction and intersection, and meaning to tax assesor or omeowner. maybe include the ineraction of pool and squarefeet instead of swimming pool and AC