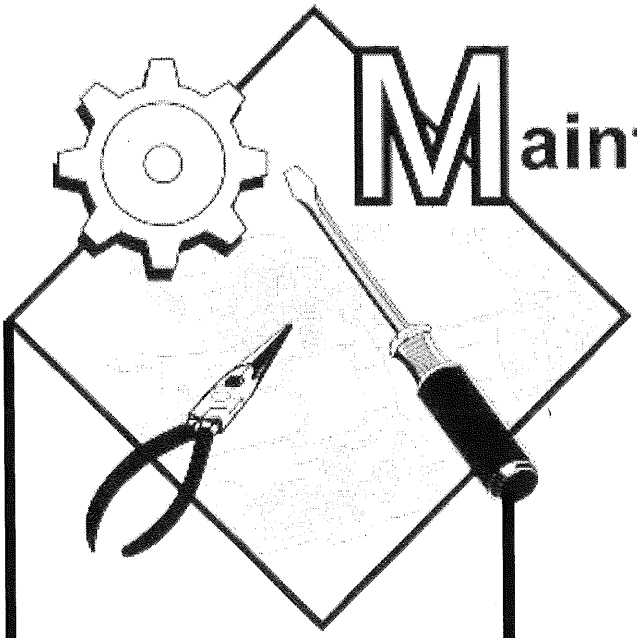

Maintenance Engineering

LESSON ONE

Mathematics — Basic Computation

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Correspondence Course

LESSON ONE

MATHEMATICS
Basic Computation

Correspondence Course
MAINTENANCE ENGINEERING

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Maintenance Engineering

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Lesson One

MATHEMATICS—Basic Computation

Introduction

1.01 A number is anything that answers the question, *How much*? and *How many*? Suppose, for example, that the question is asked, *How many inches are there in one foot*? The answer is, of course, 12 inches. Therefore, 12 is a number since it answers the question, *How many*?

Numbers may be expressed either in the form of symbols or words. For example, the number *2* is a symbol, and it may be expressed in word form as *two*.

1.02 When a number is made up of several figures, each figure is called a digit. In the number 328, for example, there are three digits: 3, 2, and 8. The number 67 is a two-digit number. The two digits are 6 and 7.

1.03 The ability to count is very important. Counting is one of the first things to be learned in arithmetic. We count by naming the numbers in the order in which they increase in value, or get greater in amount. In the following table, (*Table 1*), you will find a list of numbers from 1 to 21, given in both symbol and word form. If we recite these numbers in the order in which they are given, that is 0, 1, 2, 3, 4, etc., we are counting.

1.04 The value represented by a digit depends on its position in relation to other digits. The simple value of a digit is the value it has when it stands alone; for example, the digit 2 standing alone has a value one more than 1 and one less than 3. But in the number 21, 2 does not have the same value as when it stands alone. Now, because of the digit 1 placed after it, the 2 has a value of 2 tens, or twenty. The difference between the simple and relative value of a digit can be explained as follows:

Table 1
Arabic Numbers
Symbol and Word Form

NUMBER	NAME	NUMBER	NAME
0	zero	11	eleven
1	one	12	twelve
2	two	13	thirteen
3	three	14	fourteen
4	four	15	fifteen
5	five	16	sixteen
6	six	17	seventeen
7	seven	18	eighteen
8	eight	19	nineteen
9	nine	20	twenty
10	ten	21	twenty-one

If the figure 8 stands alone, thus: 8

The 8 is simply EIGHT UNITS, or EIGHT.

But place a 2 to the right of the 8, thus: 82

The 2 is now TWO UNITS, but the 8 has moved one place to the left so that it is not longer EIGHT UNITS. Instead, it is, EIGHT TENS or TEN TIMES 8.

Now place a 5 to the right of 82, thus: 825

The 8 is now moved another place to the left, and its value is again increased ten times more, or TEN TIMES EIGHT TENS, making EIGHT HUNDREDS. At the same time, the 2 is moved one place to the left, and its value is increased to TWO TENS.

Now place a 6 to the right of 825, thus: 8256

The 8 is now moved still another place to the left, and its value is increased ten times more, or to EIGHT THOUSANDS. The 2 is increased to TWO HUNDREDS, and the 5 to FIVE TENS. The last number of four figures is read EIGHT THOUSAND TWO HUNDRED FIFTY-SIX.

1.05 When writing numbers that contain more than three digits, it is common practice to set them off into groups of three digits each. A comma (,) is used for this purpose. This is done by counting from the right and placing a comma after every group of three digits. For example, the four-digit number 2738 would be grouped like this: 2,738. The seven-digit number, 1736982, would look like this: 1,736,982.

1.06 After a number has been marked off into groups of three digits each, it is read from the left. Each group is read in turn, with the name of the group placed after it. The group names, as well as place names, are given in *Table 2*.

In *Table 2*, as you can see, there are four groups, namely Units, Thousands, Millions, and Billions. These four groups are broken down into twelve places; or three places for each group. At the bottom of *Table 2*, the number 348,417,385,926 is shown with each of its digits in its proper group and place. If you are able to read this number, you should be able to read any other. It is read as follows: three hundred forty-eight billion, four hundred seventeen million, three hundred eighty-five thousand, nine hundred twenty-six.

Note to the Student

For mathematical computation, we recommend the purchase of an inexpensive scientific calculator, similar to a Casio FX-85, Texas Instruments TI-25, or any other equivalent make. These sell in discount stores at low cost, and are very useful in engineering calculations. It is necessary to develop good mental math skills so you understand what you are doing and have a feel for what the answer should be; however, a calculator will be useful to check your answers. Once you have developed your mental skills, the calculator will remove the drudgery many people associate with math.

Table 2
Reading a 12-digit number

Billions Group			Millions Group			Thousands Group			Units Group		
Hundreds of billions	Tens of billions	Billions	Hundreds of millions	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds of units	Tens of units	Units
3	**4**	**8**	**4**	**1**	**7**	**3**	**8**	**5**	**9**	**2**	**6**

Addition

1.07 Addition is the process of finding a number whose value is equal to the combined values of two or more other numbers. The number obtained by adding is called the **sum**, or the **total**. For example, when 3 and 2 are added, the sum, or total, is 5.

The sign which is used to indicate addition is called a **plus sign** and looks like this: +. The plus sign is placed between numbers which are to be added; thus 6 + 4 indicates that the sum of 6 and 4 is to be found, and this expression is read as: SIX PLUS FOUR.

1.08 When you know some numbers and you want to find out what number you would get if all the known numbers were combined, you should add the known numbers.

1.09 In *Table 3*, you will find the sums of all pairs of numbers from 0 to 9. In order to be able to add quickly and accurately you should be able to give the sums of any of these pairs of numbers without hesitation.

Notice that the sum of two numbers is the same regardless of which of the two numbers is first. For example, 2 + 4 = 6 and 4 + 2 = 6.

Table 3
Basic Addition Table

1+0 = 1	2+0 = 2	3+0 = 3
1+1 = 2	2+1 = 3	3+1 = 4
1+2 = 3	2+2 = 4	3+2 = 5
1+3 = 4	2+3 = 5	3+3 = 6
1+4 = 5	2+4 = 6	3+4 = 7
1+5 = 6	2+5 = 7	3+5 = 8
1+6 = 7	2+6 = 8	3+6 = 9
1+7 = 8	2+7 = 9	3+7 = 10
1+8 = 9	2+8 = 10	3+8 = 11
1+9 = 10	2+9 = 11	3+9 = 12
4+0 = 4	5+0 = 5	6+0 = 6
4+1 = 5	5+1 = 6	6+1 = 7
4+2 = 6	5+2 = 7	6+2 = 8
4+3 = 7	5+3 = 8	6+3 = 9
4+4 = 8	5+4 = 9	6+4 = 10
4+5 = 9	5+5 = 10	6+5 = 11
4+6 = 10	5+6 = 11	6+6 = 12
4+7 = 11	5+7 = 12	6+7 = 13
4+8 = 12	5+8 = 13	6+8 = 14
4+9 = 13	5+9 = 14	6+9 = 15
7+0 = 7	8+0 = 8	9+0 = 9
7+1 = 8	8+1 = 9	9+1 = 10
7+2 = 9	8+2 = 10	9+2 = 11
7+3 = 10	8+3 = 11	9+3 = 12
7+4 = 11	8+4 = 12	9+4 = 13
7+5 = 12	8+5 = 13	9+5 = 14
7+6 = 13	8+6 = 14	9+6 = 15
7+7 = 14	8+7 = 15	9+7 = 16
7+8 = 15	8+8 = 16	9+8 = 17
7+9 = 16	8+9 = 17	9+9 = 18

1.10 All numbers to be added must be arranged in columns, with the right-hand digit of each number directly below the right-hand digit of the number before it. When the numbers have been arranged in columns, the right-hand column is the **units** column, the second column from the right is the **tens** column, the third column from the right is the **hundreds** column, and so on, for as many columns as are needed.

1.11 After the numbers to be added have been arranged in columns, you are ready to add. First add the digits in the units column. If the sum of this column is greater than 9, only the right-hand digit is written; the rest are carried to the tens column. This procedure is followed for each column until the extreme left-hand column is reached. When this column is added, its entire sum is written, nothing is carried.

Sample Problem:
Find the sum of the following numbers: 272,130; 67,777; 158,309; and 4,034.

Solution:
The complete work would usually be shown as in the arrangement below. The digits in small type at the top are those that are carried from the next column at the right. Thus the sum of the digits in the units column is 20, and the 2 is put at the top of the tens column to be added with the digits of the given numbers in that column. You should add the digits in each column yourself, to see that you obtain the correct sums. They are 15 for the tens column, 12 for the hundreds column, 22 for the thousands column and 20 for the tens of thousands column.

```
2 2 1 1 2
272,130
067,777
158,309
004,034
502,250
```

Where there is a blank space in a column, as in the tens of thousand place in the number 4,034 you simply skip that space or imagine that it is occupied by a zero. Thus the digits to be added in the tens of thousands column are 2 (*the carried digit*), 7, 6, and 5. Their sum is 20.

To get practice and to make sure your result is correct, you should add the digits in each column both upward and downward. As shown, the desired total is 502,250.

Subtraction

1.12 Subtraction is the process of finding how much is left when one number is taken from another. For example, if you have 10 dollars and you buy an article costing 8 dollars, you would subtract 8 from 10 to find out how much money you have left.

You can also apply subtraction when you have a certain number of things and you want to find out how many more you need for a desired number. Suppose that you have been saving money to buy an item that costs 50 dollars, and you have 32 dollars. To find out how much more you need, subtract 32 from 50.

1.13 Each number in a subtraction problem has a certain name. The larger number, from which something is to be subtracted, is called the **minuend**. The smaller number, which is to be subtracted from the minuend is called the **subtrahend**. The number left after you have subtracted the subtrahend from the minuend is called the **remainder**. The remainder is also referred to as the **difference**. For example, if we subtract 4 from 7, we have 3. In this problem, 7 is the minuend, 4 is the subtrahend, and 3 is the remainder or difference.

The sign that indicates one number to be subtracted from another is called the **minus sign**, and looks like this: (-). The minus sign is placed between the two numbers to be subtracted. Since minus means *decreased by* the larger number must be placed first, then the minus sign, then smaller number. A problem would look like this: 7 - 4 = 3.

1.14 Although a larger number cannot be subtracted from a smaller number, it sometimes happens that one or more digits in the smaller number will be larger than the corresponding digits in the larger number. For example, we know that it is possible to subtract 24 from 52, because 24 is smaller than 52. However, 4 (*the units digit in the smaller number*) is larger than 2 (*the units digit in the larger number*). Therefore, we must use a process called **borrowing** to subtract these numbers.

To do this, we must *borrow* 10 from the 50, thereby increasing 2 to 12 and reducing 50 to 40. Now, of course, we can subtract 4 from 12, which leaves 8. Then we subtract 2 from 4 leaving 2, and the remainder is 28. This process of borrowing can be performed as many times as necessary in a problem.

52	MINUEND
24	SUBTRAHEND
28	REMAINDER

1.15 To determine whether or not you have solved a subtraction problem correctly, you simply add the subtrahend (*the smaller number*) to the remainder (*your answer*). The sum of these two numbers should be equal to the minuend (*the larger number*). For example, to check the problem that we solve previously, 52 - 24 = 28, we simply add 24 (*the subtrahend*) to 28 (*the remainder*). Since 24 + 28 = 52, which is equal to the minuend, the problem is correct.

Multiplication

1.16 Multiplication is actually a short-cut method of adding equal numbers. It eliminates long, tedious addition problems.

Each number in a multiplication problem has a certain name. The number to be multiplied is called the **multiplicand**, the number by which the multiplicand is multiplied is called the **multiplier**, and the answer which results from the process of multiplication is called the **product**. For example, if we multiply 7 by 6, the answer is 42. In this problem, 7 is the multiplicand, 6 is the multiplier and 42 is the product.

The sign used to indicate multiplication is called a **multiplication sign** or a **times sign** and looks like this: x. It is placed between the multiplicand and the multiplier in this manner: 7 x 6 = 42. This is read, 7 times 6 equals 42.

1.17 The multiplication tables contain the products of all possible pairs of numbers from 0 to 12 inclusive. To be able to multiply larger numbers quickly and correctly, you should be able to give the products of any numbers in the multiplication tables without hesitating. Notice that any number multiplied by 0 is equal to 0, and any number multiplied by 1 is equal to itself.

Table 4
Multiplication Table
(Zero times any number equals zero)

1	x	1 =	1	2	x	1 =	2	3	x	1 =	3	4	x	1 =	4
1	x	2 =	2	2	x	2 =	4	3	x	2 =	6	4	x	2 =	8
1	x	3 =	3	2	x	3 =	6	3	x	3 =	9	4	x	3 =	12
1	x	4 =	4	2	x	4 =	8	3	x	4 =	12	4	x	4 =	16
1	x	5 =	5	2	x	5 =	10	3	x	5 =	15	4	x	5 =	20
1	x	6 =	6	2	x	6 =	12	3	x	6 =	18	4	x	6 =	24
1	x	7 =	7	2	x	7 =	14	3	x	7 =	21	4	x	7 =	28
1	x	8 =	8	2	x	8 =	16	3	x	8 =	24	4	x	8 =	32
1	x	9 =	9	2	x	9 =	18	3	x	9 =	27	4	x	9 =	36
1	x	10 =	10	2	x	10 =	20	3	x	10 =	30	4	x	10 =	40
1	x	11 =	11	2	x	11 =	22	3	x	11 =	33	4	x	11 =	44
1	x	12 =	12	2	x	12 =	24	3	x	12 =	36	4	x	12 =	48
5	x	1 =	5	6	x	1 =	6	7	x	1 =	7	8	x	1 =	8
5	x	2 =	10	6	x	2 =	12	7	x	2 =	14	8	x	2 =	16
5	x	3 =	15	6	x	3 =	18	7	x	3 =	21	8	x	3 =	24
5	x	4 =	20	6	x	4 =	24	7	x	4 =	28	8	x	4 =	32
5	x	5 =	25	6	x	5 =	30	7	x	5 =	35	8	x	5 =	40
5	x	6 =	30	6	x	6 =	36	7	x	6 =	42	8	x	6 =	48
5	x	7 =	35	6	x	7 =	42	7	x	7 =	49	8	x	7 =	56
5	x	8 =	40	6	x	8 =	48	7	x	8 =	56	8	x	8 =	64
5	x	9 =	45	6	x	9 =	54	7	x	9 =	63	8	x	9 =	72
5	x	10 =	50	6	x	10 =	60	7	x	10 =	70	8	x	10 =	80
5	x	11 =	55	6	x	11 =	66	7	x	11 =	77	8	x	11 =	88
5	x	12 =	60	6	x	12 =	72	7	x	12 =	84	8	x	12 =	96
9	x	1 =	9	10	x	1 =	10	11	x	1 =	11	12	x	1 =	12
9	x	2 =	18	10	x	2 =	20	11	x	2 =	22	12	x	2 =	24
9	x	3 =	27	10	x	3 =	30	11	x	3 =	33	12	x	3 =	36
9	x	4 =	36	10	x	4 =	40	11	x	4 =	44	12	x	4 =	48
9	x	5 =	45	10	x	5 =	50	11	x	5 =	55	12	x	5 =	60
9	x	6 =	54	10	x	6 =	60	11	x	6 =	66	12	x	6 =	72
9	x	7 =	63	10	x	7 =	70	11	x	7 =	77	12	x	7 =	84
9	x	8 =	72	10	x	8 =	80	11	x	8 =	88	12	x	8 =	96
9	x	9 =	81	10	x	9 =	90	11	x	9 =	99	12	x	9 =	108
9	x	10 =	90	10	x	10 =	100	11	x	10 =	110	12	x	10 =	120
9	x	11 =	99	10	x	11 =	110	11	x	11 =	121	12	x	11 =	132
9	x	12 =	108	10	x	12 =	120	11	x	12 =	132	12	x	12 =	144

1.18 Multiplication problems are written with the multiplier directly under the multiplicand and the units digit of each in the same column. For example, let's determine the product of 358 and 714.

The first partial product, obtained by multiplying 358 by 4, is 1432. The partial product of 358 and 1, which is 358, is written below 1432 so that the 8 is in the same column as the 1 in 714. The partial product 358 and 7, which is 2506, is written below the second partial product, 358, so that the 6 is in the column with the 7 in 714. Now that we have multiplied by each digit in the multiplier, we add the three partial products, the sum of which is 255,612, the complete product of the problem.

```
   358    MULTIPLICAND
   714    MULTIPLIER
  1432    FIRST PARTIAL PRODUCT
   358    SECOND PARTIAL PRODUCT
  2506    THIRD PARTIAL PRODUCT
255612
```

1.19 Multiplication problems may be checked by reversing the order of the multiplicand and the multiplier. In the problem that we just solved, for example, we multiplied 358 by 714. To check the work, we would multiply 714 by 358, making 714 the multiplicand and 358 the multiplier. In solving this problem, the partial products would be different, but the answer, or complete product would be the same.

Division

1.20 The process of finding how many times one number is contained in another is called division. It is also the process of separating a number into a given number of equal parts. Division is considered to be the reverse or opposite of multiplication.

Suppose you wanted to know how many 2-inch pieces could be cut from 32-inch rod. You would simply divide 32 by 2.

1.21 The number which is to be divided or separated into equal parts is called the **dividend**. The number by which the dividend is divided is called the **divisor**. The solution, or answer to a division problem is called the **quotient**. For example, when 8 is divided by 2, the answer is 4. The dividend is 8, the divisor is 2, and 4 is the quotient.

The sign used to indicate division is called a **division sign** and is read divided by. It looks like this: ÷. Thus, the problem 54 ÷ 9 is read, 54 divided by 9. The divisor, in this case 9, must always come after the division sign.

1.22 In a short division problem, the divisor has only one digit, but the dividend may have several. For example, suppose you want to divide 936 by 3. First the numbers must be arranged as follows:

```
    312
3 | 936
```

Since 3 is divided into 9 three times, 3 is placed in the quotient directly above the 9. Then we divide 3 into 3. The answer is 1; therefore a 1 is placed in the quotient above the 3. Now we divide 3 into 6, and the answer is 2. The 2 is placed in the quotient above the 6. As you can see, the quotient is 312. This is the simplest type of division.

1.23 When solving a problem in which the divisor has more than one digit, long division is used. The steps used in long division are similar to those in short divisions, as you will see in the following problem. In this problem it is necessary to divide 289456 by 458.

```
                 632   QUOTIENT
DIVISOR    458 | 289456   DIVIDEND
               2748
                 1465
                 1374
                   916
                   916
```

Since 458 cannot be divided into 2, 28, or 289, it must be divided into 2894. This makes the first digit of the quotient 6, which is written directly above the 4 in the dividend. Write the product of 458 and 6, which is 2748, below the first four digits in the dividend and subtract; you get 146. Bring down the next digit in the dividend, 5, and

place it after the 146, changing it to 1465. Dividing 458 into 1465 leaves 3 as the next digit of the quotient. Then 458 is multiplied by 3 and the product, 1374 is subtracted from 1465, leaving 91. The last figure of the dividend, 6, is brought down and annexed to 91, making it 916. Since 458 is contained in 916 twice, the last digit in the quotient is 2. The product of 458 and 2 is 916, which subtracted from 916, leaves no remainder.

1.24 To determine whether or not a solution to a division problem is correct, multiply the quotient by the divisor. The product of these two quantities should equal the dividend.

For example, let's take the problem we just solved. The number 458 was divided into 289,456 and the resulting quotient was 632. To check this work, we multiply the quotient, 632, by the divisor, 458.

632	QUOTIENT
458	DIVISOR
4956	
3160	
2528	
289456	DIVIDEND

Since the product of these two numbers, 289,456, is also the dividend, the solution is correct.

Exercises

Use these exercises to help check your learning progress.

Questions	Answers
1.01 Numbers may be expressed either in the form of ____ or ____.	**1.01** symbols words Ref. 1.01
1.02 The value represented by a digit depends on its ____ in relation to other digits.	**1.02** position Ref. 1.04
1.03 When grouping digits, common practice is to group them in ____ digits.	**1.03** three Ref. 1.05
1.04 The number obtained by adding is called the ____ or the ____.	**1.04** sum total Ref. 1.07
1.05 The sign which is used to indicate addition is called a ____ sign and looks like this: ____.	**1.05** plus + Ref. 1.07
1.06 The sign which indicates that one number is to be subtracted from another is called the ____ sign, and it looks like this: ____.	**1.06** minus - Ref. 1.08
1.07 The larger number from which something is to be subtracted, is called the ____.	**1.07** minuend Ref. 1.13

1.08 The smaller number, which is to be subtracted from the larger number is called the _____.	**1.08** subtrahend Ref. 1.13
1.09 The sign used to indicate multiplication is called a _____ sign or a _____ _____ and looks like this: _____	**1.09** multiplication times sign x Ref. 1.16
1.10 The number by which the dividend is divided is called the _____.	**1.10** divisor Ref. 1.21

Fractions

1.25 When any object is divided or cut into parts, each of the resulting parts is smaller than the whole object. These parts are called **fractional parts** of the object and they are presented numerically by **fractions**. The bar shown in the following illustration is divided into a different number of parts. Each part is a fractional part of the bar, smaller than the whole bar, and is represented by a fraction.

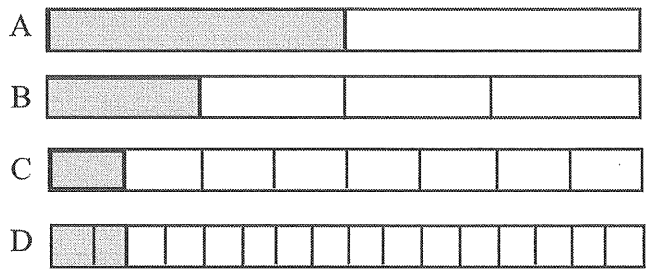

Figure 1

1.26 A fraction consists of two numbers written one below the other and separated by a short straight line. For example, in the above illustration, the bar in (**A**) is divided into two parts. Each part is called one half and is written in this manner: $^1/_2$. This section of the bar in (**D**) is divided into sixteen parts. Each part is called one sixteenth and is written as $^1/_{16}$. The shaded parts in (**D**) represent two of the sixteen parts and together they are called two sixteenths and this is written as $^2/_{16}$.

1.27 Each of the two numbers which make up a fraction has a definite name. The number above the line is called the **numerator**, and the number below the line is called the **denominator**.

The denominator, or the bottom number, tells us how many equal parts the object is divided into. The numerator indicates how many of these equal parts are being considered. For example, in *Fig. 1* the bar at (**B**) is divided into four parts. Therefore a fraction relating to this part of the bar will have denominator of 4. We can see that one of these four parts is shaded, therefore the shaded section considered alone, makes up $^1/_4$ of the bar. If we want to know what part of this bar is represented by the unshaded portion, the answer is $^3/_4$ of the bar; the numerator 3 showing that there are 3 unshaded parts, and the denominator 4 showing that there are, altogether, 4 parts.

1.28 Every fraction is actually a division problem in which the numerator is the dividend and the denominator is the divisor. For example, the fraction $^1/_4$ is another way of saying 1 divided by 4 or $1 \div 4$. The fraction $^2/_4$ means $2 \div 4$. In fact, the line used to separate the two numbers of a fraction is another way to indicate division.

Any division problem can be expressed as a fraction, just as any fraction can be expressed as a division problem. Suppose we have the problem, $320 \div 765$. This can be written as the fraction $^{320}/_{765}$. Knowing that every fraction is a division problem, it is a simple matter to determine the value of a fraction by simply dividing the numerator by the denominator.

You can see that changing the value of either the numerator or the denominator changes the value of the entire fraction. When the denominator is increased, the value of the fraction becomes smaller, but when the numerator is increased, the value of the fraction becomes larger. This can be easily seen by referring again to the illustration we considered previously. In (**D**) the bar is divided into 16 parts, each part being $^1/_{16}$. In (**B**) the bar is divided into 4 parts, each part being $^1/_4$. Now, although 4 is larger than 2, you can see that $^1/_2$ of the bar is much larger than $^1/_4$. Also, you can see that two portions of the bar in (**D**), or $^2/_{16}$, are larger than one portion, or $^1/_{16}$. Therefore, increasing the numerator increases the value of the fraction.

1.29 When the numerator of a fraction is a smaller number than the denominator, the fraction is called a **proper fraction**. Thus, $^1/_2$, $^3/_4$ and $^4/_8$, are proper fractions. If the numerator is equal to or greater than the denominator, the fraction is called an **improper fraction**. Thus, $^2/_2$, $^3/_2$ and $^4/_2$, are improper fractions. Improper fractions are always equal to or greater than 1.

1.30 A number composed of a whole number and a fraction is called a **mixed number**. For example, the number $5\,^2/_8$ is a mixed number since it consists of the whole number 5 and a fraction $^2/_8$. This number is read as five and two eighths. It is equal to $5 + ^2/_8$, but the plus sign is omitted and it appears as simply $5\,^2/_8$.

1.31 RULE: *If both the numerator and the denominator of a fraction are multiplied or divided by the same number, the value of the resulting fraction is the same as that of the original fraction.*

For example, suppose that both terms of the fraction $^1/_2$ are multiplied by 2. Then, $^1/_2 \times {}^2/_2 = {}^2/_4$. The fraction $^2/_4$ has the same value as $^1/_2$, as you can see from the following illustration.

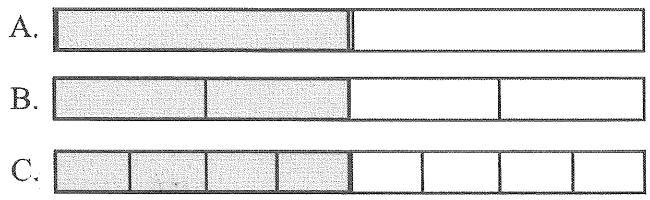

Figure 2

In *Figure 2* the bar in (**A**) is divided into 2 parts, each part being $^1/_2$ of the bar, and the bar in (**B**) is divided into 4 parts, the two shaded parts being equal to $^2/_4$ of the bar. You can see that $^2/_4$ of the bar in (**B**) covers the same space as $^1/_2$ of the bar in (**A**), therefore $^2/_4$ has the same value as $^1/_2$.

Again, take $^1/_2$ and multiply both terms by 4. Then, $^1/_2 \times {}^4/_4 = {}^4/_8$. From the bar in (**C**), which is divided into 8 parts, you can see that the 4 shaded parts, or $^4/_8$ is the same as $^2/_4$ of (**B**) and $^1/_2$ of (**A**). Therefore, $^1/_2 = {}^2/_4 = {}^4/_8$, and all of these fractions have the same value.

1.32 Just as a multiplying both terms of a fraction by the same number does not change its value, neither does dividing them by the same number. Dividing the terms of a fraction by the same number reduces it to lower terms. For example, take the fraction $^{12}/_{16}$. This fraction can be reduced to lower terms by dividing each of the numbers by 4. Thus, $^{12}/_{16}$, $^4/_4 = {}^3/_4$. Fractions are reduced to their lowest terms when both numbers cannot be divided by the same number without a remainder. The fractions $^3/_4$, $^5/_8$, and $^{11}/_{16}$ are reduced to their lowest terms.

1.33 A mixed number can be changed to an improper fraction by multiplying the whole number by the denominator, adding the product to the numerator, and placing the sum over the denominator. For example, let us change the mixed number $5\,^3/_4$ to an improper fraction. First multiply the whole number 5 by the denominator 4. The product is 20. Next, add the product 20 to the numerator 3; the sum is 23. The sum 23 is placed over the denominator 4, giving the improper fraction $^{23}/_4$.

1.34 An improper fraction is changed to a mixed number by dividing the numerator by the denominator. For example, the improper fraction $^{23}/_4$ is changed by dividing 23 by 4. 4 is divided into 23 five times with a remainder of 3. To write this, the 5 is written as a whole number and the remainder 3 is written as a fraction with 4 as the denominator, resulting in $5\,^3/_4$.

Addition of Fractions

1.35 When fractions are to be added, they must all have the same denominator. If they have different denominators, they must be changed, not in value but in form so that all their denominators are alike. Thus, $^3/_4$ may be added to $^1/_4$ to produce or give $^4/_4$, but $^3/_4$ cannot be added to $^7/_8$ as the fractions stand, because it is not possible to add fourths and eights, just as it is not possible to add feet and dollars.

It is possible to add $^3/_4$ and $^7/_8$, however, after we have changed the form of one of them so that they both have the same denominator. Since we can change fourths to eighths, this is the first step. $^3/_4 \times {}^2/_2 = {}^6/_8$, and $^6/_8$ and $^7/_8$ can be added, resulting in $^{13}/_8$.

1.36 When several fractions have the same denominator, they are said to have a **common denominator**. For example, $^1/_6$, $^5/_6$, $^3/_6$, $^4/_6$, all have the common denominator, 6. Fractions having different denominators can be changed to other forms having like denominators by changing each fraction to higher or lower terms. For example, $^1/_2$ and $^1/_3$ can each be changed to sixths. Thus, $^1/_2 = {}^3/_6$, because $^1/_2 \times {}^3/_3 = {}^3/_6$, and $^1/_3 = {}^2/_6$, because $^1/_3 \times {}^2/_2 = {}^2/_6$.

1.37 In solving problems involving fractions, it is customary to find the smallest number exactly divisible by all the denominators of the given fractions. This number is called the **least common denominator**. Suppose that the fractions $^1/_2$, $^2/_3$, and $^3/_4$ are to be changed to fractions having the same denominator. The number 24 could be used as a common denominator here, but it isn't the least

common denominator since these three numbers, 2, 3, and 4, will be evenly divided into 12. Thus, 12 is the least common denominator. Changing these fractions to twelfths, we have $^1/_2 = {}^1/_2 \times {}^6/_6 = {}^6/_{12}$, $^2/_3 = {}^2/_3 \times {}^4/_4 = {}^8/_{12}$ and $^3/_4 = {}^3/_4 \times {}^3/_3 = {}^9/_{12}$.

1.38 It is not always possible to determine the least common denominator of a group of fractions simply by looking at them. Sometimes calculations are necessary. The method of doing this is explained with the aid of the following problem.

Problem:
Find the least common denominator of $^1/_4$, $^1/_3$, $^1/_9$, and $^1/_{16}$.

Solution:
The denominators of the fractions are placed in a line, as in short division, and separated by commas. In this case they are 4, 3, 9, and 16, as shown. The denominators are now divided by some prime number other than 1 that will be contained in at least two of the numbers without a remainder. The number 2 is contained in 4 and 16; hence, dividing by 2 and placing the quotients in the second line, under the numbers divided, the quotients 2 and 8 are obtained. The numbers 3 and 9, which will not contain the divisor without a remainder, are transferred to the second line, as shown in *Fig. 3*.

Repeat the process on the second line by dividing by 2, which will result in the numbers 1, 3, 9, and 4 on the third line. Next divide by 3, the common divisor of 3 and 9. Now solve by finding the product of this fourth line and the three divisors.

1)	4,	3,	9,	16
2)	2,	3,	9,	8,
3)	1,	3	9	4,
4)	1,	1,	3,	4

Figure 3

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator.

1.39 The first step in adding common fractions, is to change them to a form in which they have a common denominator, as previously explained. When this has been done, add the numerators of the fractions and place the sum above the common denominator. We will solve a sample problem to demonstrate these steps.

Suppose we are to find the sum of the fractions $^1/_2$, $^3/_4$, and $^5/_{12}$.

The least common denominator of these fractions is 12. Then, $^1/_2 = {}^6/_{12}$, $^3/_4 = {}^9/_{12}$, and $^5/_{12}$ remains the same. Adding the numerators, we have $6+9+5$ which is equal to 20. This sum is written above the common denominator 12, resulting in $^{20}/_{12}$ which is an improper fraction. Changing the improper fraction to a mixed number, we have $20 : 12 = 1\,{}^8/_{12}$. The $^8/_{12}$ can be further reduced by dividing each term by 4, resulting in $^2/_3$. Therefore the sum of the three fractions is $1\,{}^2/_3$.

1.40 When mixed numbers are to be added, the sum of the whole numbers is found, then the sum of the fractions, and the two results are added. For example, suppose we are to find the sum of $5\,{}^3/_4$, $8\,{}^1/_2$ and $2\,{}^1/_8$. First we add the whole numbers: $5+8+2=15$. Then we add the fractions after first find the least common denominator, which is 8. Thus, $^3/_4 = {}^6/_8$, $^1/_2 = {}^4/_8$, and $^1/_8 = {}^1/_8$. Adding $6 + 4 + 1 = 11$, and the sum of the fractions is $^{11}/_8$. Changing this to a mixed number we have $11 \div 8 = 1\,{}^3/_8$. Now we must add the sum of the whole numbers, 15, to the sum of the mixed number $1\,{}^3/_8$. Thus, $15 + 1\,{}^3/_8 = 16\,{}^3/_8$.

Subtraction of Fractions

1.41 You already know that to add fractions, each fraction must have the same denominator called a common denominator. The same rule holds true for subtracting fractions. We cannot subtract $^1/_8$ from $^1/_4$ any more than we can add $^1/_8$ and $^1/_4$. Find the common denominator for a subtraction problem is exactly the same as finding it for an addition problem.

1.42 After both fractions have been changed to a common denominator, subtract the smaller numerator from the larger numerator. Then write the remainder above the common denominator.

Let's take a sample problem. Suppose we want to subtract $^1/_4$ from $^{25}/_{64}$. The least common denominator will in this case be 64. In order to change $^1/_4$ to sixty-fourths, it is necessary to multiply each term of the fraction by 16. Thus, $^1/_4 = {}^1/_4 \times {}^{16}/_{16} = {}^{16}/_{64}$. Subtracting 16 from 25 gives us a remainder of 9. Write the remainder 9 over the common denominator 64, and the answer to the problem is $^9/_{64}$. As you can see, this fraction cannot be reduced to lower terms.

1.43 When subtracting mixed numbers, first subtract the whole numbers and then subtract the fractions, placing the remainder side by side. Suppose, for example, that we must subtract $2\,^3/_8$ from $7\,^5/_8$. First subtract the whole numbers: 7 - 2 = 5. Then subtract the numerators, 5 - 3 = 2, and write the remainder over the denominator, which leaves $^2/_8$. Writing the remainders together, we have $5\,^2/_8$ (which would reduce to $5\,^1/_4$). *In this case it was not necessary find the least common denominator since both denominators are the same.*

Sometimes you will find that the fraction of the larger mixed number or minuend is smaller than the fraction of the smaller mixed number or subtrahend. In this case, it is necessary to borrow 1 from the whole number, just as was done previously in the subtraction of whole numbers.

For example, suppose that $6\,^7/_8$ is to be subtracted from $10\,^1/_4$. The least common denominator is 8, and $10\,^1/_4$ changed to eighths is $10\,^2/_8$. However, it is still impossible to subtract $^7/_8$ from $^2/_8$, therefore it is necessary to borrow 1 from the 10 *in the form of* $^8/_8$ and add it to the $^2/_8$. This would be done as follows:

$$\begin{array}{lll} 10\,^2/_8 & = 9\,^{10}/_8 & \text{MINUEND} \\ -6\,^7/_8 & = -6\,^7/_8 & \text{SUBTRAHEND} \\ & \overline{\quad 3\,^3/_8} & \text{REMAINDER} \end{array}$$

Borrowing 1 from 10 leaves 9, and adding the $1\,^8/_8$ to the fraction $^2/_8$, makes the minuend $9\,^{10}/_8$. As you can see, $6\,^7/_8$ can be subtracted from $9\,^{10}/_8$, and the remainder is $3\,^3/_8$.

1.44 When a fraction (*or a mixed number*) is to be subtracted from a whole number, there will of course be no fraction in the minuend. When this situation occurs, it is necessary to borrow 1 from the whole number and change it to a fraction having the same denominator as the fraction in the subtrahend. For example, suppose that we must subtract $^7/_{12}$ from 8. First, borrow 1 from the whole number 8, making it 7 and add the 1 to it in the form of $^{12}/_{12}$.

$$\begin{array}{lll} 8 & = 7\,^{12}/_{12} & \text{MINUEND} \\ -\,^7/_{12} & = -\,^7/_{12} & \text{SUBTRAHEND} \\ & \overline{= 7\,^5/_{12}} & \text{REMAINDER} \end{array}$$

Now we can subtract $^7/_{12}$ from $7\,^{12}/_{12}$, and the remainder is $7\,^5/_{12}$.

Multiplication of Fractions

1.45 Multiplication of fractions is a much simpler process than either addition of subtraction of fractions. This is due mainly to the fact that it is not necessary to find a common denominator before multiplying.

To multiply either proper or improper fractions, we simply multiply the numerators to form a new numerator, and then multiply the denominators to form a new denominator. For example, suppose that we are to multiply the fractions $^3/_8$ and $^1/_2$.

$$\frac{3}{8} \times \frac{1}{2} = \frac{3 \times 1}{8 \times 2} = \frac{3}{16} \quad \text{Product}$$

1.46 To find the product of a fraction and a whole number, multiply the whole number by the numerator of the fraction and place the result over the denominator of the fraction. For example, let's multiply 8 by $^2/_3$.

$$8 \times \frac{2}{3} = \frac{8 \times 2}{3} = \frac{16}{3}$$

Reducing the improper fraction $^{16}/_3$ to its lowest terms, we have $16 \div 3$ which is equal to $5\,^1/_3$.

1.47 When it is necessary to multiply a fraction and a mixed number, the mixed number must first be reduced to an improper fraction. After this has been done, the process is the same as that for multiplying two fractions. Suppose, for example, that we are to multiply $3\,^2/_3$ by $^7/_8$.

First change $3\,{}^{2}/_{3}$ to an improper fraction:

$$3\frac{2}{3}=\frac{3\times3+2}{3}=\frac{11}{3}$$

Now multiply ${}^{11}/_{3}$ by ${}^{7}/_{8}$:

$$\frac{11}{3}\times\frac{7}{8}=\frac{11\times7}{3\times8}=\frac{77}{24}$$

Reducing the improper fraction ${}^{77}/_{24}$ to its lowest terms, we have:

$$\frac{77}{24}=77\div24=3\frac{5}{24}\ \text{Product}$$

1.48 The same method used to multiply two fractions is used to multiply several fractions. For example, let's multiply ${}^{2}/_{3}$ x ${}^{3}/_{4}$ x ${}^{1}/_{2}$.

$$\frac{2}{3}\times\frac{3}{4}\times\frac{1}{2}=\frac{2\times3\times1}{3\times4\times2}=\frac{6}{24}\ \text{or}\ \frac{1}{4}\ \text{Product}$$

Several mixed numbers, whole numbers, and fractions are multiplied in the same way. Suppose we want to multiply $6\,{}^{1}/_{3}$ x 3 x ${}^{2}/_{5}$. Here we have a mixed number, a whole number, and a fraction. The first step is to change the mixed number to an improper fraction.

$$6\frac{1}{3}=\frac{3\times6\times1}{3}=\frac{19}{3}$$

Now we are ready to multiply.

$$\frac{19}{3}\times3\times\frac{2}{5}=\frac{19\times3\times2}{3\times5}=\frac{114}{15}$$

Reducing ${}^{114}/_{15}$ to its lowest terms, we have:

$$\frac{114}{15}=114\div15=7\frac{9}{15}\ \text{Product}$$

1.49 The process of multiplying fractions can often be simplified and shortened by a method called **cancellation**. Instead of multiplying the numerators and denominators as they are, it is often possible to divide them by the same number just as we do in reducing a fraction to its lowest terms. For example, suppose the fractions to be multiplied are ${}^{3}/_{8}$, ${}^{2}/_{3}$ and ${}^{1}/_{4}$.

Setting up the problem, we have:

$$\frac{3\times2\times1}{8\times3\times4}$$

You can see that 3 can be divided into both the 3 in the numerator and the 3 in the denominator.

$$\frac{\overset{1}{\cancel{3}}\times2\times1}{8\times\underset{1}{\cancel{3}}\times4}$$

There is also a 2 and an 8 which are divisible by 2.

$$\frac{\overset{1}{\cancel{3}}\times\overset{1}{\cancel{2}}\times1}{\underset{4}{\cancel{8}}\times\underset{1}{\cancel{3}}\times4}$$

Since further cancellation is not possible, all of the uncanceled numbers are multiplied.

$$\frac{1\times1\times1}{4\times1\times4}=\frac{1}{16}\ \text{Product}$$

Remember that cancellation can be used only when all of the numbers in both the numerator and denominator are **multiplied** together. If addition or subtraction is included in the problem you cannot cancel. For example, in the expression

$$\frac{2\times3-4+6}{7-5}$$

it is not possible to cancel. The indicated addition and subtraction must be performed. This problem would be solved as follows:

$$\frac{2\times3-4+6}{7-5}=\frac{6-4+6}{2}=\frac{2+6}{2}=\frac{8}{2}=4$$

Division of Fractions

1.50 Once you have mastered multiplication of fractions, you have practically mastered division of fractions. Fractions may be divided by or into whole numbers, mixed numbers, or other fractions. The process is quite simple, and can be stated briefly: INVERT THE DIVISOR AND MULTIPLY.

The phrase *invert the divisor* simply means turn it upside down, or reverse the order of numerator and denominator. For example, suppose that the divisor of a problem is $^5/_8$. To invert the fraction $^5/_8$, we change it to $^8/_5$.

Suppose that the divisor is a whole number, 6. Every whole number can be thought of as being divided by 1, thus 6 is actually $^6/_1$. To invert the whole number 6 then, we simply make it $^1/_6$.

To invert a mixed number, we must first change it to an improper fraction. If the divisor is $3\,^4/_5$, changing it to an improper fraction we have $^{19}/_5$, and inverting it gives $^5/_{19}$.

1.51 Suppose the problem to be solved is as follows:

$$\frac{4}{5}\div\frac{3}{2}=?$$

First it is necessary to invert the divisor. *Remember that the divisor is always the number following the division sign.*

$$\frac{4}{5}\times\frac{2}{3}$$

Multiplying, we have:

$$\frac{4\times2}{5\times3}=\frac{8}{15}\ \text{Quotient}$$

Notice that although the final step of the problem is multiplication, the problem is actually one of division and the answer is labeled QUOTIENT.

1.52 Suppose that it is necessary to divide the fraction $^3/_8$ by the mixed number $5\,^6/_7$.

$$\frac{3}{8}\div5\frac{6}{7}=?$$

Changing the mixed number to an improper fraction, we have:

$$\frac{3}{8}\div\frac{41}{7}$$

Inverting the divisor and multiplying, we have:

$$\frac{3}{8}\times\frac{7}{41}=\frac{3\times7}{8\times41}=\frac{21}{328}\ \text{Quotient}$$

1.53 The reciprocal of any number is 1 divided by the number. This holds true for whole numbers, fractions, and mixed numbers. For example, the reciprocal of the whole number 8 is $^1/_8$. The reciprocal of the fraction $^2/_3$ is $1\div{}^2/_3$ or $^3/_2$. The reciprocal of any fraction is simply the fraction inverted.

To find the reciprocal of a mixed number, it is first necessary to change it to an improper fraction. The reciprocal of $6\,^3/_7$ is $^7/_{45}$. This was found by first changing $6\,^3/_7$ to $^{45}/_7$. Then: $4^5/_7 = {}^7/_{45}$

Decimals

1.54 Values between 0 and 1 can be expressed by means of either fractions or decimals. All fractions can be written in decimal form, just as all decimals can be expressed in the form of fractions.

A decimal is a number with a period, or decimal point, in front of it, such as .25, .376, and .8. The decimal point is very important, since the value of the decimal depends upon the position of the digits with respect to it.

When a decimal is to be written as a proper fraction, the denominator consists of 1 followed by as many zeros as there are decimal places in the decimal. The number of digits to the right of the decimal point determines the number of decimal places in a decimal. For example, the decimal .235 has three decimal places since there are three digits to the right of the decimal point. The decimal .235 is expressed as a proper fraction as $^{235}/_{1000}$. As you can see, the numerator of the fraction is simply the decimal without a decimal point.

To change a fraction to a decimal, the process is simply reversed. Suppose that it is necessary to change the fraction $^{263}/_{1000}$ to a decimal. The decimal will have as many places as there are zeros in the denominator of the fraction, therefore the decimal will be .263. If the fraction had been $^{263}/_{10,000}$, the decimal would be .0263 since there are four zeros in the denominator.

The conversion of a fraction to a decimal becomes more complicated when the denominator is not a multiple of ten. The method for converting a fraction of this type into a decimal is simply to divide the denominator into the numerator and carry out the quotient to the desired number of decimal places.

The procedure for changing the fraction $^{7}/_{8}$ to a decimal is as follows: 7 divided by 8 equals .87 when carried out two decimal places; .875 when carried out three decimal places; and .8750 with no remainder when carried out four decimal places. Since the last digit in the four place decimal means nothing the appropriate decimal for $^{7}/_{8}$ is .875. The guide that is used in changing fractions to decimals is usually to carry out the decimal no more than four places.

1.55 Each digit to the right of a decimal point occupies a decimal place. The decimal places are of great importance because they indicate the value of the denominator.

The following series of decimals show how the number of decimal places affects the value of the denominator, when the same numerator 3 is used in each decimal:

$$.3 = \frac{3}{10} = 3 \text{ tenths}$$

$$.03 = \frac{3}{100} = 3 \text{ hundredths}$$

$$.003 = \frac{3}{1000} = 3 \text{ thousandths}$$

$$.0003 = \frac{3}{10,000} = 3 \text{ ten - thousandths}$$

$$.00003 = \frac{3}{100,000} = 3 \text{ hundred thousandths}$$

$$.000003 = \frac{3}{1,000,000} = 3 \text{ millionths}$$

1.56 When digits are written both to the left and to the right of a decimal point, such as 38.25, the number is called a **mixed decimal**. It consists of a whole number *38* and a decimal *.25*. This mixed decimal is read as thirty-eight and twenty-five hundredths.

A mixed decimal can be changed to a whole number and a fraction. For example, the mixed decimal 38.25 would be $38\,^{25}/_{100}$. The whole number remains as it is and the decimal is placed over 1 with as many zeros as there are decimal places, in this case, two.

1.57 The first decimal place to the right of the decimal point is the tenths place, the second decimal place the hundredths, the third decimal place the thousandths, the fourth decimal place the ten thousandths and so on. In reading a decimal, the decimal is treated as if were a proper fraction. The numerator is read as if it were a whole number, and then the denominator is stated. For example, the decimal .25 is read as twenty-five hundredths; the decimal .736 is read as seven hundred thirty-six thousandths.

Addition of Decimals

1.58 Decimals are added exactly as whole numbers, except that the decimal points must all be in the same column. If the decimal points are in the same vertical line, tenths will be under tenths, hundredths will be under hundredths, and so. The decimal point in the sum is also placed directly under those in the numbers to be added. For example, suppose that we are to add the following decimals: .25, .376, .1, and .23. First they must be arranged in a column.

```
.25
.376
.1
.23
.956          SUM
```

1.59 The same rule concerning decimal points applies to adding whole numbers and decimals. Whole numbers may be considered as having a decimal point after the last digit. Let's take a sample problem. Add the numbers 263, .28, 3, and .124.

Arranging them in a column with the decimal points in a vertical line, we have:

```
263.
    .28
  3.
   .124
266.404
```

The empty spaces may be considered as zeros. If it will make your work easier, put them in. The problem would then look like this:

```
263.000
000.280
003.000
000.124
266.404       SUM
```

You can see that the inclusion of zeros does not change the value of the numbers.

1.60 The same procedure is followed when adding decimals and mixed decimals. For example, add the following: 23.8, 76.72, 236.5, and 2.323.

Arranging the numbers in a column we have:

```
  23.8
  76.72
 236.5
   2.323
 339.343      SUM
```

You can, of course, include zeros if it will make your work easier.

Subtraction of Decimals

1.61 The subtraction of two numbers when there is a decimal point in either or both of them, is the same as subtraction of whole numbers, except that the decimal points must be in a direct line just as in addition of decimals. For example, suppose that we are to subtract the decimal .314 from the decimal .789. First they must be arranged with the larger number first and the decimal points in line.

```
.789      MINUEND
.314      SUBTRAHEND
.475      REMAINDER
```

1.62 Subtracting mixed decimals follows the same procedure as decimals. Let's take a sample problem. Subtract 36.21 from 98.72. Arranging them with the decimal points vertical we have:

```
98.72  MINUEND
36.21  SUBTRAHEND
62.51  REMAINDER
```

1.63 In order to subtract a decimal from a whole number, it is necessary to add as many zeros to the whole number as there are decimal places in the decimal. As you know, adding zeros to a whole number *after the decimal point* does not change the value of the number. For example, 10, 10.00, and 10.0000 all have the same value, which is, of course, 10.

Suppose that it is necessary to subtract the decimal .238 from the whole number 25. First they must be set up with the decimal points in a vertical line.

```
25.
  .238
```

Then we must add zeros to the whole number. In this case, since there are three decimal places, three zeros will be added.

```
25.000  MINUEND
 .238   SUBTRAHEND
24.762  REMAINDER
```

1.64 The same rule applies to checking a problem involving decimals as one with only whole numbers. Add the subtrahend and the remainder, and the sum of these two numbers should equal the minuend. In the problem just solved, we subtracted .238 from 25 and the remainder was 24.762. To prove this we add the remainder and the subtrahend.

```
24.762  REMAINDER
 .238   SUBTRAHEND
25.000  MINUEND
```

Since the sum of the remainder and subtrahend is 25, which is also the minuend, the solution was correct.

Multiplication of Decimals

1.65 Decimals and mixed decimals are multiplied in the same manner as whole numbers. The two numbers to be multiplied are written one under the other with the **right-hand** digit of each in the same column. The decimal points do not have to be lined up for multiplication. They are disregarded until the product has been found.

For example, if 23.41 is to be multiplied by .223, the problem is set up as follows:

```
  23.41  MULTIPLICAND
   .223  MULTIPLIER
   7023
  46820
 468200
5.22043  PRODUCT
```

1.66 After the product of two numbers has been found, it is necessary to point off as many decimal places in the product as there are in both the multiplicand and the multiplier. In the above problem, for example, there are two decimal places in the multiplicand and three decimal places in the multiplier. Therefore, there will be 2 + 3, or 5, decimal places in the product. Decimal places in the product are pointed off beginning with the right-hand digit and counting off toward the left. The decimal point is placed to the left of the last place counted off. In the above problem the product is 522043, and, since there are five decimal places, the decimal point is placed at the left of the fifth digit, making the product 5.22043. The product of .2341 and .223 would have seven decimal places. This would necessitate adding a zero in front of the first digit to have the required number of decimal places and would result in a product of .0522043.

1.67 When a decimal or a mixed decimal which is to be multiplied contains one or more zeros immediately after the decimal point, as .0023, for example, these zeros have no effect on the digits forming the product. However, they do have an effect on the position of the decimal point in the product, since they must be counted in finding the number of decimal places to be pointed off in the product.

Zeros at the end of a decimal or a mixed decimal may be dropped because they have no effect on the value of the number. For example, 37.00 is the same as 37, .2500 has the same value as .25, and 1.30 is equal to 1.3

This can be illustrated by solving a *sample problem*:

Multiply .92500 by .313.

Solution.
According to the preceding statement the last zeros in the decimal .92500 could be omitted without affecting the product. The correctness of this statement will be proved by first multiplying the decimals in their present forms and then multiplying without the zeros in the multiplicand, the products being the same in each case.

In the solution (**A**) there must be eight decimal places in the product, eight being the sum of the decimal places in the multiplicand multiplier. If the two zeros are omitted from the multiplicand, as in (**B**), only six decimal places are required in the product. The two products are equal, because the two zeros to the right of the decimal in the solution (**A**) have no value.

A.

```
     .92500 MULTIPLICAND
       .313 MULTIPLIER
     277500
     925000
   27750000
  .28952500 PRODUCT
```

B.

```
       .925 MULTIPLICAND
       .313 MULTIPLIER
       2775
       9250
     277500
    .289525 PRODUCT
```

Division of Decimals

1.68 The methods to be used in the division of decimals are the same as those used for the division of whole numbers. The only point to be considered is the location of the decimal point in the quotient.

1.69 When the divisor is a whole number, the decimal point in the quotient is directly over the decimal point in the dividend. For example, divide the mixed decimal 3.69 by 3 which is a whole number.

```
    1.23
3 | 3.69
```

You can see that the decimal point has been placed in the quotient directly above the decimal point in the dividend.

In some cases it is necessary to place a zero in the first decimal place of the quotient. This is done when the divisor will not be contained in the first digit of the dividend. The following problem is an example of this.

Divide .875 by 25.

Solution:
Since 25 is not contained in 8, zero must be place in the quotient immediately after the decimal point. 25 is contained in 87, however, and the division is carried on as in any other problem. It is essential that this zero be placed in the quotient, since the quotient must have the same number of decimal places as the dividend.

```
      .035        QUOTIENT
25 | .875
       750
       125
       125
```

1.70 When the divisor is not a whole number, it must be changed to a whole number by moving the decimal point to the right as many places as necessary. When the decimal point in the divisor is moved, the decimal point in the dividend must also be moved the same number of places. To indicate the change in the position of the decimal point, write the dividend and divisor as they are given, and insert a caret ∧ in the dividend where the decimal point would be if it were actually moved. Suppose, for example, that we are to divide the mixed decimal 56.7055 by the mixed decimal 2.35.

Since there are two decimal places in the divisor, the decimal point in the dividend must be moved two places to the right. A caret is inserted at this point and a decimal point is placed in the quotient directly above it. The division is then carried out through the divisor was 235 and the dividend was 5670.55.

```
        24.13          QUOTIENT
2.35 | 56.70∧55
       47 0
        9 70
        9 40
          30 5
          23 5
           7 05
           7 05
```

1.71 When the dividend has fewer decimal places than the divisor, zeros must be added to the dividend so that it is possible to move the decimal point the required number of places. For example, in order to divide .235 into 67.2, two zeros must be added since the dividend has only one decimal place and three are required.

```
.235 | 67.200 ∧
```

1.72 More often than not the divisor does not divide into the dividend an even number of times. When this occurs you must determine how far you wish to carry out

the division process to get the desired number of decimal places in the quotient. When this situation occurs carry the quotient one more digit than the number of digits required. This extra digit, which you will discard later, is used to obtain a more accurate value for the digit preceding it. If the extra digit is 0, 1, 2, 3, or 4, drop it and use the other digits as they were found. In other words, if the extra digit is less than 5, the quotient is rounded off merely by dropping the extra digit and using the other digits. On the other hand, if the extra digit is 5, 6, 7, 8, or 9, still drop it, but the preceding digit is increased by adding 1 to it. That is, if the extra digit is 5 or a digit greater than 5, the result is rounded by dropping the extra digit and adding 1 to the digit just to the left of it.

This procedure can be used for rounding no matter what operation is involved.

Example Problem:
Perform the following division operation.

```
.703 | 200.
```

Solution:
In this example at least three zeros must be added to the dividend in order to move the decimal point three places.

```
.703 | 200.000^
```

Upon inspection it can be found that the quotient can be carried out almost indefinitely. For this reason we will choose to round our quotient to the nearest whole number. Therefore, we will carry out the quotient to the tenths place in order to round to the nearest whole number.

```
        0000284.4
.703 | 200.000  0
       140 6
        59 40
        56 24
         3 160
         2 812
           348 0
           281 2
            66 8
```

Since the tenths digit is 4 the quotient can be rounded to 284.

1.73 You will recall that a decimal can be changed to a common fraction by placing it over 1 followed by as many zeros as there are decimal places. For example, the decimal .25 is equal to the common fraction $^{25}/_{100}$.

A common fraction is changed to a decimal by dividing the denominator into the numerator. The fraction $^{1}/_{2}$ is changed to a decimal by dividing 2 into 1.

```
    .5
2 | 1.0
```

Following is a table of decimal equivalents (*Table 5*) all of which are calculated by method just described. *Table 5* is convenient for reference purposes.

Table 5
Table of Decimal Equivalents

$^{1}/_{8}$	=	.125	$^{1}/_{32}$	= .03125
$^{1}/_{4}$	=	.25	$^{3}/_{32}$	= .09375
$^{3}/_{8}$	=	.375	$^{5}/_{32}$	= .15625
$^{1}/_{2}$	=	.5	$^{7}/_{32}$	= .21875
$^{5}/_{8}$	=	.625	$^{9}/_{32}$	= .28125
$^{3}/_{4}$	=	.75	$^{11}/_{32}$	= .34375
$^{7}/_{8}$	=	.875	$^{13}/_{32}$	= .40625
			$^{15}/_{32}$	= .46875
$^{1}/_{16}$	=	.0625	$^{17}/_{32}$	= .53125
$^{3}/_{16}$	=	.1875	$^{19}/_{32}$	= .59375
$^{5}/_{16}$	=	.3125	$^{21}/_{32}$	= .65625
$^{7}/_{16}$	=	.4375	$^{23}/_{32}$	= .71875
$^{9}/_{16}$	=	.5625	$^{25}/_{32}$	= .78125
$^{11}/_{16}$	=	.6875	$^{27}/_{32}$	= .84375
$^{13}/_{16}$	=	.8125	$^{29}/_{32}$	= .90625
$^{15}/_{16}$	=	.9375	$^{31}/_{32}$	= .96875

Exercises

Use these exercises to help check your learning progress

Questions	Answers
1.11 In a fraction the number above the line is called the _____, and the number below the line is called the ____.	**1.11** numerator denominator Ref. 1.27
1.12 When the numerator of a fraction is a smaller number than the denominator the fraction is called a _____ _____.	**1.12** proper fraction Ref. 1.29
1.13 If the numerator is equal to or greater than the denominator, the fraction is called an ____.	**1.13** improper fraction Ref. 1.29
1.14 An improper fraction is changed to a mixed number by dividing the _____ by the ____.	**1.14** numerator denominator Ref. 1.34
1.15 When fractions are to be added, they must all have the same ____.	**1.15** denominator Ref. 1.35
1.16 In solving problems involving fractions it is customary to find the smallest number exactly divisible by all the denominators. This number is called _____ _____ _____.	**1.16** least common denominator Ref. 1.37
1.17 The process of multiplying fractions can often be simplified and shortened by a method called ____.	**1.17** cancellation Ref. 1.49

1.18 The phrase "_____ _____ _____" simply means turn it upside down, or reverse the order of numerator and denominator.	**1.18** invert the divisor Ref. 1.50
1.19 A decimal is a number with a _____ _____ in front of it, such as .25, or .8.	**1.19** decimal point Ref. 1.54
1.20 .3 = _____ = _____ .03 = _____ = _____ .003 = _____ = _____	**1.20** $^3/_{10}$, 3 tenths $^3/_{100}$, 3 hundredths $^3/_{1000}$, 3 thousandths Ref. 1.55

Summary

In this lesson we have studied what numbers are, how to write them, and how to read them. We have studied basic computation involving addition, subtraction, multiplication, and division. We then studied the rules for working with fractions, and decimals. Then we learned how to convert from fractions to decimals and from decimals to fractions.

You will find that mathematics is a useful tool in your engineering studies. Like any tool, mathematics must be understood to be used properly.

Glossary of Terms

Borrowing - The act of moving a number from one column, such as the tens column to the next lower column, such as the units column. This is done to increase the value of the number in the units column so that an originally larger number may be subtracted from it.

Cancellation - A procedure used to simplify the multiplication of fractions. If a number in the numerator is equal to or a whole multiple of a number in the denominator, one of the numbers may be divided into the other. If the two numbers are of equal value, they cancel out. If one is larger than the other, then there is a remainder.

Common Denominator - When fractions are added or subtracted they must be transformed so that they all have the same or common denominator.

Denominator - The number in the bottom half of the fraction.

Difference - Another name used for the remainder in a subtraction problem.

Dividend - The number which is to be divided into equal portions by the divisor.

Division Sign (÷) - A symbol indicating one number is to be divided into another. Division is also sometimes indicated by placing a slash bar between two numbers. i.e., $^{15}/_{3} = 5$.

Divisor - The number which is used to divide another number.

Fractional Parts - Portions of an object or quantity which has been divided into smaller pieces.

Fractions - Numbers used to indicate fractional parts.

Hundreds Column - The third column of numbers to the left of the decimal.

Improper Fraction - A fraction in which the numerator is larger than the denominator.

Least Common Denominator - The smallest number that can be used as the common denominator for a group of fractions.

Minuend - The number or quantity from which another number is to be subtracted.

Minus Sign - The symbol "-" used to indicate one number is to be subtracted from another.

Mixed Decimal - A number with digits on both sides of the decimal point. Such a number can be considered to be the combination of a whole number plus a decimal number.

Mixed Number - A number consisting of both whole numbers and fractions.

Multiplicand - A number which is to be multiplied by another.

Multiplication Sign - There are several symbols used to indicate multiplication. Probably the most common is the symbol "x", sometimes a dot is used, provided one is not multiplying decimal numbers. Placing numbers in parentheses next to each other indicates multiplication i.e., (5)(2) = 10. Also in algebra 5xy indicates multiplying 5 times x times y.

Multiplier - The number which is multiplied times the multiplicand.

Numerator - The number in the top half of the fraction.

Plus Sign - The symbol "+" used to indicate addition of numbers.

Product - The answer or result of multiplication.

Proper Fraction - A fraction in which the numerator is smaller than the denominator.

Quotient - The answer or solution to a division problem.

Remainder - The number or quantity left or remaining following subtraction. In subtraction, the answer is the remainder. Remainder is also used to describe the quantity left over or remaining following division of one number by another.

Subtrahend - A number or quantity which is to be subtracted from another number.

Sum - The answer or result of addition.

Tens Column - The second column of numbers to the left of the decimal.

Total - Used in the same sense as sum. Total is the entire or complete quantity of something.

Units Column - The first column of numbers to the left of the decimal.

Self-Check Quiz

Questions	Answers
1.01 By what process do we get a sum of two or more numbers?	**1.01** The process of addition
1.02 Add 719, 218, 45, 5629, 29451, 666, 1778, and 294.	**1.02** 38,800
1.03 Subtract 3456 from 6507.	**1.03** 3,051
1.04 Multiply 5316 by 4.	**1.04** 21,264
1.05 Divide 985,736 by 2.	**1.05** 492,868
1.06 What must be done to the denominators of fractions before we can add?	**1.06** They must be identical.
1.07 What is a fraction?	**1.07** A fraction is a method of expressing part of a unit.
1.08 Express the following decimals as fractions: .25, .2, .75.	**1.08** $^1/_4$, $^1/_5$, $^3/_4$
1.09 Add .25, .376, .1 and .23.	**1.09** .956
1.10 Subtract .782 from .983.	**1.10** .201

1

Mathematics - Basic Computation

Objective

To provide the student with a review of basic mathematics.

Introduction

Reference 1.01 through 1.06

A number is anything that answers the question, _____? and _____?

A number is made up of several figures, each figure is called a _____.

One of the first things to be learned in arithmetic is _____. ***(Table 1)***

The value represented by a digit depends on its _____ in relation to other digits.

When writing numbers, set numbers off in groups of _____ digits

Read numbers from the _____. ***(Table 2)***

A _____ calculator is recommended for use in performing mathematical computations.

Addition

Reference 1.07 through 1.11

The process of finding a number whose value is equal to the combined values of two or more other numbers is called _____. ***(Table 3)***

Define the following ***Glossary Terms***:

Plus sign_____

Sum_____

Total_____

All numbers to be added must be arranged in _____.

Practice:

In the number 1,526:

the digit in the units column is _____

the digit in the tens column is _____

the digit in the hundreds column is _____

Define the following ***Glossary Terms***:

Hundreds column_____

Tens column _____

Units column_____

Subtraction

Reference 1.12 through 1.15

_____ is the process of finding how much is left when one number is taken from another.

When you have a certain number of things and you want to find out how many more you need for a desired number, you _____.

Define the following ***Glossary Terms***:

Borrowing _____

Difference _____

Minuend _____

Minus sign _____

Remainder _____

Subtrahead _____

Practice:

$$\begin{array}{r} 52 \\ \underline{-\,24} \end{array}$$

Multiplication

Reference 1.16 through 1.19

A shortcut method of adding equal numbers is called _____. ***(Table 4)***

Define the following ***Glossary Terms***:

Multiplicand _____

Multiplication sign (times sign) _____

Multiplier _____

Product _____

Writing multiplication problems

Practice:

$$\begin{array}{r} 358 \\ \times\,\underline{714} \end{array}$$

Check multiplication problems by reversing the order of the _____

and _____.

1

Division

Reference 1.20 through 1.24

The process of finding how many times one number is contained in another is called _____.

Division is the _____ of multiplication.

Define the following ***Glossary Terms***:

Dividend _____

Division sign _____

Divisor _____

Quotient _____

Solving division problems

Practice:

Short division:

Divide 936 by 3 (936 ÷ 3)

Long division:

Divide 289,456 by 458 (289,456 ÷ 458)

Check division problems by multiplying the quotient by the divisor

Practice: (refer to the long division practice problem above)

_____ x 458 = 289,456

Fractions

Reference 1.25 through 1.34

When any object is divided or cut into parts, each of the resulting parts is _____ than the whole object. ***(Figure 1)***

Define the following ***Glossary Terms***:

Denominator _____

Fractional parts _____

Fractions _____

Numerator _____

Fractions

Reference 1.25 through 1.34

Fractions are actually division problems:

1/4 = 1 ÷ _____

Write 320 ÷ 765 as a fraction _____

Types of fractions

Define the following ***Glossary Terms***:

Improper fraction _____

Mixed number _____

Proper fraction _____

Match the following terms with the proper expression

Proper fraction	5 2/8
improper fraction	3/4
Mixed number	5/2

RULE: If both the numerator and the denominator of a fraction are multiplied or divided by the same number, the value of the resulting fraction is the same as that of the original fraction. ***(Figure 2)***

Changing a mixed number to an improper fraction and vice versa

Practice: Change 5 3/4 to an improper fraction

Step 1:

Step 2:

Step 3:

_____

Addition of Fractions

Reference 1.35 through 1.40

Define the following ***Glossary Terms***:

Common denominator _____

Least common denominator (Figure 3) _____

Practice: Find the least common denominator for 1/4, 1/3, 1/9, and 1/16

_____

1

Subtraction of Fractions

Reference 1.41 through 1.44

Subtraction of Fractions

Practice:

Subtract 1/4 from 25/64

Subtract 6 7/8 from 10 2/3

Subtract 7/12 from 8

Multiplication of Fractions

Reference1.45 through 1.49

Multiplication of Fractions

Practice:

3/8 x 1/2 =

8 x 2/3 =

3 2/3 x 7/8 =

2/3 x 3/4 x 1/2 =

Cancellation

Practice: use cancellation to solve the following problem

$$\frac{3 \times 2 \times 1}{8 \times 3 \times 4} =$$

Division of Fractions

Reference 1.50 through 1.53

Division of Fractions

Practice: invert the divisor and multiply

4/5 ÷ 3/2 =

3/8 ÷ 5 6/7 =

Decimals

Reference 1.54 through 1.57

Values between _____ and _____ can be expressed by means of either fractions or decimals.

All fractions can be written in _____ form and vice versa.

Practice: Write the following fractions as decimal.

1/4 = _____

5/10 = 1/2 = _____.

Practice: Write the following decimals as proper fractions.

.35 = _____

.4 = _____

Decimal places

Practice:

.3 = 3/10 = 3 _____

.003 = 3/100 = 3 _____

.000003 = 3/1,000,000 = 3 _____

Mixed decimals

Practice: Change 38.25 to a whole number and a fraction:

_____

Addition of Decimals

Reference 1.58 through 1.60

Addition of Decimals

Practice:

.25 + .376 + .1 + .23 =

263 + .28 + 3 + .124 =

23.8 + 76.72 + 236.5 + 2.323 =

Subtraction of Decimals

Reference 1.61 through 1.64

Subtraction of Decimals

Practice:

.789 - .314 =

98.72 - 36.21 =

25 - .238 =

Multiplication of Decimals

Reference 1.65 through 1.67

Multiplication of Decimals

Practice:

23.41 x .223 =

.92500 x .313 =

Division of Decimals

Reference 1.68 through 1.73

Division of Decimals

Practice:

3.69 ÷ 3 =

.875 ÷ 25 =

56.7055 ÷ 2.35 =

Table of Decimal Equivalents ***(Table 5)***