The Role of Inequality in the Response of Consumption to a Credit Deepening*

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Abstract

We study how income and wealth inequality affect the transmission mechanism of a credit deepening to consumption in a heterogeneous-agents incomplete market model. After a one-time unexpected shock that increases the borrowing capacity of households, there is a short-run consumption "boom" and a subsequent persistent "bust" due to household leverage. The reaction of consumption to the credit deepening is decomposed by two channels: a direct channel measured by credit MPCs of households and the debt tradeoff; and an indirect channel led by reactions to prices. The direct channel is quantitatively more relevant, driving the bulk of the consumption response, and amplifies in the presence of bigger wealth inequality. On the other hand, the indirect channel is relevant for the recovery of household demand, and also amplifies with wealth inequality, leading to a quicker recovery. Moreover, the empirical analysis concludes that credit-induced consumption cycles are at least three times more elastic when accounting for higher inequality.

JEL Codes: D52, D3, E21, E44, E30.

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1 Introduction

Increasing access to credit in the economy has been a global phenomenon in the past five decades (Müller and Verner 2023), particularly with the vertiginous rise of credit as a share of GDP in the 21st century. This phenomenon is known as "credit deepening", which Carvalho et al. (2023) describe as the permanent increase in the level of private credit in the economy that results from institutional changes, and is often cited as a source of above-trend growth in emerging-market economies[1]. Given that most of this process was led by household credit, as argued by Müller and Verner (2023), household demand for more consumption fostered above-trend growth.

Considering the widely mapped relationship between inequality and its effects on aggregate consumption behavior, we turn to investigate whether the fact of emerging-markets having more inequality could drive above-trend consumption growth[2] in a credit deepening when compared to less unequal peers. This is motivated by the idea that in these countries, less developed credit markets and a higher share of constrained households facing income growth will benefit more from an expansion of credit. Moreover, it is also based on the widely studied importance of household inequality in the propagation of macroeconomic shocks. Therefore, this paper seeks to address whether inequality affects the transmission mechanism of credit deepening to aggregate consumption, and if it does, scrutinize which sources of inequality matter, and through which channels do they act.

To tackle the question at hand, this paper relies on an incomplete-markets *Heterogeneous Agents* model with household heterogeneity in terms of preferences, income and wealth. Within this framework, a credit deepening is a one-time unexpected shock to the borrowing constraint of all households, which react in distinct ways. We take advantage of the rich setting of household heterogeneity to study the channels of a credit deepening to consumption and how they interact with different dimensions of inequality.

At the individual level, we show the role of inequality in the response of household consumption to a credit deepening with a proposition that answers how much will a household consume out of an increase in the supply of credit (Credit MPCs). This response is composed of two components: the current marginal propensity to consume (MPC) of the household, a widely used statistic in the *Heterogeneous Agents* literature, and its response to future consumption loss. The former captures the importance of individual wealth holdings, saving behavior and precautionary savings, whereas the latter

[1] As in the case of Latin America in the 21st century, as documented by Carvalho et al. (2023)

[2] In the 21st century, emerging markets have been contributing more to global consumption growth than advanced economies. Mirae Asset apud IMF, available here

captures the intertemporal consumption smoothing trade-off. Thus, the intuition for the role of inequality in credit deepening becomes simple, as inequality affects the response of consumption through the distribution of credit MPCs and the share of constrained households. Hence, a source of inequality matters inasmuch as it affects these two components.

The intuition aforementioned describes the role of inequality in the transmission of credit to consumption through the *direct channel* of credit. However, heterogeneity also affects the response of prices in the economy, which in turn explains the *indirect channel* of credit. Hence, we study both channels in a simple general equilibrium framework of Aiyagari (1994), relying on the methodology of Kaplan et al. (2018) to decompose their effects on aggregate consumption. We employ alternative calibrations of the initial distribution of income and wealth in the economy to study the impact of higher inequality on the response of consumption throughout the transition path through each of its channels. This analysis concludes that wealth inequality drives the amplification of the response of consumption to a credit deepening through the direct channel of credit, but is also responsible for a stronger recovery of consumption through the intertemporal substitution effect.

We base our analysis on high-quality micro data on income and wealth to develop quantitative assessments. Moreover, we take advantage of their cross-country availability and comparability to extend our exercise to a cross-country setting by calibrating our model to advanced and emerging markets. Such an approach is novel in the literature and made possible by the availability of micro data on income by the Global Repository of Income Dynamics (GRID) (Guvenen et al. 2022) as well as wealth data by the World Inequality Database (WID) (Chancel et al. 2021).

We conclude by investigating whether the quantitative results and the channels we investigated through our model have empirical validation. By using Jordà (2005) Local Projections adjusted to capture the nonlinear relationship between credit and inequality, we conclude that credit-induced consumption cycles are at least three times more elastic (in expansions and recessions) when accounting for higher inequality. Such results also hold in our robustness exercise, where we study the same relationship on consumption growth trends via single-equation estimations.

This article finds its way into the literature of credit variation as a source of macroeconomic shocks, focusing on a credit deepening instead of a credit crunch, and making a bridge with inequality that was previously unexplored. It also uses state-of-the-art modeling of household heterogeneity calibrated with high-quality micro data to developing and developed economies, in contrast with previous models which were oriented towards developed economies and/or had limited heterogeneity. Most of the literature on the effects of credit as a macroeconomic shock was born in the aftermath of the Great Financial Crisis (GFC), with one branch of the literature focusing on spending multipliers the the liquidity trap - see Christiano et al. (2009), Woodford (2010) and Werning (2011); whilst other was focused households' balance sheet adjustment and consumer spending.

This branch was focused on explaining to which extent the rapid increase of household debt in the years leading up to the 2008 GFC was a factor that led to the recession and its slow recovery. Jones et al. (2011) argues that the role of credit was more of an amplification of macroeconomic shocks, as it played a lesser role in explaining changes in employment but a major role in a slow recovery. Mian and Sufi (2011) use geographic variation to show how the home equity borrowing channel[3] drove the rise in household leverage when house prices soared before the GFC, while Mian et al. (2013) estimate MPCs with respect to housing wealth and explores how a contraction in households borrowing capacity, caused by a fall in house prices, led to a slump on consumer spending and a rise of unemployment.

The canonical article of Eggertsson and Krugman (2012) tries to conciliate the two branches in one that studies credit shocks as a driver of aggregate demand on business cycles, and does such by establishing a relation between credit crisis and liquidity trap. It was also one of the first to incorporate heterogeneity, albeit limited (the presence of borrowers and lenders with distinct discount rates), whose usage in the literature further evolved from that point onward. Justiniano et al. (2013) use a similar heterogeneity in the model (working with two types of households) with the intent of matching debt dynamics over a credit cycle to U.S. data, and does such by shocking preferences for housing services to generate credit cycles with leveraging and deleveraging.

In the spirit of understanding the GFC and housing crisis, Huo and Ríos-Rull (2016) further incorporate heterogeneity by establishing a Bewley model with goods, labor and financial market frictions, as well as capital and housing as assets (despite housing also being in the utility function) but without transaction costs. In their model, Financial shocks increase the difficulty of house-collateralized credit, which rebounds on prices and depresses even more credit conditions. In this context, wealth distribution plays an important role in the effect of a credit shock depending on the quantity of households constrained. Guerrieri and Lorenzoni (2017) use a similar Bewley model with price rigidities, but one asset only and without collateralized debt, to delve into the effects of a one-time credit crunch. Similar to us, they capture two channels in the consumer's

[3] borrowing with housing as collateral

response to a credit shock: direct and precautionary channels. Following a credit shock, consumers in the lower end of the wealth distribution are forced to deleverage while others increase their precautionary savings, reducing consumption and increasing employment. The shock is also amplified by the movement in prices, and composition changes in magnitude when dissecting consumption of durables and non-durables.

Plentiful research has also been conducted on credit and debt as a source of amplification of macroeconomic shocks on aggregate demand, as in Cloyne et al. (2016), where the response of consumption to monetary policy is driven by households with mortgages as mortgagors hold sizable illiquid assets but small liquid assets, being more sensitive to changes in interest rates. Flodén et al. (2016) instead examine how monetary policy is driven by households whose debt is linked to short-term rates and thus are more sensible to debt service. Others took a more empirical approach, as for instance Schularick and Taylor (2012) use a historically long cross-country database to document that credit booms are indicative of a heightened risk of financial crisis. Whereas Mian et al. (2017) focus on how credit and household debt influence business cycles. They document that an increase in household debt predicts short-run growth booms and a subsequent reversal in debt and lower GDP growth, as well as other facts about household booms and possible theories behind them. However, Müller and Verner (2023) look at the factors why some credit cycles lead to busts and others not. They document that credit expansions focused on firms of non-tradable sectors and households lead to busts, whereas those focused on firms of tradable sectors lead to sustained growth without financial risk.

As heterogeneity gets even more present in the literature that studies credit, household balance sheets and business cycles, MPCs have become a core statistic of the literature. Auclert et al. (2018) demonstrate how this statistic is the key element that disciplines general equilibrium models with heterogeneous agents and nominal rigidities, and that dynamic effects are led by intertemporal MPCs. However, he emphasizes that only two-asset models have managed to match empirical estimates of iMPCs successfully. This is explained by Kaplan and Violante (2022) as one asset models have a trade-off when matching empirical counterparts: either it matches high average MPCs by generating an excessively polarized wealth distribution[4] or a realistic level of aggregate wealth. The solution so far, as he points out, is the usage of heterogeneous models with liquid and illiquid assets with a return differential. For a comprehensive survey on MPCs and their usage in calibration and study of macro shocks in business cycles, see Kaplan and Violante (2018) and Kaplan and Violante (2022).

[4]In the sense that it understates the wealth attained by households in the middle of the distribution

The rest of the article is organized as follows. In Section 2, we present the model and study analytically the relationship between credit, consumption and inequality at the household and aggregate level. In Section 3, we present the calibration of the model and analyze distribution of income, wealth and micro-level consumption behavior. Section 4 we study the response of aggregate consumption to a credit deepening, quantifying the response through each channel and using counterfactual analysis to assess the role of inequality in the transmission mechanisms. Section 5 we study whether short and long-run patterns of consumption "boom" and "bust" cycles are present after household debt shocks, and if they amplify with inequality as in the model. Finally, Section 6 concludes and lays down the next steps of the project

2 Model

In this section, I describe the general equilibrium model used to study how does aggregate and household-level consumption reacts to a credit deepening when controlling for different sources of heterogeneity. Time is continuous and the economy is closed with neither aggregate risk nor uncertainty. The risk is at the individual level, where a continuum of households faces idiosyncratic income risk and incomplete credit markets. They supply labor inelastically to consume and save in liquid assets (capital) for precautionary measures. Besides them, there are two agents in the economy: a representative firm that interacts with the continuum of households through capital and labor markets, hiring labor and capital to produce a final consumption good; and a government with an insurance role, which taxes labor earning redistributes it through lump-sum transfers.

As one of the contributions of this work is to map which dimension of inequality is relevant in driving the relationship between credit and consumption, I employ a model with a rich setting in terms of the cross-section heterogeneity of households: There is preference heterogeneity through discount rates; individual income fixed effects drawn at birth; and multidimensional uninsurable idiosyncratic risk. These elements generate endogenous dispersion in the distribution of asset holdings of households and consumption-savings behavior, both of which are important for the propensity of households to take credit and consume out of it.

2.1 Households

The economy is populated with a continuum of infinitely-lived households with time-separable CRRA preferences. Each household is born with a fixed effect ω_i, $\forall i \in \mathbb{I}$, and a discount rate ρ_j, $\forall j \in \mathbb{J}$. Throughout his life, the household is subject to idiosyncratic income risk through its labor productivity z, which follows a Markov process and reflects permanent and transitory changes to income. Consequentially, households have precautionary motives and save in the form of liquid assets (capital), but do so in the presence of incomplete markets. In addition, they supply labor inelastically, which is taxed by the government at the tax rate τ_t, but also receive lump-sum transfers T_t as a form of universal cash transfers.

Households are indexed by their individual fixed effect i, their discount rate j, their idiosyncratic labor productivity z and holding of liquid assets a, with preferences given by

$$\mathbb{E}_0 \left[\int_0^\infty e^{-\rho_j t} u(c_{ijt}) dt \right] \tag{1}$$

where the expectations are taken over the households' realization of idiosyncratic productivity shocks. In the aggregate, idiosyncratic risks disappear, so there is no economy-wide uncertainty. Each household accumulates a liquid asset $a_{i,j,t}$ which, given consumption, follows the law of motion:

$$\begin{aligned} \dot{a}_{i,j,t} &= (1-\tau) w_t z_{i,t} + r_t a_{i,j,t} - c_{i,j,t} + T_t \\ a_{i,j,t} &\geq -\underline{a}_t \end{aligned} \tag{2}$$

where $\underline{a}_t \geq 0$ is an ad-hoc borrowing constraint due to incomplete credit markets. Households maximize (1) subject to prices of the economy $\{r_t, w_t\}$, their individual wealth law of motion and borrowing constraint (2), as well as the law of motion for idiosyncratic labor productivity $z_{i,t}$ (19) detailed in Section 3. Define the value function of the household of fixed effect type i and discount rate j as:

$$V_{i,j}(a, z, t) = \mathbb{E}_t \left[\int_t^\infty e^{-\rho_j s} u(c_{i,j,s}) ds \right] \tag{3}$$

due to the fully markovian structure of the problem of the household, this value function satisfies the following HJB equation:

$$\rho_j V_{i,j}(a,z,t) = \max_{c_{i,j} \geq 0} \left\{ u(c_{i,j,t}) + \partial_a V_{i,j}(a,z,t) s_{i,j}(a,z,t) + \mathcal{A} V_{i,j}(a,z,t) + \partial_t V_{i,j}(a,z,t) \right\} \tag{4}$$

where $\mathcal{A}$ is an arbitrary infinitesimal generator for the income process related to z. Moreover, the HJB equation is subject to the following state constraint boundary conditions:

$$\partial_a V_{i,j}(\underline{a}, z) \geq u'(y_{i,t} + r_t \underline{a}_t) \ \forall z, i, t \tag{5}$$

where $y_{i,t} = (1 - \tau_t) w_t z_i$ is the income of the household. The function $s_{i,j}(a,z)$ is the savings policy function, which is the optimally chosen drift of wealth, and $c_{i,j}(a,z)$ is the consumption policy function. These two can be obtained by the envelope condition, and are characterized by:

$$\begin{aligned} s_{i,j}(a,z,t) &= (1 - \tau_t) w_t z_i + r_t a + T_t - c_{i,j}(a,z,t) \\ c_{i,j}(a,z,t) &= (u')^{-1} \left(V_{i,j}(a,z,t) \right) \end{aligned} \tag{6}$$

From the individual decisions of agents with regards to consumption and savings in (4), and the dynamics of the idiosyncratic income risk in (19), the joint distribution of individual productivity and wealth for a given type $(i,j) \in \mathbb{I} \times \mathbb{J}$, $g_{i,j}(a,z,t)$ evolves in time as determined by the Kolmogorov-Forward equation:

$$\partial_t g_{i,j}(a,z,t) = -\partial_a (s_{i,j}(a,z,t) g(a,z,t)) + \mathcal{A}^* g_{i,j}(a,z,t) \tag{7}$$

where $\mathcal{A}^*$ is the adjoint operator of the infinitesimal generator $\mathcal{A}$. The Mean-Field Game system in (4), (5) and (7) dictates the evolution of micro-level consumption and saving decisions, as well as the evolution of joint distribution of wealth and prices of the economy[5]. In the aggregate, the measure of households $\mu_t(di, dj, da, dz)$ describes the state of economy at time t, and is comprised of aggregating all the distributions $g_{i,j}(a,z,t)$.

2.2 Credit deepening and household consumption

In this subsection, I answer two questions that are relevant for the role of inequality for household consumption in the presence of a credit deepening: (i) how much does the household consume out of an infinitesimal increase in the access to credit; and (ii) how does each source of heterogeneity in the cross-section affects this propensity of the household to take extra credit and consume. To proceed with these questions, it is necessary to first define what is a credit deepening:

Definition 1 (Credit Deepening)**.** A credit deepening is defined a path of borrowing constraints that increases the access of liquidity in the economy. In formal terms, for any

[5]The full forms of the HJB and KF equations with the income process (19) described in Section 3, as well as their respective derivations, are present in the appendix

given t, is defined as $\{\underline{a}_{t+s}\}_{s\geq 0}$ such that $\forall s > 0$, $\underline{a}_{t+s} \geq \underline{a}_t$

That is, a credit deepening is when there is an economy-wide increased access to more credit by households, independent of their individual types, income or asset holdings. To answer the question (i), we resort to Proposition 1, which maps the marginal change in the consumption of the household given a credit deepening to the current marginal propensity to consume (MPC) of the household corrected by a term which "captures" future consumption loss through debt. For the sake of simplicity, I proceed dropping time subscripts t, as it applies for any period of time.

Proposition 1 (Credit MPCs)**.** *The household policy function $c_{i,j}(a,z)$ that solves the Household problem satisfies the following identity with respect to the borrowing constraint $\underline{a}$:*

$$\frac{\partial c_{i,j}}{\partial \underline{a}}(a,z;\underline{a}) = MPC_{i,j}(a,z;\underline{a}) - r\frac{\partial c_{i,j}}{\partial T}(a,z;\underline{a})$$

where $MPC_{i,j}(a,z) = \frac{\partial c_{i,j}}{\partial a}(a,z)$ is the current MPC of the household, and T is a lump-sum transfer.

Proof. See appendix A.1.1 □

Therefore, the static response of consumption to credit is heterogeneous and driven by a key statistic of the literature, the current MPC. On the other hand, the second term has its force weighted down by the interest rate, only becoming relevant as the individual gets wealthy, which is when $MPC \to r$. The second term also captures that wealthy individuals may increase savings in the presence of a credit deepening, implying that not only responses are heterogeneous in magnitude, but in sign as well. The Figure 4 plots the distribution of Credit MPCs calibrated to the US, showing a more broad perspective of the forces behind the heterogeneity of responses.

Given the importance of MPCs in driving the household response of consumption to credit, we borrow from (Achdou et al. 2022) to study how each source of heterogeneity affects Credit MPCs for heavily constrained individuals.

Proposition 2 (Decomposing MPCs and heterogeneity)**.** *If $r < \rho$ and the coefficient of absolute risk aversion $R(c)$ remains finite at the borrowing constraint, then as the asset holdings of the household approach the borrowing constraint, $a \to \underline{a}$, the MPC of the household is asymptotically* *equivalent to:*

$$iMPC_{i,j}(a,z) \sim r + \frac{1}{2}\sqrt{\frac{2\nu_{i,j}(z)}{a - \underline{a}}}$$

where $\nu_{i,j}(z)$ *can be decomposed in two parts:*

$$\nu_{i,j}(z) = (\rho_j - r) \times IES \times \underline{c}_{i,j}(z) + \frac{\mathcal{A}(u'(\underline{c}_{i,j}))}{u''(\underline{c}_{i,j}(z))}$$

where $\mathcal{A}$ *is the infinitesimal generator of the stochastic process,* $IES = -\frac{u'(c)}{cu''(c)}$ *the intertemporal elasticity of substitution and* $\underline{c}_{i,j}(z) := c_{i,j}(\underline{a}, z)$

The mapping between sources of ex-ante heterogeneity and income inequality to consumption and credit behavior are done by the object $\nu_{i,j}(z)$, which can be decomposed in two terms: the certainty component, which does not involve risk and tells how the household adapts its consumption to its current state variables; and the uncertainty component, which captures the precautionary effect of income risk on consumption decisions.

$$\nu_{i,j}(z) = \underbrace{(\rho_j - r) \times IES \times \underline{c}_{i,j}(z)}_{\text{Certainty Component}} + \underbrace{\frac{1}{u''(\underline{c}_{i,j}(z))}\mathcal{A}(u'(\underline{c}_{i,j}))}_{\text{Uncertainty Component}}$$

In the certainty component lies the most of the influence of ex-ante fixed effects over consumption and savings behavior: A higher discount rate ρ_j will, for instance, imply a uniformly higher MPC of the household, in line with a lower wealth-to-income target. On the other hand, the relationship with ex-ante income fixed effects becomes more complicated: with respect to the certainty component, it depends on the absolute risk aversion $\mathcal{R}(\underline{c})$ and the absolute prudence $\mathcal{P}(\underline{c})$ at the borrowing constraint[6]

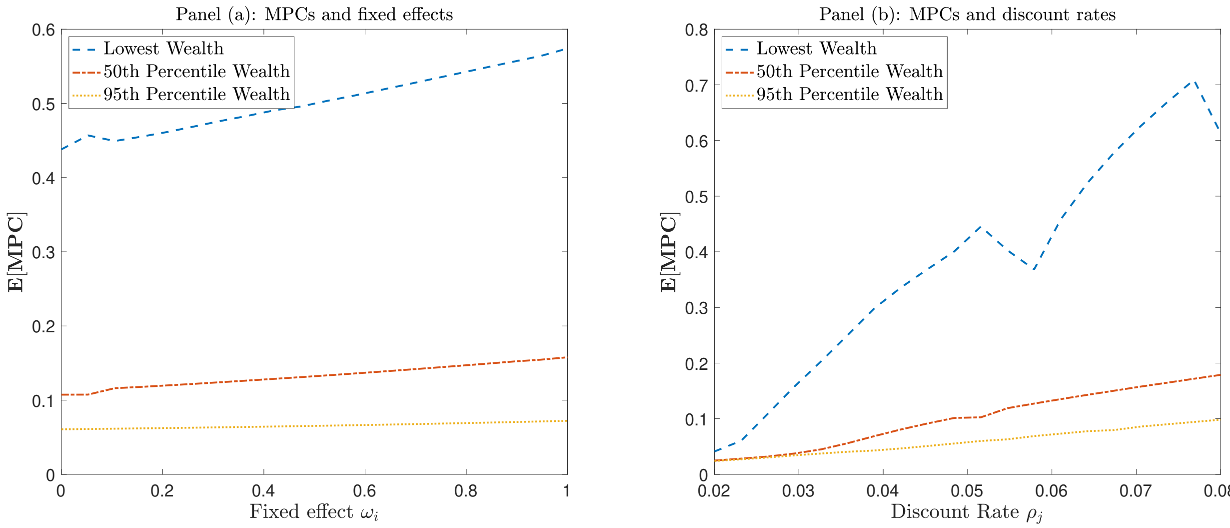
$$\frac{\partial \nu_{ij}(z)}{\partial \omega_i} = (\rho_j - r) \times \frac{\partial \underline{c}_{i,j}}{\partial \omega_i}(1 - \mathcal{P}(\underline{c})\mathcal{R}(\underline{c}))$$

such that for some types of utility functions, it is increasing (e.g. CRRA); whereas for others (e.g. exponential)[7], it does no impact. We can extend our analysis of ex-ante

[6] Follows by differentiating directly $\nu_{i,j}(z)$. The absolute risk aversion is given by $\mathcal{R}(\underline{c}) = -\frac{u'((\underline{c})}{u''(\underline{c})}$, whereas the absolute prudence is $\mathcal{P}(\underline{c}) = -\frac{u'''(\underline{c})}{u''(\underline{c})}$

[7] In the case of CRRA flow utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, $\mathcal{P}(c) = \frac{\gamma+1}{c}$ and $\mathcal{R}(c) = \frac{c}{\gamma}$. Alternately, for exponential

Figure 1: MPCs and Ex-Ante Heterogeneity



Note: On the Panel (a), we compute the average MPC with respect to income varying by income fixed effects; whereas on Panel (b), we compute the same object but varying by discount rate. In panel (b), drop in the "Lowest Wealth" are due to possible numerical errors. Both responses are partial equilibrium objects for the US calibration, with a log felicity utility function

heterogeneity by computing a numerical exercise, in which we vary income fixed effects ω_i and discount rates ρ_j in partial equilibrium and see the effect over the average MPC with respect to productivity z. As indicated by the Figure 1, MPCs are monotonically increasing in income fixed effects ω_i and discount rates ρ_j, indicating that these components also have a role in the consumption-credit behavior. Moreover, this relationship fades as the individuals have higher levels of wealth, in line with what we would expect.

On the other hand, the uncertainty component captures the role of income risk and inequality on the consumption-saving behavior of the household. The response of the MPC will depend on the evolution of the process, given by the functional operator $\mathcal{A}$, which dictates the size of the precautionary savings. In the case of the income process we use (19), both of its properties (the persistence of income and the income shocks)

utility $u(c) = -\frac{1}{\theta}e^{-\theta c}$, $\mathcal{P}(c) = \theta$ and $\mathcal{R}(c) = \frac{1}{\theta}$

have similar effects. Up to a first-order approach[8], they are characterized as:

$$\frac{\mathcal{A}u'(\underline{c}_{i,j}(z^P,z^T)}{u''(\underline{c}_{i,j}(z^P,z^T))} \approx \underbrace{\mu(z^T)\partial_{z^T}\underline{c}_{i,j}(z^T,z^P) + \mu(z^P)\partial_{z^P}\underline{c}_{i,j}(z^T,z^P)}_{\text{Persistence of income shocks}}$$
$$+ \underbrace{\lambda_\varepsilon \int_{-\infty}^{\infty} (\underline{c}_{i,j}(z^T,u) - \underline{c}_{i,j}(z^T,z^P))\phi_\varepsilon(u)du + \lambda_\eta \int_{-\infty}^{\infty} (\underline{c}_{i,j}(s,z^P) - \underline{c}_{i,j}(z^T,z^P))\phi_\eta(s)ds}_{\text{Potential consumption gains/losses of receiving an income shock}} \tag{8}$$

both terms are forward-looking as the agent adjusts its consumption marginally to the perceived law of motion of his income. This change in behavior is affected by the relative position of the household in the income distribution: households with low income perceive persistence negatively, as it implies lower income for longer periods. As a consequence, they anticipate this and decrease MPCs. Instead, they regard income shocks as a way of increasing future consumption, anticipating this effect and increasing current MPCs. On the other hand, households with higher income act just the opposite as persistence implies longer periods of higher income, whereas income shocks imply loss of consumption.

It is through both of these channels of precautionary savings that income inequality will affect the distribution of MPCs, as it implies more potential income gains or losses of receiving a shock while higher persistence of income will imply that there is lower mobility[9]. However, it is important to notice that, as pointed out by Proposition 2, all of these effects are scaled by wealth. Therefore, the more assets the individual holds, the better he can insure himself of second-order effects over his consumption. This is in line with the results of Benhabib et al. (2015) and Achdou et al. (2022), that wealthy households consume linearly with respect to asset holdings.

[8]Up to a first order approximation as, using a Taylor series approximation of first order:

$$u'(\underline{c}^s) - u'(\underline{c}) \approx u''(\underline{c})(\underline{c}^s - \underline{c})$$

Therefore,

$$\frac{1}{u''(\underline{c})}\lambda_y \int_{-\infty}^{\infty} (u'(\underline{c}^s) - u'(\underline{c}))\phi(s)ds \approx \lambda_y \int_{-\infty}^{\infty} (\underline{c}^s - \underline{c})\phi(s)ds$$

[9]As persistence decreases, both components get closer to behaving as an income fixed effect

2.3 Credit deepening and aggregate consumption

So far, we have seen that the micro-level response of consumption to a credit relief is heterogeneous and depends on how each dimension of inequality drives the consumption-saving behavior of households. Of those dimensions, wealth is the most relevant in the aggregate but other sources become relevant when households are constrained. The next step is to extend the question to understand how inequality affects the response of aggregate consumption to a credit deepening

To undertake this task, we proceed similarly to Kaplan et al. (2018): Households perceive the whole path of credit deepening and its impacts on the economy through prices, taxes and transfers. Formally, define $\{\Gamma_t\}_{t\geq 0}$ with $\Gamma_t = \{\underline{a}_t, r_t, w_t, T_t\}$ such that the aggregate consumption at period $t = 0$ is given by

$$C_t(\{\Gamma_t\}_{t\geq 0}) = \int c_{i,j,t}(a, z; \{\Gamma_t\}_{t\geq 0}) d\mu_t \tag{9}$$

where in $c_{i,j,t}(a, z; \{\Gamma_t\}_{t\geq 0})$ and $\mu_t(da, dz, di, dj; \{\Gamma_t\}_{t\geq 0})$ I emphasize the role of future and past prices on decisions and distributions. By totally differentiating (9), we can decompose the response of consumption:

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial \underline{a}_t} d\underline{a}_t dt}_{\text{PE direct effect}} + \underbrace{\int_0^\infty \left(\frac{\partial C_0}{\partial r_t} dr_t + \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial T_t} dT_t \right) dt}_{\text{GE indirect effects}} \tag{10}$$

where the first component captures the direct channel of the credit deepening over consumption - that is, how it affects consumption while all prices remain constant; whereas the other components explain the indirect channel of the credit deepening, where consumption reacts through secondary effects captured by the budget constraint. Moreover, we connect the direct channel of credit to consumption to micro-level response as follows:

$$\int_0^\infty \frac{\partial C_0}{\partial \underline{a}_t} d\underline{a}_t dt = \int_0^\infty \left(\int \frac{\partial c_{i,j,t}(a, z; \{\underline{a}_t, \bar{w}, \bar{r}, \bar{T}\}_{t\geq 0})}{\partial \underline{a}_t} d\mu_0^{\underline{a}} \right) d\underline{a}_t dt \tag{11}$$

where $\mu_0^{\underline{a}} = \mu_0(da, dz, di, dj; \{\underline{a}_t, \bar{w}, \bar{r}, \bar{T}\}_{t\geq 0})$. Therefore, for the impact of the direct channel of credit deepening on consumption, the initial measure and distribution of credit MPCs matter, as well as an extrapolation of this distribution to the impact of future changes in the borrowing constraint aggregated by the partial equilibrium evolution of wealth. Proposition 1 measures the role of inequality on this initial impact, but for the full response and for the whole path $\{C_t\}_{t\geq 0}$, we rely on numerical exercises detailed in Section 4

2.4 General Equilibrium Framework

To assess the impacts of the indirect/general equilibrium channel of a credit deepening on consumption and the role of inequality in this channel, we introduce a simple general-equilibrium framework. We opt to do such as the complexity of the model relies on the household side and its interaction with inequality.

Firms: We choose to close the model in the tradition of Aiyagari (1994), in which a representative firm hires labor and uses capital. The firm produces consumption goods with a Cobb-Douglas production function:

$$Y_t = F\left(K_t, L_t\right) = K_t^{\alpha} L_t^{1-\alpha} \tag{12}$$

where K_t and L_t denote, respectively, aggregate capital and efficient labor units and $\alpha \in (0,1)$ is the share of capital in production. We assume the presence of a competitive labor market. The problem of the representative firm is standard:

$$\max_{\{K_t, L_t\}} F\left(K_t, L_t\right) - w_t L_t - (r_t + \delta) K_t \tag{13}$$

Capital depreciates at the exogenous rate δ and since factor markets are competitive, the interest rate and the wage are given by:

$$w_t = \frac{\partial F\left(K_t, L_t\right)}{\partial L} \tag{14}$$

$$r_t = \frac{\partial F\left(K_t, L_t\right)}{\partial K} - \delta \tag{15}$$

Government: The government has the role of universal insurer of households as they face idiosyncratic income risk, partially insuring them against income fluctuations. It exerts this role by taxing linearly labor income through a tax rate τ_t, and rebating all of its funds as lump-sum transfers T_t. For each period, the government budget constraint must be satisfied:

$$\tau_t w_t L_t = \int T_t d\mu_t \tag{16}$$

Competitive Equilibrium

Definition 2 (Competitive Equilibrium)**.** A competitive equilibrium is defined as the set of paths for household value functions and decisions $\{c_{i,j,t}(a,z), s_{i,j,t}(a,z), V_{i,j,t}(a,z)\}_{t\geq 0}$, input prices $\{r_t, w_t\}_{t\geq 0}$, fiscal variables $\{\tau_t, T_t\}$, measures $\{\mu_t(da, dz, di, dj), g_{i,j}(a,z,t)\}_{t\geq 0}$, aggregate quantities $\{Y_t, C_t, L_t, K_t\}_{t\geq 0}$ and borrowing constraints $\{\underline{a}_t\}_{t\geq 0}$ such that, given the exogenous stochastic process for $\{z\}_{t\geq 0}$ and cross-section heterogeneity $(i,j) \in \mathbb{I} \times \mathbb{J}$, at every period t:

1. *Households optimize*: Given (i) prices $\{r_t, w_t\}_{t\geq 0}$; (ii) fiscal policy $\{\tau_t, T_t\}_{t\geq 0}$; and (iii) borrowing constraints $\{\underline{a}_t\}_{t\geq 0}$; the value functions $\{V_{i,j,t}(a,z)\}_{t\geq 0}$ solve the HJB (4) given the set of state constraints (5) and the infinitesimal generator $\mathcal{A}$ of the stochastic process $\{z\}_{t\geq 0}$. Moreover, policy functions $\{c_{i,j,t}(a,z), s_{i,j,t}(a,z)\}_{t\geq 0}$ satisfy (6)

2. *Firms Optimize*: Given prices $\{r_t, w_t\}_{t\geq 0}$, allocations $\{L_t, K_t\}_{t\geq 0}$ solve the Firm's problem (13)

3. *Fiscal Policy*: Fiscal policy $\{\tau_t, T_t\}$ satisfies the government budget constraint (16) at all periods

4. *Markets Clear*: Markets for goods, labor and capital clear

    - Goods Market: $Y_t = C_t + K_t$
    - Capital Market: $K_t = \int a_t d\mu_t$
    - Labor Market: $L_t = \int z_t d\mu_t$

5. *Measures satisfy consistency dynamics*: For every pair $(i,j) \in \mathbb{I} \times \mathbb{J}$, distributions $g_{i,j}(a,z,t)$ satisfy the KF equations (7). Moreover, by aggregating $(i,j) \in \mathbb{I} \times \mathbb{J}$, we obtain measures $\{\mu_t(da, dz, di, dj)\}_{t\geq 0}$

3 Calibration

There are two main goals to be carried out in the calibration strategy. First, to ensure that we calibrate the model to several countries of the sample in the most comparable way, so as to infer differences in the average Credit MPCs, as well as perform cross-country credit expansion quantitative exercises. Second, to use assumptions over exogenous heterogeneity in the cross-section of households and fiscal policy to match key moments of the income and wealth distribution. We focus on these moments as they determine two key objects of the model: (i) the distribution of Credit MPCs; and (ii) the aggregate measure of the economy μ_t. Together, these objects can deliver a quantitatively accurate response of aggregate consumption to a credit deepening.

The full set of parameters in the economy are displayed in the Table 1. The three initial parameters $\{\gamma, \alpha, \delta\}$ are the same in all alternative country calibrations and follow estimates commonly used in the literature, whereas the rest of the parameters are calibrated internally to match moments of the income and wealth distribution. All the details of the internal calibration are explained as follows, while details on the data are explained in the appendix

Table 1: Parameter Values for Country-Specific Calibration

	Scope	Description	Value	Source/Target
$1/\gamma$	Cross-country	IES	1	(Kaplan et al, 2018)
α	Cross-country	Capital share	0.33	(Kaplan et al, 2018)
δ	Cross-country	Depreciation rate	10% (p.a.)	Literature
Cross-Country parameters				
$\underline{a}$	Country specific	Borrowing Limit	-2.541	HH Debt-to-GDP
$\bar{\rho}$	Country specific	Avg. Discount Rate	0.074	Net Wealth to Income ratio
$\{\nabla_1, \nabla_2\}$	Country specific	Discount rate Heterogeneity	$\{0.0033, 0.002\}$	Wealth Quintiles
τ	Country specific	Labor Income Tax	21.7%	Cash transfers to GDP
β_{z^T}	Country specific	Persistence Transitory inc.	0.791	Log Earnings dist.
β_{z^P}	Country specific	Persistence Permanent inc.	0.002	Log Earnings dist.
λ_{z^P}	Country specific	Arrival rate Permanent	0.008	Log Earnings dist.
λ_{z^T}	Country specific	Arrival rate Transitory	0.177	Log Earnings dist.
σ_ε	Country specific	Variance Permanent	1.171	Log Earnings dist.
σ_η	Country specific	Variance Transitory	1.692	Log Earnings dist.

Note: The values displayed in the country-specific rows are for the Brazilian calibration

3.1 Internal calibration strategy

The internal calibration strategy is carried out in two steps, taking advantage of the exogenous distribution of income: In the first step, I calibrate the parameters that govern the income process exogenously; to then use their estimates and calibrate moments of the wealth distribution in the steady-state of the model.

3.1.1 Fiscal Policy

Given the insurance role of fiscal policy in the model, we first assume that tax rates are unchanged in the model $\tau_t = \tau \ \forall t$, to then use them to match the total size of cash transfers to households as a share of output.

3.1.2 Preference Heterogeneity

The idea of using heterogeneity in the degree of patience of households is as old as Krusell and Smith (1997), but has been used in the literature as an instrument for generating higher wealth inequality ever since (Krueger et al. 2016). What explains this is that discount rate heterogeneity captures much more than different degrees of impatience in the economy: it reflects a portion of differences in saving and life-cycle behavior, different income growth expectations, diverse risk aversion and other factors that imply different wealth-to-income targets within the population[10]. Our approach is similar to Carroll et al. (2017): The continuum of discount factors $\mathbb{J}$ is distributed uniformly between five different discount rates

$$\{\bar{\rho} - (\nabla_1 + \nabla_2), \bar{\rho} - \nabla_1, \bar{\rho}, \bar{\rho} + \nabla_1, \bar{\rho} + (\nabla_1 + \nabla_2)\} \tag{17}$$

where $\bar{\rho}$ is the average discount rate of the economy, used to match the target net wealth to net income ratio of the economy, and parameters $\{\nabla_1, \nabla_2\}$ determine the dispersion in heterogeneity of discount rates. Altogether, the dispersion parameters gives us degrees of freedom in better fitting the model wealth distribution to the data, targeting net-wealth quintiles ($P0P20$, $P20P40$, $P40P60$, $P60P80$, $P80P100$), as in Carroll et al. (2017).

3.1.3 Income Process and individual fixed effects

The goal of the earnings process we choose is to generate an income distribution that features a large amount of right-tail inequality, as pointed out by Guvenen et al. (2021), and capture risky earning dynamics of households - all of this while still relying on annual data, as provided in Guvenen et al. (2022). To obtain these features, the income process must have substantially more risk than the usual calibrations relying on AR processes with gaussian shocks. I introduce this with a "buffer-stock" model of permanent and transitory income components as in Kaplan et al. (2018), together with an individ-

[10]These can potentially be explicitly modelled by OLG and life-cycle models or by incorporating bequests and non-homothetic preferences, but this approach is significantly more simple and delivers quantitatively similar results

ual fixed effect to account for a degree of heterogeneity in the initial conditions, as in Heathcote et al. (2009).

Formally speaking, the idiosyncratic productivity component of the household, z_{it}, is composed of three orthogonal components: (i) a fixed effect ω_i, drawn at birth specific to that household; (ii) a transitory component z_t^T; and (iii) a permanent component of income z_t^P:

$$\ln z_{it} = \omega_i + z_t^P + z_t^T \tag{18}$$

The permanent and transitory components of earnings are stochastic and each of them evolves separately, dictated by a compound jump-drift process with idiosyncratic jumps:

$$\begin{aligned} dz_t^T &= -\beta_{z^T} z_t^T dt + \eta_t dJ_{\eta,t} \;\; \eta_t \sim \mathcal{N}(0, \sigma_\eta^2) \\ dz_t^P &= -\beta_{z^P} z^P dt + \varepsilon_t dJ_{\varepsilon,t} \;\; \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2) \end{aligned} \tag{19}$$

which differs in frequency and amplitude of shocks, as well as the degree of persistence. The terms $\{\beta_{z^T}, \beta_{z^P}\}$ govern the mean-reverting characteristics of the process, directly determining how persistent are these shocks and each component of productivity. On the other hand, the jump components dJ_t have different arrival rates $\{\lambda_{z^T}, \lambda_{z^P}\}$ which determine how frequent is to receive a shock. Moreover, the sign and magnitude of the shock are also stochastic and determined by the normally-distributed random variables ε_t, η_t, each with variance $\{\sigma_\varepsilon^2, \sigma_\eta^2\}$. Therefore, these two will determine the amplitude of the shocks. Moreover, given that we already capture the variance of log-earnings through the income process, we abstract from fixed effect heterogeneity as the income process already captures it.[11]

3.2 Step 1: Income Process Calibration

In this step, I calibrate the set of parameters $\Theta = \{\beta_{z^P}, \beta_{z^T}, \sigma_\eta, \sigma_\varepsilon, \lambda_{z^P}, \lambda_{z^T}\}$ of the two-state income process (19) such that the stationary distribution of log income and log income growth matches key moments of their empirical counterparts, thus generating realistic income dynamics on the side of the households. Within the set of targeted moments, there are the standard deviations of the distribution of log income innovations [12] and the one and five-year growth of log income[13]; the kurtosis of the distributions of

[11]This is something that may change in future versions of this paper

[12]These are obtained by taking the log income of the individual, regressing in a set of observable controls such as age, gender and education, and then taking the residual of this regression

[13]That is, the one and five-year change of the residuals of log income regressions

one and five-year growth of log income; and measures of tail dispersion of these distributions, such as the $P9050$ and $P5010$. The need for distributions of earnings growth at multiple durations relies on the necessity of properly identifying the components of permanent and transitory income.

All of these moments are available in the GRID database (Guvenen et al. 2022) in the form of comparable panel data, with different time spans. As argued by Busch et al. (2022) and Guvenen et al. (2022), some measures of idiosyncratic labor income risk[14] present business cycle variation, and may also present a trend. As the intent of our exercise is to emulate a steady-state instead of capturing short-term cycles, we abstract of trends and cycles and proceed by taking a sample average of each moment used for calibration. These values are displayed in the Table 8

To carry on with this calibration, I first approximate the continuous-time continuous state processes (19) with continuous-time discrete-state processes. To do such, I employ the methodology in Kaplan et al. (2018): Approximate the permanent income state-space with a 11-point non-linear grid[15] and the transitory income state-space with a 3-point non-linear grid, to then discretize the infinitesimal generator of the processes (19) as continuous time transition matrices based on a finite difference approximations, provided the calibration of Θ.

Given the transition matrices, I use the Kolmogorov-Forward equations of the stochastic processes to compute the ergodic distributions of permanent and transitory productivity components of the productivity of the Household. Moreover, I use the transition matrices and the ergodic distribution to simulate 5000 individuals over 20 years to reconstruct the distributions of log income and log earnings growth in time. Furthermore, as the time grid implies each period of simulation is a quarter of a year, but we observe only moments of the annual distribution, I aggregate income annually and then compute the simulated counterparts $\hat{\mu}(\Theta)_k$ of the observed empirical moments $\hat{\mu}_k$ in Table 8

For a uniform comparison of moments, I calculate the percentage deviation of the simulated moment from its empirical counterpart. For any given moment $k \in \mathcal{K}$:

$$F_k(\Theta) = \frac{\hat{\mu}(\Theta)_k - \hat{\mu}_k}{\hat{\mu}_k} \tag{20}$$

[14]Measures such as skewness of income growth have a very strong pro cyclical behavior, while volatility of income growth is mildly countercyclical. When decomposing skewness by measures of tail dispersion, such as $P9050$ and $P5010$, the latter presents strong countercylical behavior, while the former exhibits cyclical pattern

[15]Grid widths and spacing used are the same as in Kaplan et al. (2018), so as to ensure that simulating the discrete-space continuous time process is as close as possible to a continuous state process, as well as cross-country comparability

to then define the loss function from the given vector of percentage deviations. Let $F(\Theta) = (F_1(\Theta), \ldots, F_K(\Theta))'$ be the vector of stacked moments, over which I minimize to obtain the estimated parameters:

$$\widehat{\Theta} = \arg\min_{\Theta} F(\Theta)' W F(\Theta) \tag{21}$$

where W is a weighting matrix. I presuppose that each moment is equally informative, so I assign equal weights to all of them by choosing $W = I$. Finally, to solve the minimization problem, I employ a multi-start algorithm used in Mello and Martinez (2020), which is an adaptation of Guvenen et al. (2021): In the first stage of the algorithm, we randomly evaluate 10,000 initial parameter vectors (chosen based on a Sobol sequence). Afterward, based on the loss function, the 5% best guesses are selected and carried out for the second stage of the algorithm. In that stage, we perform a local search on the selected guesses using the Nelder-Mead simplex algorithm and select the $\widehat{\Theta}$ that minimizes equation (21). The estimated moments for each country are displayed in Table 10, and the numerical procedure for discretizing and estimating the income process is available at the appendix

3.3 Step 2: Wealth Calibration

Given as input the parameters $\widehat{\Theta}$ calibrated in step one, in the second step I target general moments of the wealth distribution available so as to try to recover as much information as possible about the wealth distribution as whole. To calibrate the parameters $\Omega = \{\underline{a}, \rho, \nabla_1, \nabla_2, \tau\}$, I use the following set of moments: (i) wealth shares of each quintile of the wealth distribution ($P0P20$, $P20P40$, $P40P60$, $P60P80$, $P80P100$); (ii) the net private wealth to net national income ratio; and (iii) unsecured household debt-to-GDP.

As in the case of the first step of the calibration, these data are available in the form of panel data. With regards to the wealth shares by quintile series, the first four quintiles $P0P20$, $P20P40$, $P40P60$, $P60P80$, I take the sample average and compute the last percentile $P80P100$ as a residue such that the sum of percentiles is 1. As for the case of transfers to GDP and Wealth to Income Ratio, I proceed likewise in computing sample averages. The only exception is the case of household debt-to-GDP, which is single observation of panel data due to availability of data issues. All the data used is displayed in Table 9. To calibrate the parameters Ω, I proceed just a in the first step by computing the steady state of the model and calculating the percentage deviation of the simulated moment from its empirical counterpart in (20), to then stack these moments and minimize (21). The estimated moments for each country are in Table 11, and the numerical procedure for estimating the stationary equilibrium is available at the appendix

3.4 How well does the model fare with respect to data?

3.4.1 Earning Dynamics

The general estimation fit for selected countries are displayed in Table 2. Overall, the fitted earnings process matches the nine targeted moments of Table 8 well, and generates the observed level of income inequality within the data - as evidenced by the fit of untargeted moments, such as pre-tax income gini.

Table 2: Earnings Process Estimation Fit

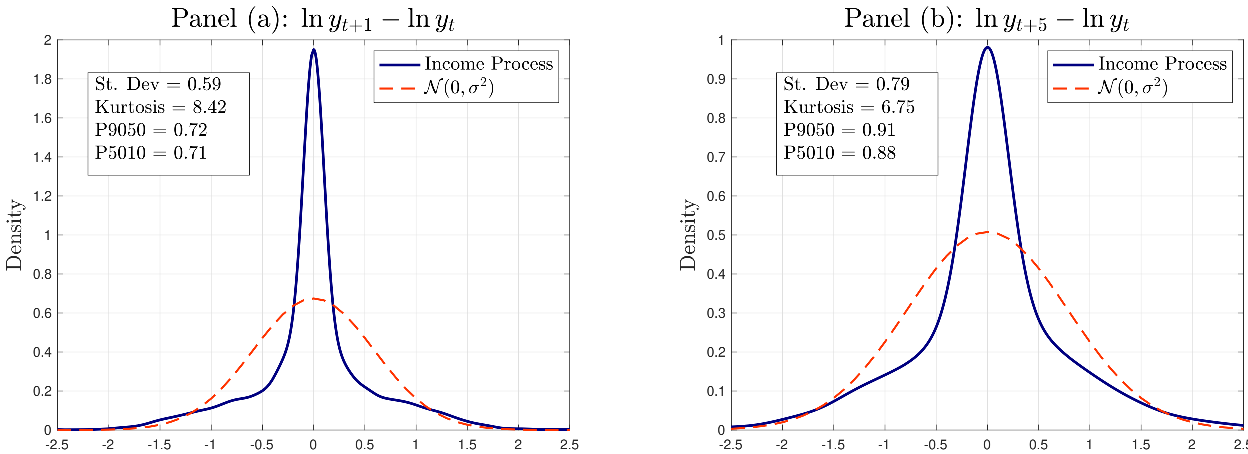
Targeted Moments	United States		Brazil	
	Model	Data	Model	Data
St. Dev. of Log Earnings	0.89	0.93	1.07	1.04
St. Dev. 1 year Log Earnings growth	0.48	0.56	0.59	0.68
St. Dev. 5 year Log Earnings growth	0.81	0.78	0.78	0.82
Kurtosis 1 year Log Earnings growth	13.37	12.86	8.42	8.53
Kurtosis 5 year Log Earnings growth	7.91	8.82	6.75	6.34
P9050 1 year Log Earnings growth	0.43	0.43	0.72	0.61
P5010 1 year Log Earnings growth	0.46	0.46	0.70	0.70
P9050 5 year Log Earnings growth	0.72	0.71	0.91	0.84
P5010 5 year Log Earnings growth	0.75	0.77	0.87	0.94
Untargeted Moment:				
Pre-tax Income Gini	0.46	0.47	0.58	0.55

Note: Pre-tax Income Gini reflects sample averages of administrative data, available in Guvenen et al. (2021)

By targeting higher-order moments of the distributions of log-earnings growth, we generate distributions of log-earnings growth that are *leptokurtic*, that is, with more mass concentrated around the mean and tails with respect to a normal distribution with same variance - as evidenced in Figure 2.

This stems from the fact that transitory earnings component concentrates mass around the mean with small but frequent shocks, whereas permanent earnings component concentrates mass on the tails with larger and infrequent shocks. In terms of the ergodic distribution of log earnings, these income dynamics will generate a skewed distribution that is a crucial element in generating the dispersion of liquid assets. Firstly, because earnings dispersion generates likewise wealth dispersion through saving decisions of

Figure 2: Calibrated Distributions of One and Five Year Log Income Changes



Note: In Panel (a), distribution of 1 year log income changes and its respective high-order moments; while in Panel (b), distribution of 5 year log income changes. Both correspond to the stationary distributions of the process (19) calibrated to match micro moments of Brazilian data, available in the Table 8. In the red-dotted lines, we have a gaussian distribution with the same variance as the leptokurtic distribution

households; secondly, as a leptokurtic distribution implies that households face substantially more precautionary risk, increases wealth accumulation specially for the wealthiest

3.4.2 Wealth Distribution and micro-level consumption & credit behavior

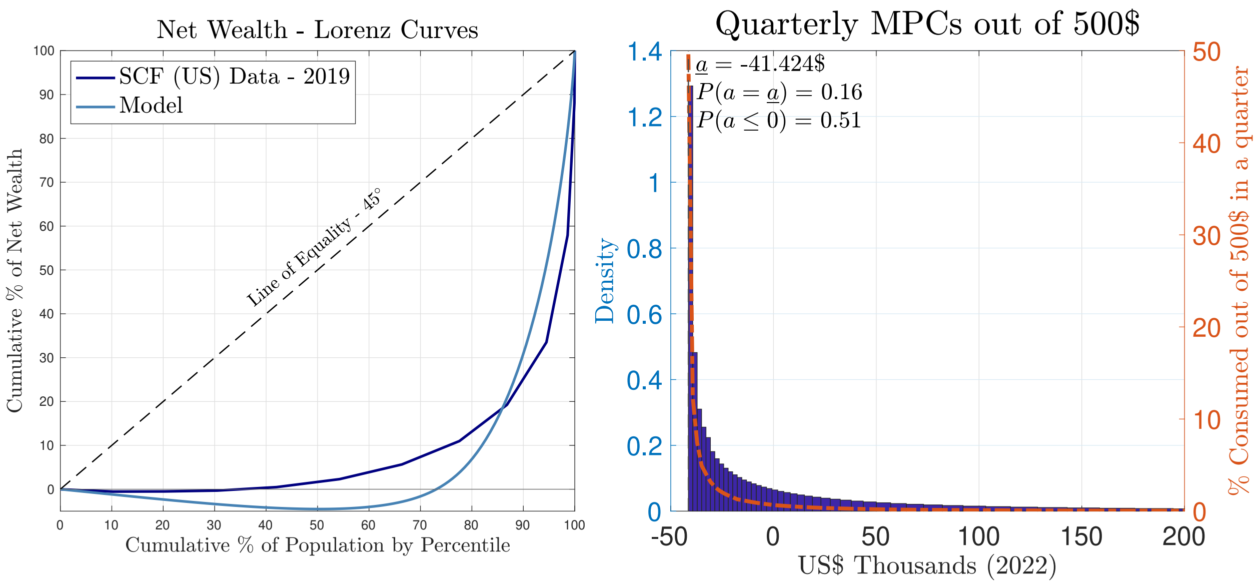
In table Table 3, we display the estimation fit for the targeted moments for selected countries. The model replicates fairly well general moments of the wealth distribution, such as the mean wealth to GDP ratio and wealth gini, as well as the size of fiscal policy in the economy. However, general wealth share quintiles ($P0P20$, $P20P40$, $P40P60$, $P60P80$) are not well fit to the data as we would expect. Several reasons arise as to why we fail to account for them properly: (i) extensions of the one-asset heterogeneous agents model that incorporates ex-ante heterogeneity (as in our case) can generate sizeable responses of consumption while still remaining consistent with aggregate wealth data, but at the cost of severely understating asset holdings of households in the middle of the distribution - what Kaplan and Violante (2022) coined as the "missing-middle" problem; (ii) introducing household debt-to-GDP moment in the calibration is conflicting with the moment of mean wealth-to-GDP ratio, as our model is not of gross-positions[16], aggravating the "missing-middle" problem, which can be clearly seen in the comparison of our model with US SCF[17] net wealth in the left plot of Figure 3

[16]For future development, it makes more sense to change this moment for another, such as . . .
[17]United States 2019 Survey of Consumer Finances

Table 3: Wealth and Fiscal Data Estimation Fit

Targeted Moments	United States		Brazil	
	Model	Data	Model	Data
Mean Wealth to GDP ratio	5.16	4.88	3.79	3.61
$P0P20$	0	-1.54	-2.15	-2.14
$P20P40$	-3.99	0.35	-3.16	1.23
$P40P60$	-0.13	3.24	1.12	3.75
$P60P80$	10.14	12.06	11.98	9.50
$P80P100$	93.99	85.88	92.21	87.65
Cash transfers to GDP (%)	13.14	12.74	14.53	14.56
Household (unsecured) Debt-to-GDP	23.35	27.09	21.15	23.20
Untargeted moments				
Average Quarterly MPC out of 500$	15.68	-	15.60	-
Share of Households with negative wealth (%)	50.54	15	45.14	-
Share of HtM in the economy	56.34	14.2		-
Wealth Gini	0.853	0.867	0.859	0.886

Figure 3: Calibrated Distributions of Wealth and Quarterly MPCs

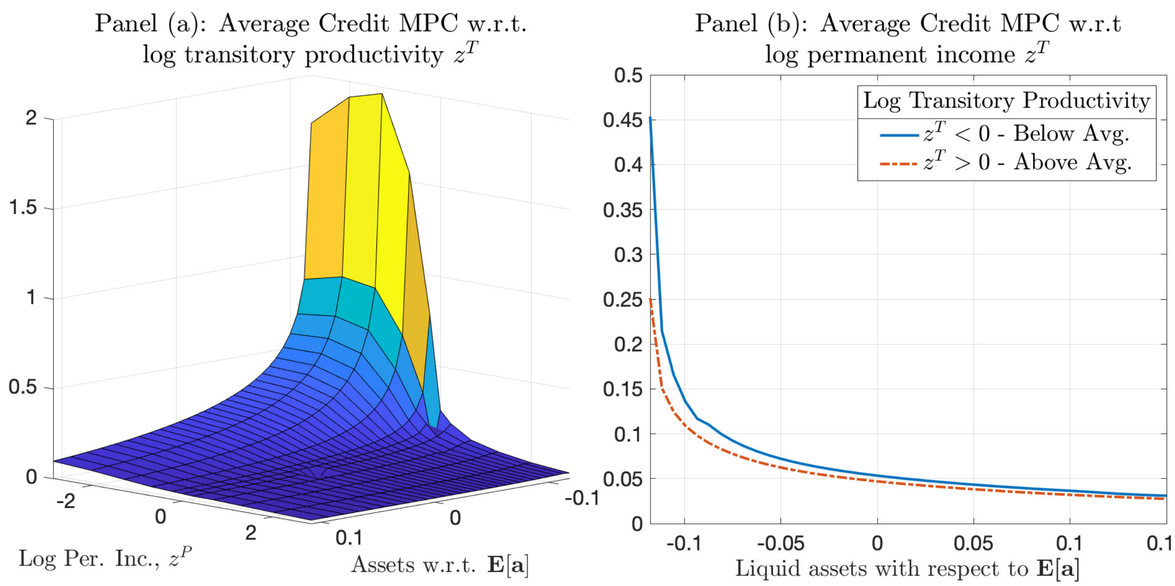


Note: Distributions are calibrated to US data

In the right plot of Figure 3, we show the US model generated wealth distribution and the distribution of empirically observed MPCs. Two things are worth mentioning: first, we generate a skewed wealth distribution with a fat right-tail, as desired, and secondly, despite some issues arising in the wealth distribution calibration, the quantitative response of consumption generated by the model is in line with the literature: the average quarterly MPC out of 500$[18] transfer is line with the literature, which is around 15% to 25% (Jappelli and Pistaferri 2010). As good as this can be, it is driven the excessive share of households with negative wealth in the economy, which inflates the size of HtM in the economy, and also the average consumption response given by MPCs.

With respect to the object of interest in our study, we plot in the figure Figure 4 the credit MPCs of Proposition 1 under the US steady-state calibration. In the panel (a), we plot the distribution of average credit MPCs with respect to the stationary distribution of z^T, and in panel (b) likewise, but with respect to the stationary distribution of z^P. Different from Proposition 2, this numerical exercise permits to visualize micro-level consumption out of credit behavior much beyond the borrowing constraint, even though the region close to the borrowing constraint is the quantitatively relevant region.

Figure 4: Calibrated Distributions of Credit MPCs



Note: On the Panel (a), we compute MPCs averaging for log transitory productivity, $\mathbb{E}[MPC^{500\$}_{Qtr}|g^*(z^T)]$; whereas on Panel (b), we compute MPCs averaging for log permanent productivity, $\mathbb{E}[MPC^{500\$}_{Qtr}|g^*(z^P)]$. Both distributions are calibrated to US data

First, it tells us that close to the borrowing constraint, the distribution of income matters substantially for determining credit-consumption behavior. That is, the lower the income of the household[19], the higher the response of consumption to a credit deepening. And that this response will be even more increased if the household has no assets to

[18]To calculate the Quarterly MPC of 500$, I first map the value of an asset to 2022 US$, to then calculate the object using the Feynman-Kac formula. For more details, see appendix

[19]That is, the credit MPC response is almost monotonically decreasing in permanent and transitory income for all levels of asset holdings. The only non-monotonicity implied by the exercise is at borrowing constraint for log permanent income, although this could be due to numerical error.

smooth consumption. Second, the relevance is much bigger with respect to permanent income rather than the transitory income component. This result reflects the permanent income hypothesis (Friedman 1957), as consumption behavior is much more sensible permanent income rather than transitory. Third and last, it strengthens the point made in Proposition 2: although income inequality is relevant for the consumption response to a credit deepening, its effects are mitigated by the relevance of the dispersion in household wealth. For already barely positive levels of assets, household credit MPC is almost insensible to permanent or transitory levels of income.

4 Credit Deepening, Aggregate Consumption and Inequality: Quantitative results

The goal of this section is to study the response of aggregate consumption in the short and long-run to a credit deepening (1), how inequality affects this response through time and through which channels does it act. To do such, we consider two experiments: First, we conduct two kinds of MIT-shocks [20] to the borrowing constraint, simulating a permanent and transitory credit deepening. In the former, we shock the borrowing constraint permanently to a level 50% higher, whereas in the second case, after the shock the borrowing constraint mean-reverts at rate ν back to the initial value $\underline{a}_0$[21]. Using the decomposition in (10), we map which channels drive the response of consumption through time.

In the second experiment, we alter the initial level of income or wealth inequality in the economy by changing marginally the calibration of one of the sources of ex-ante heterogeneity or fiscal policy, to then run transitory credit shock and see the impact of inequality in the initial income and wealth distribution to the whole path of aggregate consumption. In all the following exercises, we use the calibration for the United States. In Figure 5, we plot the Impulse Response Functions (IRFs) for aggregate consumption after a credit deepening, together with its decomposition of the response to the direct and indirect channels of credit.

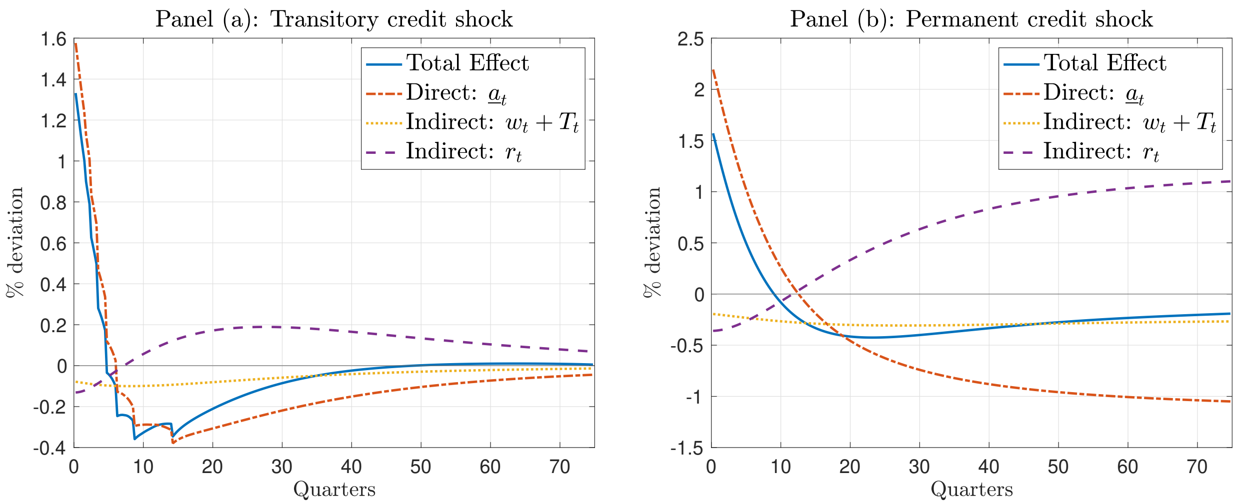
[20] That is, an unexpected shock which then follows a perfect-foresight path into the new equilibrium

[21] In the transitory credit shock, the borrowing constraint mean-reverts at rate ν back to the initial value $\underline{a}_0$, following the ODE

$$d\underline{a}_t = \nu(\underline{a}_0 - \underline{a}_t)dt$$

We set the parameter of mean-reversion $\nu = 0.2$, implying a quarterly autocorrelation of $e^{-\nu} = 0.81$

Figure 5: Aggregate Consumption IRFs to a Credit Deepening



Note: On the Panel (a), we compute the response of consumption to a transitory credit deepening; whereas on Panel (b), we compute the same response but for a permanent shock. All values are log deviations from steady state. The decomposition is given by (10)

4.1 Transmission mechanisms of a credit deepening to aggregate consumption

In response to a credit deepening, aggregate consumption is stimulated in the short run as households have access to more credit, decreasing their liquid asset positions in order to consume more. This response is short-lived, as the consumption "boom" fades after two to three years from the initial shock. Afterwards, aggregate consumption falls below its initial level, leading to a long-lasting reduction in consumption. This reduction is persistent because it is followed by a higher level of household debt (see Figure 6), as households have levered responding to the initial relief in the access to credit. Moreover, in the case of a transitory shock, as credit conditions tighten once again to their previous level, households slowly readjust their consumption through a slow process of deleveraging.

As evidenced in Figure 5 alongside with Table 4, there are different forces driving the consumption "boom" and "bust" cycles, and they also change their effects and magnitudes with time. For instance, the direct channel of credit captures the household demand expansion through constrained households (mapped by the distribution of income, wealth and credit MPCs) while controlling for distributional effects led by prices. It will be responsible for the consumption growth in the short-run, but its persistent decrease in the long-run because of higher household debt.

Table 4: Decomposition of the Effect of a Credit Deepening on Aggregate Consumption

	1yr	3yrs	5yrs	10yrs	20yrs
Elasticity of C_t	0.240	0.035	-0.012	-0.013	-0.010
Contribution of each component (%)					
Direct channel: $\underline{a}$	127	208	35	-231	-276
Indirect channel: $w_t + T_t$	-11	-82	-234	-134	-150
Indirect channel: r_t	-14	-22	103	262	325

Note: Columns display the elasticity and decomposition of aggregate of consumption over the described time-frame. The contribution elements are in % of the general equilibrium steady-state deviation

Alternately, the indirect channel of credit will reflect distributional effects of credit deepening over consumption. It is composed of two forces: a budget effect, capturing the response of wages w_t and transfers T_t on household consumption, and the intertemporal substitution effect, capturing responses on household consumption through interest rates r_t. The former depends on the aggregate impacts of the household demand shock over the labor market and the resulting fiscal policy through cash transfer. In the case of the current model, as wages are lower and fiscal policy is pro-cyclical[22], all households have lower income and consume less. On the other hand, in the case of the latter, a path of increasing interest rates in the short run lowers the current consumption of households. But, in the medium and long-run, as interest rates face a decreasing path, drives back consumption rebound through intertemporal consumption smoothing.

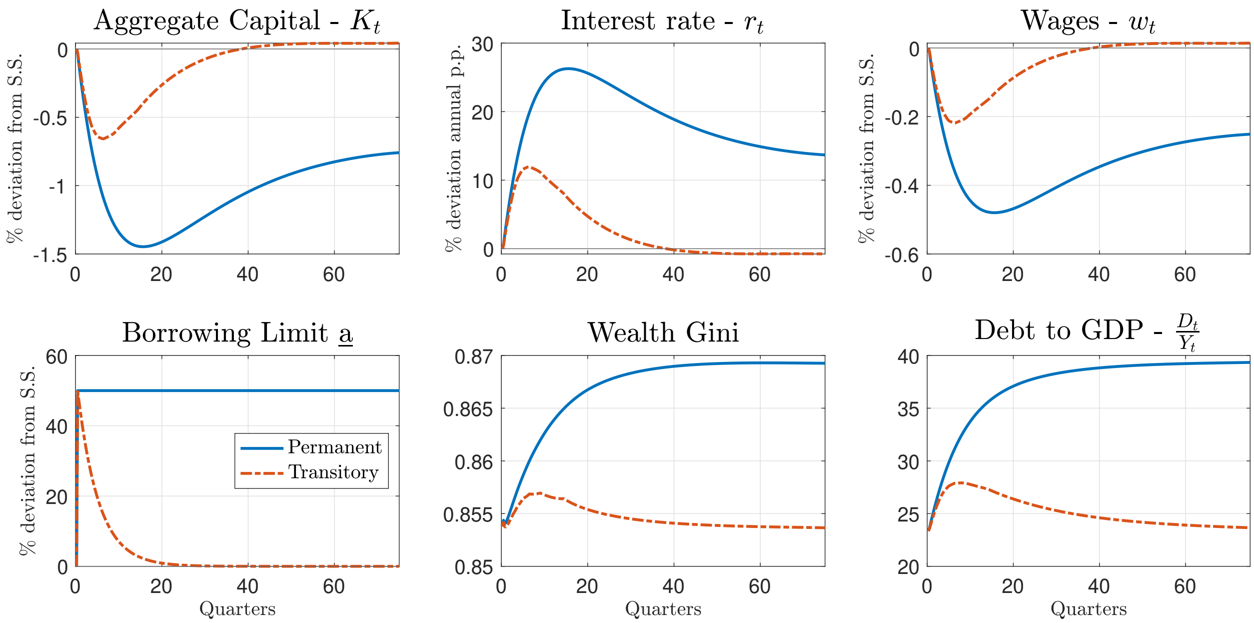
4.2 The Role of Inequality on Credit Deepening

In this subsection, we study how inequality affects the channels of transmission of a credit deepening on aggregate consumption, amplifying or reducing its response. We do such by using counterfactual credit deepening exercises in which we alter marginally the initial calibration of one of the parameters which affects the initial income and wealth distribution, to then compare the aggregate consumption IRF of the baseline calibration to these alternative scenarios of more or less income and wealth inequality.

From these exercises, we conclude beforehand that: (i) Wealth inequality is the main source of amplification of the response of consumption "boom" and "busts", doing such

[22] In the Aiyagari flexible-price economy, this household demand shock that follows a credit deepening will lead the economy to a supply-side recession as capital lowers. As a consequence, wages also lower and interest raise increases. This is counterfactual to what is observed in the data (e.g. Mian et al. (2017)), and within a Neo-Keynesian framework may have different effects

Figure 6: Prices, Debt, and Wealth Gini IRFs to a Credit Deepening



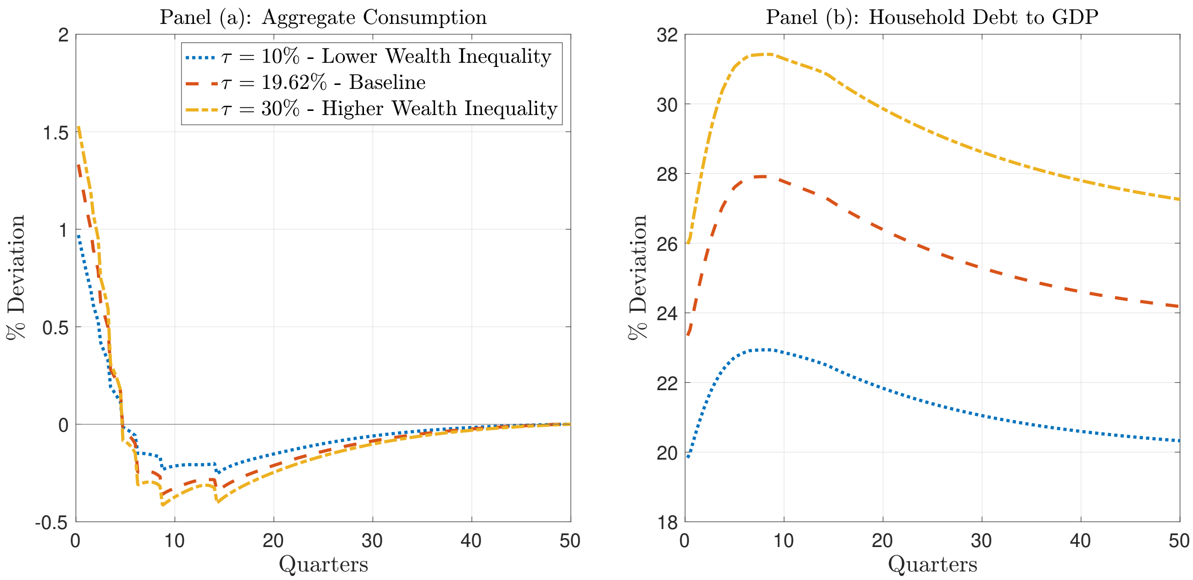
through the direct channel of credit; (ii) wealth inequality also plays a role in the intertemporal substitution channel, amplifying the rebound of consumption in the long-run through the wealthy, reducing the persistence of a consumption slowdown. In the case of the former, the higher the level of wealth inequality, the more sizable the quantitative response of Credit MPCs driven by a higher share of constrained households. Income inequality is also a determinant for credit MPCs, but its importance is overshadowed by the distribution of wealth[23]. While as for the latter, the wealthy are responsible for driving the consumption rebound as the poor save to deleverage given more tight credit conditions. Therefore, the higher the wealth inequality, the more income is allocated to those that are consuming more in the long-run.

4.2.1 Fiscal Policy Experiment

In this exercise, we alter the tax rate τ and lump-sum transfers T in the economy to simulate counterfactual steady-states with higher or lower inequality. As argued in Subsection 3.1, fiscal policy has an insurance role within the model by subsidizing those with low income by taxing those with high income. It alters the post-tax income distribution and also the wealth inequality, as it alters precautionary savings. Consequentially, post-tax income inequality is lower (higher) in economies with higher (lower) τ and T. On

[23] This point is further explored on the fiscal policy experiment

Figure 7: Credit Deepening with More and Less progressive Fiscal Policy



Note: On the Panel (a), we plot the consumption IRFs to the same MIT-Shock to the borrowing constraint, but with different levels of wealth inequality induced by different fiscal policy $\{\tau, T\}$; whereas on Panel (b) we plot the corresponding Debt to GDP IRFs

the other hand, wealth inequality will is lower (higher) in economies with lower (higher) τ and T.

On the baseline economy, the wealth gini is 0.853, whereas in the economy with lower (higher) τ and T, the wealth gini is 0.844 (0.864)[24]. Figure 7 shows our results for a transitory credit deepening, alongside Table 5. Our results suggest that economies with higher wealth inequality experience a more amplified consumption growth in the short run, succeeded by a bigger consumption "bust" that boasts a more vigorous recovery[25]. This response is led by a stronger direct channel of credit, as in the economy with lower τ, T there is less precautionary savings, altering the share of constrained households. Therefore, it amplifies the short-run consumption growth through higher average credit MPCs, but also the long-run share of indebted households. Concomitantly, an economy with higher wealth inequality also implies a stronger channel of intertemporal substitution, as wealthy households detain bigger share of the wealth distribution and will drive the consumption rebound while indebted households save to deleverage given more tight credit conditions

[24]We set the alternative tax rate to $\tau = 10\%$ and $\tau = 30\%$

[25]That is, although in the economy with higher inequality consumption decreases more, it takes the same amount of quarters to recover than the economy subject to a smaller consumption decrease

Table 5: Elasticity and Decomposition of the Consumption Response

	Tax rate τ	1yr	3yr	5yr	10yr	20yr
Elasticity of C_t	10%	0.166	0.026	-0.008	-0.009	-0.007
	19.46%	0.240	0.035	-0.012	-0.013	-0.010
	30%	0.283	0.040	-0.014	-0.016	-0.012
Contribution of each component (%)						
Direct Effect of $\underline{a}_t$	10%	125	196	33	-214	-251
	19.46%	127	208	35	-231	-276
	30%	128	216	40	-247	-300
Indirect Effect of r_t	10%	-13	-20	105	246	299
	19.46%	-14	-22	103	262	325
	30%	-15	-25	100	278	350
Indirect Effect of $w_t + T_t$	10%	-11	-74	-239	-133	-148
	19.46%	-11	-82	-234	-134	-150
	30%	-12	-87	-238	-134	-150

Note: Columns display the elasticity and decomposition of aggregate of consumption over the described time-frame. The contribution elements are in % of the general equilibrium steady-state deviation

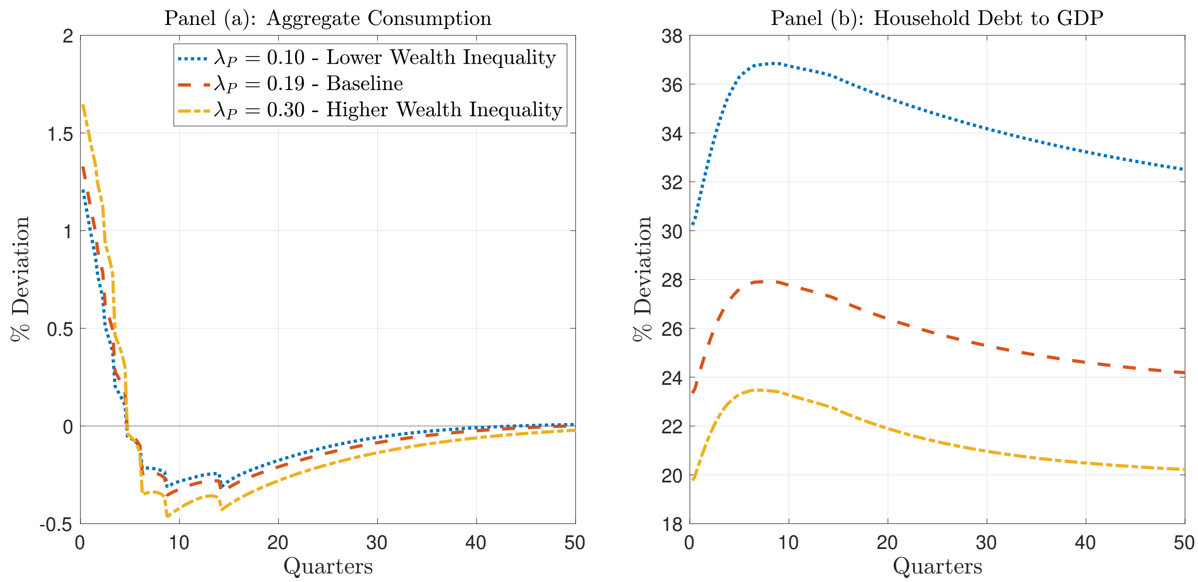
4.2.2 Permanent Income Risk Experiment

In this experiment, we alter the arrival rate of permanent income shocks of the individual income process (19), affecting the level precautionary savings of households in the economy. By the mapping proposed in Proposition 2, a marginal increase in λ_P will increase general precautionary savings in the economy. This effect is concentrated in individuals with higher MPCs, composed by those with high permanent income and low wealth. As depicted by (8), these individuals will lower their consumption to save more they have higher potential consumption losses of receiving an income shock. Therefore, the higher λ_P, the lower the wealth inequality in the economy - as well as a lower share of Hand-to-Mouth agents.

With respect to the baseline economy, the wealth gini is 0.864 (0.847) in an economy with lower (higher) λ_P[26]. As in line with the fiscal policy experiment, the economy with higher wealth inequality boasts of a more amplified consumption growth in the short run, succeeded by a bigger consumption "bust" together with a more vigorous recovery. Just as before, this result is is led by stronger channel of direct and indirect credit, as the average credit MPC increases.

[26]We set the alternative the alternative arrival rate of persistent shocks to $\lambda_P = 0.1$ and $\lambda_P = 0.3$

Figure 8: Credit Deepening with More and Less Permanent Income Shocks



Note: On the Panel (a), we plot the consumption IRFs to the same MIT-Shock to the borrowing constraint, but with different levels of wealth inequality induced by different idiosyncratic income risk λ_P; whereas on Panel (b) we plot the corresponding Debt to GDP IRFs

Table 6: Elasticity and Decomposition of the Consumption Response

Elasticity and decomposition of consumption response to a credit shock						
	Arrival Rate λ_P	1yr	3yr	5yr	10yr	20yr
Elasticity of C_t	0.100	0.330	0.051	-0.012	-0.021	-0.017
	0.197	0.240	0.035	-0.012	-0.013	-0.010
	0.300	0.206	0.027	-0.012	-0.010	-0.008
Contribution of each component (%)						
Direct Effect of $\underline{a}_t$	0.100	130	213	122	-237	-289
	0.197	127	208	35	-231	-276
	0.300	125	207	-7	-226	-265
Indirect Effect of r_t	0.100	-14	-26	118	265	329
	0.197	-14	-22	103	262	325
	0.300	-13	-18	92	260	319
Indirect Effect of $w_t + T_t$	0.100	-11	-82	-235	-134	-150
	0.197	-11	-82	-234	-134	-150
	0.300	-11	-85	-183	-136	-154

Note: Columns display the elasticity and decomposition of aggregate of consumption over the described time-frame. The contribution elements are in % of the general equilibrium steady-state deviation

5 Empirical Evidence

The objective of this section is twofold: First, to check whether the short and long-run patterns of aggregate consumption IRFs observed in the model are present in credit expansions in the data. That is, if we can identify short-run “booms” in which there is consumption growth and household debt overshoot, followed by long-run consumption “bust” and a slow deleveraging. And second, whether these “Boom and Bust” cycles amplify with inequality, just as we observed in the model. We want to be clear that the following analysis is not meant to identify any causal relationship but to document the relationship between credit expansions and consumption in the presence of higher or lower inequality.

5.1 Data and Summary Statistics

To tackle the question at hand, we build a country-level unbalanced panel data set with information on household and nonfinancial corporate debt to GDP, inequality measures of income and wealth gini, and national accounts data. The data frequency is annual and covers the time span of 82 countries from 1961-2021, with an average time-span of 25.5 years per country. Moreover, we also take 1 and 3 year differences of these variables, and measure the change in household, firm debt and consumption from year $t-k$ to year t as $\Delta_k d_{it}^{HH}, \Delta_k d_{it}^{F}, \Delta_k c_{it}$. Summary statistics of the sample are displayed in Table 7

The household and nonfinancial corporate debt measures are taken from the IMF global debt database (Mbaye et al. 2018), which is drawn from individual country national statistic sources. We use debt instead of sectoral credit because of the higher availability of the data. For the inequality measures, we take income gini from the SWIID database (Solt 2019), which standardizes the World Institute for Development Economics Research (WIDER) data and other sources while minimizing reliance on problematic assumptions by using as much information as possible from proximate years within the same country. For the wealth gini, we use the World Inequality Database (Chancel et al. 2021), which relies upon data sources on national accounts, survey data, fiscal data, and wealth rankings. Moreover, for aggregate consumption, we rely on the World Bank's World Development Indicators (WDI) database (Bank 2016), and use the deflated time-series.

Table 7: Summary Statistics

	Description	Source	N	Mean	St. Dev.	Min	Median	Max
d^{HH}	Household Debt to GDP	IMF Debt Db.	2,091	39.439	29.533	0.183	34.086	137.939
d^{F}	Firm Debt to GDP	IMF Debt Db.	2,091	71.124	51.299	1.094	64.615	566.649
g^{Inc}	Income Gini	SWIID	2,091	0.337	0.079	0.203	0.320	0.635
g^{W}	Wealth Gini	WID	1,713	0.757	0.067	0.577	0.744	1.002
c	Log Real Consumption	World Bank	2,005	27.285	2.830	21.065	27.216	36.343
Δc	1yr growth Cons.	World Bank	1,988	2.975	4.471	-44.433	3.033	27.041
Δd^{HH}	1yr diff. HH debt	IMF Debt Db.	2,020	0.965	2.780	-24.619	0.713	31.203
Δd^{F}	1yr diff. Firm debt	IMF Debt Db.	2,020	1.133	9.190	-162.367	0.746	128.294
$\Delta_3 c$	3yr growth Cons.	World Bank	1,951	9.181	9.114	-54.332	8.963	63.185
$\Delta_3 d^{HH}$	3yr diff. HH debt	IMF Debt Db.	1,883	2.861	6.586	-42.335	2.323	34.463
$\Delta_3 d^{F}$	3yr diff. Firm debt	IMF Debt Db.	1,883	3.441	20.030	-281.134	2.372	292.198

Note: Log changes and ratios are multiplied by 100 to report changes in percentages or percentage points, Δ and Δ_3 denote one-year and three-year changes respectively.

5.2 Full Dynamic Relation

In this section, we document the relationship between credit expansions, debt and consumption. Given the lack of data on sectoral household credit, we use debt instead motivated by the comovement of credit and debt, the response of household debt after a transitory credit shock in our model and by other papers, as Mian et al. (2017). As a first approach to gauge the dynamic relationship between consumption growth, household debt and inequality, we employ the methodology of Jordà (2005) Local Projections (LPs henceforth). The advantages of LPs reside in higher robustness to misspecification and direct inference of the estimated impulse response function Jordà (2023), as well as flexibility for inclusion of control variables and nonlinearities - which is crucial for our study. However, as argued by Li et al. (2021), these come at the cost of higher variance than its other counterpart, the finite-order VAR.

Our specification is based on Mian et al. (2017), which also studies the relationship between credit, debt and growth, but adapted to include inequality in levels and a nonlinear relation between household debt, consumption growth and inequality. Let c_{it} be the log real consumption and d^{HH}_{it-j} the household debt to GDP. To capture the role of inequality, we alter in the specification $d^{\theta}_{it-j} \in \{d^{HH}_{it-j}, (d^{HH}_{it-j} * g_{it-j})\}$, where $(d^{HH}_{it-j} * g_{it-j})$ is an interaction term between household debt-to-GDP and an inequality measure. The local projection impulse responses to household debt shocks are given by the sequence of coefficients $\{\hat{\beta}^{h}_{\theta,1}\}_{h=1,\ldots,9}$, $\forall\, \theta$, estimated for the following specification:

$$c_{it+h-1} = \alpha_i^h + X_{it-1}\Gamma^h + \sum_{j=1}^{p} \beta_{\theta,j}^h d_{it-j}^{\theta} + \sum_{j=1}^{p} \beta_{F,j}^h d_{it-j}^{F} + \sum_{j=1}^{p} \delta_j^h c_{it-j} + \epsilon_{it+h-1}^h \tag{22}$$

where we include nonfinancial firm debt d_{it-j}^F as a control, lagged consumption c_{it-j} and other controls X_{it-1}, such as: (i) a time trend; (ii) an inequality variable in levels and (iii) country fixed-effects α_i^h. The time trend is to control for the expansion of private credit over the past four decades (Mian et al. 2017), while the inequality term g_{it-1} to observe the impact in levels of household debt on consumption growth when controlling for inequality. As in Mian et al. (2017), we choose the quantity of lags $p = 5$ based on the Akaike criterium.

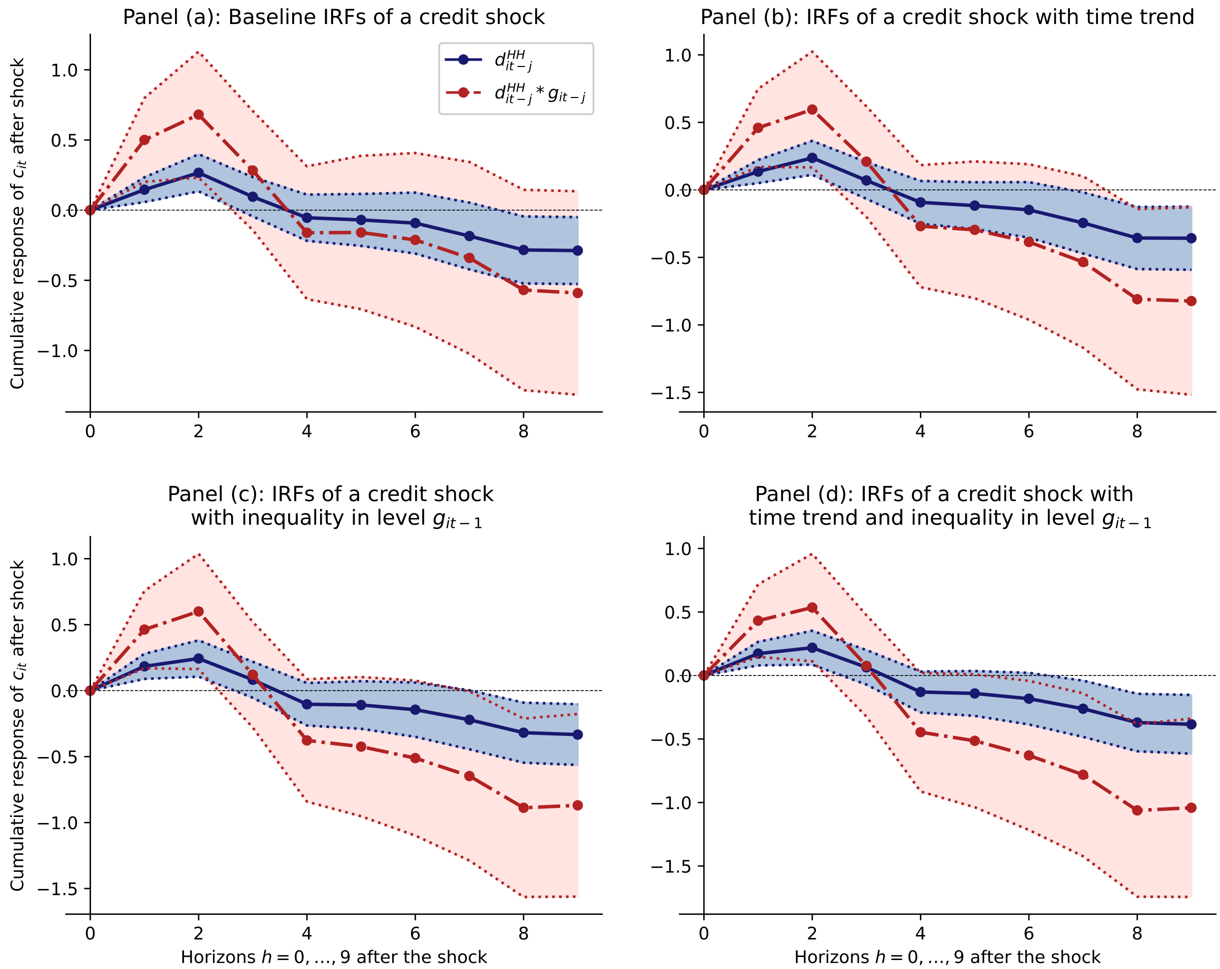
All panels in Figure 9 reiterate the main story studied in our model, that is, that inequality has a substantial impact on the sensibility of consumption growth to a credit deepening: it amplifies consumption booms in the short run and busts in the long run. In the short run, the positive impact of a household debt shock over consumption lasts for 3-4 years (timing similar to the model simulated consumption IRF in Figure 5) and, when accounting for inequality, the impact is three times as large as the specification which does not account for this nonlinearity. However, in the medium and long run (4 or 5 years after the shock), the specification with the non-linearity accounting for inequality predicts a consumption decrease at least twice as large. Moreover, both specifications exhibit a persistent decrease in consumption. As we include other controls, the results are robust in sign and magnitude.

5.3 Robustness using Single Equation Estimation

The exercise in Section 5.2 goes in hand with what we expect from the model, predicting that in countries with higher income or wealth inequality, the relationship between a household debt shock and the response of aggregate consumption is amplified. However, how robust is this relationship? Can we study it through a different specification? To address this question, we turn to a single-equation estimation of the impact of household debt growth trends on consumption growth trends when controlling for income and wealth inequality measures.

The variables are the same as before, with log real consumption c_{it}, household and firm debt to GDP d_{it}^{HH}, d_{it}^F and g_{it} an inequality measure. Given the presence of country fixed effects α_i and a set of controls X_{it}, we estimate the following regression based on Mian et al. (2017), but now accounting for inequality by estimating the specification with

Figure 9: Aggregate Consumption IRFs from a Household Debt Shock



Note: Impulse response functions (IRFs) from Jordà (2005) estimated in levels. The specification is given by $c_{it+h-1} = \alpha_i^h + X_{it-1}\Gamma^h + \sum_{j=1}^{p} \beta^h_{\theta,j} d^{\theta}_{it-j} + \sum_{j=1}^{p} \beta^h_{F,j} d^F_{it-j} + \sum_{j=1}^{p} \delta^h_j c_{it-j} + \epsilon^h_{it+h-1}$ for horizons $h = 1, \ldots, 9$. The blue line stands for the cumulative response of aggregate consumption to a household debt shock d^{HH}_{it}, while the red one stands for the response taking into account the sensibility with respect to income gini, $(d^{HH}_{it-j} * g_{it-j})$. Panel (a) stands for the baseline regression; (b) including a time trend; (c) including inequality in level g_{it-1} and (d) both controls. Shaded regions correspond for 95% confidence intervals computed using dually clustered standard errors (country and year)

and without interaction, $d^{\theta}_{it-j} \in \{d^{HH}_{it-j}, (d^{HH}_{it-j} * g_{it-j})\}$:

$$\Delta_3 c_{it+k} = \alpha_i + \beta_{\theta,k} \Delta_3 d^{\theta}_{it-1} + \beta_{F,k} \Delta_3 d^F_{it-1} + X_{it-1}\Gamma_k + u_{it+k} \tag{23}$$

for $k = -1, 0, \ldots, 5$. Therefore, we fix the right-hand side variables to be the change in household debt from 4 years ago to last year, and vary output growth trends from contemporaneous (same period of the household debt growth) from up to 5 years ahead. For instance, when $k = 2$, $\beta_{\theta,k}$ captures the effect of growth from 4 to one year ago into 3 years ahead. Results are displayed in Table 7, depicting clearly the "Boom" and "bust" cycle, as predicted in our model. In columns (1)-(4) of the table, there is a positive and robust correlation at the short term of Household debt growth and aggregate consumption growth, which extends for $k = -1, 0, 1$ and is almost three times higher when accounting for inequality interaction with household debt $d^{HH}_{it-j} * g_{it-j}$. For further periods ahead shown in columns (5)-(12) ($k \geq 2$), the relationship inverts and debt growth predicts a consumption growth slump, which is also intensified 3 to 4 times when accounting for the interaction with inequality. All the results show a story that goes hand in hand with our model: Inequality predicts stronger growth in the short run, but at the cost of a more intense recession in the long-run.

Table 8: Single Equation Estimation of Consumption Growth Trends to Household Debt Growth Accounting for Inequality

	Dependent variable: $\Delta_3 c_{it+k}, k=-1,0,\ldots,5$											
	$\Delta_3 c_{it-1} = c_{it-1} - c_{it-4}$				$\Delta_3 c_{it+2} = c_{it+2} - c_{it-1}$				$\Delta_3 c_{it+5} = c_{it+5} - c_{it+2}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$\Delta_3 d^{HH}_{it-1}$	0.231**	0.254***		0.290	−0.067†	−0.061		−0.013	−0.117*	−0.130**		−0.067*
	(0.056)	(0.056)		(0.272)	(0.038)	(0.039)		(0.199)	(0.199)	(0.199)		(0.199)
$\Delta_3 d^{HH}_{it-1} * g_{it-4}$			0.783***	−0.092			−0.171	−0.129			−0.411**	−0.207
			(0.811)	(0.811)			(0.115)	(0.566)			(0.139)	(0.566)
$\Delta_3 d^{F}_{it-1}$	−0.007	−0.010	−0.003	−0.006	−0.037***	−0.038***	−0.039***	−0.039***	−0.021**	−0.020**	−0.021**	−0.020**
	(0.013)	(0.013)	(0.178)	(0.014)	(0.008)	(0.008)	(0.009)	(0.009)	(0.007)	(0.007)	(0.007)	(0.007)
g_{it-4}		0.058	0.085	0.097		0.071	0.071	0.070		−0.058	−0.053	−0.054
		(0.186)	(0.178)	(0.175)		(0.237)	(0.238)	(0.239)		(0.252)	(0.251)	(0.252)
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
R^2	0.030	0.043	0.049	0.047	0.029	0.033	0.039	0.018	0.023	0.027	0.029	0.019
Observations	1969	1880	1770	1770	1846	1772	1722	1722	1638	1581	1555	1555

Note: The current table presents estimates for the specification $\Delta_3 c_{it+k} = \alpha_i + \beta_{HH,k}\Delta_3 d^{HH}_{it-1} + \beta_{F,k}\Delta_3 d^{F}_{it-1} + \beta_{IN,k} g_{it-4} + +\beta_{HH-IN,k}(g_{it-4} * \Delta_3 d^{HH}_{it-1}) + u_{it+k}$ for $k = -1, 2, 3$. All specifications include country fixed effects. Reported R^2 values are from within-country variation. Standard errors in parentheses are dually clustered on country and year. The symbols ***, **, *, † indicate significance at 0.1%,1%,5%,10% levels respectively.

6 Conclusion and Next Steps

In this paper, we study how inequality affects the response of consumption to a credit deepening. To do such, we relied on an incomplete-market *Heterogeneous Agents* model, relying on preference and idiosyncratic income risk to generate income and wealth inequality. From this framework, we have derived analytic results mapping inequality to consumption and credit behavior, and also we estimated it with high-quality micro and macro data to generate quantitatively reliable consumption responses.

By conducting credit expansion experiments under different levels of initial inequality, we have concluded that the response of consumption to a credit deepening is composed by a direct and indirect channel, and that wealth inequality amplifies it. The direct channel is quantitatively more relevant and is matched to micro-level consumption and leverage behavior, whereas the indirect channel captures budget and intertemporal substitution effects. Moreover, we also tested our model results empirically in a cross-country dataset. It corroborated the idea that inequality amplifies the response of consumption to a credit deepening, indicating that when accounting for inequality, the variation of the cycle can be three times as large.

There are several areas to be addressed in the future within this project. First, Proposition 1 only maps the static response of consumption to the credit deepening, whereas the relevant object is given by the intertemporal response. An expansion of that analytic result would be necessary to fully understand the role of inequality in a credit deepening. Second, empirical analysis capturing heterogeneous responses of consumption to identified credit shocks could validate our theoretical results empirically.

Third, as we focused on a richer household heterogeneity setting, the general equilibrium framework is simplified and may display results that are counterfactual. For instance, a credit shock generating just a recession with consumption booms is at odds with the data. A New-Keynesian framework could capture these dynamics. Fourth, other complementary analysis can be done with respect to a credit deepening. Mian et al. (2017) associate credit expansions with lower borrowing spreads, a feature which can also be studied in this framework. Fifth, state-dependent models may be a better alternative to capture the nonlinear relationship between inequality, consumption and credit expansions in the data. By doing such, we may move one step further in avoiding misspecification and biases that may overstate the role of inequality measured in the data.

References

Yves Achdou, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. *The review of economic studies*, 89(1):45–86, 2022.

S Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994.

Adrien Auclert, Matthew Rognlie, and Ludwig Straub. The intertemporal keynesian cross. *ERN: Other Macroeconomics: Monetary & Fiscal Policies (Topic)*, 2018.

World Bank. *World Development Indicators 2016*. Number 23969 in World Bank Publications - Books. The World Bank Group, October 2016. ISBN ARRAY(0x414fd4f0). URL https://ideas.repec.org/b/wbk/wbpubs/23969.html.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The wealth distribution in bewley economies with capital income risk. *Journal of Economic Theory*, 159:489–515, 2015. ISSN 0022-0531. doi: https://doi.org/10.1016/j.jet.2015.07.013. URL https://www.sciencedirect.com/science/article/pii/S0022053115001362.

Christopher Busch, David Domeij, Fatih Guvenen, and Rocio Madera. Skewed idiosyncratic income risk over the business cycle: Sources and insurance. *American Economic Journal: Macroeconomics*, 14(2):207–42, April 2022. doi: 10.1257/mac.20190019. URL https://www.aeaweb.org/articles?id=10.1257/mac.20190019.

Christopher Carroll, Jiri Slacalek, Kiichi Tokuoka, and Matthew N. White. The distribution of wealth and the marginal propensity to consume. *Quantitative Economics*, 8(3): 977–1020, 2017. doi: https://doi.org/10.3982/QE694. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/QE694.

Carlos Carvalho, Nilda Pasca, Laura Souza, and Eduardo Zilberman. Macroeconomic effects of credit deepening in latin america. *Journal of Money, Credit and Banking*, 55(7):1817–1855, 2023. doi: https://doi.org/10.1111/jmcb.12936. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jmcb.12936.

Lucas Chancel, Thomas Piketty, Emmanuel Saez, and Gabriel Zucman. World Inequality Report 2022. PSE-Ecole d'économie de Paris (Postprint) halshs-03693233, HAL, December 2021. URL https://ideas.repec.org/p/hal/pseptp/halshs-03693233.html.

Lawrence J. Christiano, Martin Eichenbaum, and Sergio Rebelo. When is the government spending multiplier large? *Journal of Political Economy*, 119:78 – 121, 2009.

James S. Cloyne, Clodomiro Ferreira, and Paolo Surico. Monetary policy when households have debt: New evidence on the transmission mechanism. *Macroeconomics: Monetary & Fiscal Policies eJournal*, 2016.

Gauti B Eggertsson and Paul Krugman. Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *The Quarterly Journal of Economics*, 127(3):1469–1513, 2012.

Martin Flodén, Matilda Kilström, Jósef Sigurdsson, and Roine Vestman. Household debt and monetary policy: Revealing the cash-flow channel. *ERN: Other Microeconomics: Intertemporal Consumer Choice & Savings (Topic)*, 2016.

Milton Friedman. *Theory of the Consumption Function*. Princeton University Press, 1957. URL http://www.jstor.org/stable/j.ctv39x7zh.

Veronica Guerrieri and Guido Lorenzoni. Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467, 2017.

Fatih Guvenen, Fatih Karahan, Serdar Ozkan, and Jae Song. What do data on millions of u.s. workers reveal about lifecycle earnings dynamics? *Econometrica*, 89(5):2303–2339, 2021. doi: https://doi.org/10.3982/ECTA14603. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA14603.

Fatih Guvenen, Luigi Pistaferri, and Giovanni L. Violante. Global Trends in Income Inequality and Income Dynamics: New Insights from GRID. NBER Working Papers 30524, National Bureau of Economic Research, Inc, September 2022. URL https://ideas.repec.org/p/nbr/nberwo/30524.html.

Jonathan Heathcote, Kjetil Storesletten, and Giovanni L. Violante. Quantitative macroeconomics with heterogeneous households. *Annual Review of Economics*, 1(1):319–354, 2009. doi: 10.1146/annurev.economics.050708.142922.

Zhengxin Huo and José-Víctor Ríos-Rull. Financial frictions, asset prices, and the great recession. *Macroeconomics: Consumption*, 2016.

Tullio Jappelli and Luigi Pistaferri. The consumption response to income changes. *Annual Review of Economics*, 2(1):479–506, 2010. doi: 10.1146/annurev.economics.050708.142933.

Callum Jones, Virgiliu Midrigan, and Thomas Philippon. Household leverage and the recession. Technical report, National Bureau of Economic Research, 2011.

Òscar Jordà. Estimation and inference of impulse responses by local projections. *American Economic Review*, 95(1):161–182, March 2005. doi: 10.1257/0002828053828518. URL https://www.aeaweb.org/articles?id=10.1257/0002828053828518.

Òscar Jordà. Local projections for applied economics. *Annual Review of Economics*, 15(1): 607–631, 2023. doi: 10.1146/annurev-economics-082222-065846.

Alejandro Justiniano, Giorgio E. Primiceri, and Andrea Tambalotti. Household leveraging and deleveraging. *Household Finance eJournal*, 2013.

Greg Kaplan and Giovanni L. Violante. Microeconomic heterogeneity and macroeconomic shocks. *NBER Working Paper Series*, 2018.

Greg Kaplan and Giovanni L. Violante. The marginal propensity to consume in heterogeneous agent models. *SSRN Electronic Journal*, 2022.

Greg Kaplan, Benjamin Moll, and Giovanni L. Violante. Monetary Policy According to HANK. *American Economic Review*, 108(3):697–743, March 2018. URL https://ideas.repec.org/a/aea/aecrev/v108y2018i3p697-743.html.

D. Krueger, K. Mitman, and F. Perri. Chapter 11 - macroeconomics and household heterogeneity. volume 2 of *Handbook of Macroeconomics*, pages 843–921. Elsevier, 2016. doi: https://doi.org/10.1016/bs.hesmac.2016.04.003. URL https://www.sciencedirect.com/science/article/pii/S1574004816300039.

Per Krusell and Anthony A. Smith. Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. *Macroeconomic Dynamics*, 1(2):387–422, 1997. doi: 10.1017/S1365100597003052.

Dake Li, Mikkel Plagborg-Möller, and Christian K. Wolf. Local Projections vs. VARs: Lessons From Thousands of DGPs. Working Papers 2021-55, Princeton University. Economics Department., March 2021. URL https://ideas.repec.org/p/pri/econom/2021-55.html.

Samba Mbaye, Ms. Marialuz Moreno Badia, and Kyungla Chae. Global Debt Database: Methodology and Sources. IMF Working Papers 2018/111, International Monetary Fund, May 2018. URL https://ideas.repec.org/p/imf/imfwpa/2018-111.html.

Ursula Mello and Tomas Rodriguez Martinez. Trade-induced Local Labor Market Shocks and Asymmetrical Labor Income Risk. Working Papers 1230, Barcelona School of Economics, December 2020. URL https://ideas.repec.org/p/bge/wpaper/1230.html.

Atif Mian and Amir Sufi. House prices, home equity-based borrowing, and the us household leverage crisis. *American Economic Review*, 101(5):2132–56, 2011.

Atif Mian, Amir Sufi, and Emil Verner. Household Debt and Business Cycles Worldwide*. *The Quarterly Journal of Economics*, 132(4):1755–1817, 05 2017. ISSN 0033-5533. doi: 10.1093/qje/qjx017. URL https://doi.org/10.1093/qje/qjx017.

Atif R. Mian, Kamalesh Rao, and Amir Sufi. Household balance sheets, consumption, and the economic slump*. *Quarterly Journal of Economics*, 128:1687–1726, 2013.

Karsten Müller and Emil Verner. Credit allocation and macroeconomic fluctuations. *Available at SSRN: https://ssrn.com/abstract=3781981*, 2023.

Moritz Schularick and Alan M. Taylor. Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review*, 102(2):1029–61, April 2012. doi: 10.1257/aer.102.2.1029. URL https://www.aeaweb.org/articles?id=10.1257/aer.102.2.1029.

Frederick Solt. The Standardized World Income Inequality Database, Versions 8-9. Technical report, 2019. URL https://doi.org/10.7910/DVN/LM4OWF.

Frank van Hoenselaar, Boris Cournède, Federica De Pace, and Volker Ziemann. Mortgage finance across OECD countries. OECD Economics Department Working Papers 1693, OECD Publishing, December 2021. URL https://ideas.repec.org/p/oec/ecoaaa/1693-en.html.

Iván Werning. Managing a liquidity trap: Monetary and fiscal policy. *Macroeconomics: Monetary & Fiscal Policies eJournal*, 2011.

Michael Woodford. Simple analytics of the government expenditure multiplier. *Political Economy: Government Expenditures & Related Policies eJournal*, 2010.

A Appendix

A.1 Proofs

A.1.1 Proof of Proposition 1

Proof. We write the proof for a general consumption-savings problem in partial equilibrium, to then adapt specifically to the model at hand. The agent has stochastic income y_t dependent on the state, θ_t, which follows any arbitrary process dictated by an infinitesimal generator $\mathcal{A}$. We write its problem as:

$$\max_{\{c_t,a_t\}_{s\geq 0}} \mathbb{E}_t \left[\int_t^\infty e^{-\rho s} u(c_s) ds \right]$$
$$\text{s.t. } \dot{a}_t = y_t(\theta_t) + r_t a_t - c_t$$
$$a_t \geq -\underline{a}$$

The proof relies on showing that any path for consumption that is feasible at $(a, y, \theta, t; \underline{a})$, is also feasible at $(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon)$. To do such, we show equivalence of consumption-saving decisions given the budget and borrowing constraints. For any $\varepsilon > 0$:

$$\begin{aligned} \dot{a}_t &= y_t(\theta_t) + r_t a_t - c_t \\ &= y_t(\theta_t) + r_t(a_t - \varepsilon) + r_t\varepsilon - c_t \\ &= (y_t(\theta_t) + r_t\varepsilon) + r_t(a_t - \varepsilon) - c_t \end{aligned} \tag{24}$$

Moreover, this infinitesimal change will also satisfy the borrowing constraint:

$$a_t \geq -\underline{a} \Leftrightarrow a_t - \varepsilon \geq -(\underline{a} + \varepsilon) \tag{25}$$

Therefore, via 24 and 25, the consumption policy function that satisfies $(a, y, \theta, t; \underline{a})$, must also satisfy $(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon)$, which implies:

$$c(a, y, \theta, t; \underline{a}) = c(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon) \tag{26}$$

Since this is valid $\varepsilon > 0$, we differentiate 26 with respect to ε to obtain:

$$\frac{\partial c}{\partial \underline{a}}(a, y, \theta, t; \underline{a}) = \frac{\partial c}{\partial a}(a, y, \theta, t; \underline{a}) - r_t \frac{\partial c}{\partial y}(a, y, \theta, t; \underline{a}) \tag{27}$$

In the case of out model, we have that $\theta_t = z_t$ and $y_t = (1-\tau_t)w_t z_t + T_t$. Therefore, infinitesimal changes in y that do not come from $\log z_t$ come from a lump-sum transfer, adapting the result to the formula in 1:

$$\frac{\partial c_t}{\partial \underline{a}}(a,z;\underline{a}) = \frac{\partial c_t}{\partial a}(a,z;\underline{a}) - r_t \frac{\partial c_t}{\partial T}(a,z;\underline{a})$$

□

A.1.2 Proof of Proposition 2

Proof. Follows directly from Achdou et al. (2022) □

A.1.3 HJB and KF equations

A.2 Calibration Data and Estimates

A.2.1 Micro data on Income

For calibration of the income process 19, we need to rely on information of high-order moments of the income distribution. We rely on the novel Global Repository of Income Dynamics (GRID) database, which contains a wide range of micro statistics on income inequality and dynamics. Its quality is based on two pillars: (i) it is build on micro panel data drawn from administrative records; and (ii) it is cross-country comparable. The data is provided in the form of annual panel data, over which we take average samples. The moments we target in the calibration of step 1 are shown in Table 8 below

Table 9: Targeted Moments of the distribution of Log Earnings Growth

	Standard Deviation			Kurtosis		P9050		P5010	
	$\Delta t = 0$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$
Argentina	0.992	0.622	0.824	10.472	7.262	0.517	0.855	0.512	0.814
Brazil	1.041	0.682	0.825	8.535	6.347	0.614	0.845	0.708	0.943
Canada	0.799	0.503	0.696	14.926	10.044	0.379	0.653	0.352	0.601
Denmark	0.581	0.407	0.561	17.634	11.622	0.255	0.447	0.246	0.430
France	0.671	0.453	0.579	15.863	11.682	0.254	0.422	0.258	0.447
Germany	0.767	0.384	0.533	17.803	11.210	0.241	0.454	0.183	0.382
Italy	0.775	0.448	0.560	16.980	13.323	0.260	0.434	0.233	0.386
Mexico	1.123	0.650	0.902	8.293	6.034	0.617	0.946	0.649	1.066
Norway	0.771	0.517	0.747	17.047	12.574	0.302	0.540	0.317	0.547
Sweden	0.603	0.425	0.604	15.522	10.070	0.300	0.520	0.278	0.496
Spain	0.776	0.482	0.681	14.084	9.152	0.333	0.514	0.329	0.731
US	0.937	0.567	0.785	12.865	8.827	0.437	0.716	0.461	0.771

Note: Sample averages of the Standard Deviation of log residual income (i); (ii)-(iii) Standard Deviation of residual 1 and 5-year log income changes; (iv)-(v) Kurtosis of residual 1 and 5-year log income changes; and (vi)-(ix) 90^{th} to 50^{th} and 50^{th} to 10^{th} Percentile difference of residual 1 and 5-year log income changes. Values expressed are sample averages of the time-series of moments

A.2.2 Macro Data on Wealth and Fiscal Policy

For the calibration of the steady state of the model, we focus on matching moments of wealth data and also cash transfers as fiscal data. For the former, Wealth to Income Ratio and Wealth Shares per Quintile are drawn from World Inequality Database (Chancel et al. 2021). For the latter, we use cash transfers as a share of GDP, which is available at the OECD database[27] Moreover, for household debt, we focus on non-secured household debt, which is drawn from van Hoenselaar et al. (2021). All data is in annual frequency, and WID and OECD data are providaded as panel data, whereas household debt excluding mortgages has only one observation (2018). All the targeted moments are sample averages, reported in the Table 9 below:

[27] https://data.oecd.org/socialexp/social-benefits-to-households.htm

Table 10: Targeted Moments of the Wealth Distribution & Fiscal Policy

	Wealth to Income Ratio	Wealth Shares per Quintile $P0P20$	$P20P40$	$P40P60$	$P60P80$	$P80P100$	Household Debt to GDP	Transfers to GDP
Argentina	2.115	-0.011	0.024	0.067	0.147	0.773	-	-
Brazil	3.613	-0.021	0.012	0.038	0.095	0.877	23.20	14.565
Canada	5.255	-0.010	0.028	0.078	0.167	0.737	40.15	9.296
Denmark	4.676	0.002	0.014	0.060	0.205	0.718	21.16	16.755
France	6.090	0.004	0.021	0.078	0.166	0.731	25.91	18.359
Germany	5.221	-0.009	0.017	0.073	0.176	0.743	14.87	16.827
Italy	6.277	-0.013	0.033	0.082	0.185	0.713	21.72	17.563
Mexico	4.210	-0.022	0.012	0.037	0.094	0.879	-	2.085
Norway	6.371	-0.041	0.028	0.109	0.218	0.687	-	13.829
Spain	7.331	0.000	0.033	0.095	0.161	0.711	19.96	13.550
Sweden	3.288	-0.010	0.027	0.078	0.165	0.740	23.11	14.303
US	4.883	-0.015	0.004	0.032	0.121	0.859	27.09	12.774

Note: Targeted moments in the calibration of the steady state of the model. All series besides Household debt to GDP are sample averages of the available data. Household Debt to GDP stands for non-mortgage debt

A.2.3 Parameter Estimates

The cross-country parameters estimated with micro and macro data on income and wealth are depicted below. For those of the first step, which refer to the estimates of the income process, see Table 10. For those of the second step, referring to the steady-state parameters, see Table 11

Table 11: Estimated Set of Paremeters of the Income Process for each Country

	Persistence β_{z^T}	β_{z^P}	Frequency λ_{z^P}	λ_{z^T}	Variance σ_{z^P}	σ_{z^T}
Argentina	0.6948	0.0184	0.0101	0.1752	0.9840	1.8053
Brazil	0.7910	0.0019	0.0082	0.1776	1.1716	1.6927
Canada	•	•	•	•	•	•
Denmark	0.0136	0.5415	0.1901	0.0169	0.6973	0.9557
France	0.0245	0.9771	0.0666	0.0147	1.3584	0.8556
Germany	•	•	•	•	•	•
Italy	0.0145	0.2981	0.1685	0.0046	1.8668	0.4494
Mexico	0.7487	0.0018	0.0087	0.1792	1.1874	1.7951
Norway	•	•	•	•	•	•
Sweden	•	•	•	•	•	•
Spain	0.0266	0.4396	0.1076	0.0371	0.6503	0.7762
United States	0.0115	0.5005	0.1978	0.0170	0.8164	1.0257

Note: For the values with •, there has not been estimates yet. To be done in the future

Table 12: Estimated Set of Paremeters of the Steady-State for each Country

	ρ	∇_1	∇_2	τ	$\underline{a}$
Argentina	•	•	•	•	•
Brazil	0.0743	0.0033	0.0020	0.2170	-2.5414
Canada	•	•	•	•	•
Denmark	•	•	•	•	•
France	•	•	•	•	•
Germany	•	•	•	•	•
Italy	•	•	•	•	•
Mexico	•	•	•	•	•
Norway	•	•	•	•	•
Sweden	•	•	•	•	•
Spain	•	•	•	•	•
United States	0.0521	0.0065	0.0003	0.1962	-2.3044

Note: For the values with •, there has not been estimates yet. To be done in the future

A.3 Numerical Appendix

A.3.1 HJB equation - Steady State

For the sake of simplicity, I drop ex-ante heterogeneity $(i, j) \in \mathbb{I} \times \mathbb{J}$ subscripts in the value function:

$$\begin{aligned} \rho V(a, z^T, z^P) = & \max_{c \geq 0} u(c) + \partial_a V(a, z^T, z^P) s(a, z^T, z^P) \\ & + \partial_{z^P} V(a, z^T, z^P)(-\beta_P z^P) + \partial_{z^T} V(-\beta_T z^T) \\ & + \lambda_\eta \int_{-\infty}^{\infty} \Big(V(a, z^P, s) - V(a, z^P, z^T) \Big) \phi_\eta(s) ds \\ & + \lambda_\varepsilon \int_{-\infty}^{\infty} \Big(V(a, u, z^T) - V(a, z^P, z^T) \Big) \phi_\varepsilon(u) du \end{aligned} \tag{28}$$

which can be rewritten in reduced form by using the inifnitesimal generator $\mathcal{L}$ notation

$$\rho V(a, z^T, z^P) = \max_{c \geq 0} \Big\{ u(c) + \mathcal{L}(a, z^T, z^P, c)[V] \Big\} \tag{29}$$

The problem is initially cast onto a tridimensional grid of individual liquid asset holdings a, log permanent productivity z^P and log transitory productivity z^T. For the income components, we work with a symmetric power grid around zero based on the fact that, as we have a compound poisson process with a normally distributed jump around zero, transitions to regions nearby are more likely. For the asset grid, we work work with a non-symmetric power grid with more points closer to the borrowing constraint as the value and policy functions are strictly concave these regions.

Let the (a, z^P, z^T) gridpoints be represented by, respectively, the indexes i, j, k such that we define $V(a, z^P, z^T) \equiv V_{i,j,k}$. We proceed by using a finite-difference discretization method of the HJB equation (28), which is a Partial Differential Equation, ensuring that the solution of the equation converges to the real one - or at least to a weak solution (a viscosity solution). To guarantee this convergence in general terms - that is, when discretizing the PDE as a whole, we resort to Barles & Souganidis (1991):

Definition 3 (Barles & Souganidis (1991) convergence conditions)**.** A numerical finite difference scheme converges to the viscosity solution as long as it satisfies the three following conditions:

1. **Consistency**: when approximating derivatives by finite differences, the approximation will converge to the true value as the grid gets finer
2. **Stability**: The numerical scheme doesn't explode
3. **Monotonicity**: The values that we are solving for depend positively on those that we have already determined

To ensure that three conditions are satisfied and that we can use an arbitrarily large step size Δ in the iterative step, we rely on a implicit numerical scheme with the upwind method. The forward and backward approximations of the partial derivatives with respect to assets and log income components are defined as:

$$\partial_{a,F} V_{i,j,k} = \frac{V_{i+1,j,k} - V_{i,j,k}}{\Delta a_{i,+}}; \quad \partial_{z^P,F} V_{i,j,k} = \frac{V_{i,j+1,k} - V_{i,j,k}}{\Delta z^P_{j,+}}; \quad \partial_{z^T,F} V_{i,j,k} = \frac{V_{i,J,k+1} - V_{i,j,k}}{\Delta z^T_{k,+}}$$
$$\partial_{a,B} V_{i,j,k} = \frac{V_{i,j,k} - V_{i-1,j,k}}{\Delta a_{i,-}}; \quad \partial_{z^P,F} V_{i,j,k} = \frac{V_{i,j,k} - V_{i,j-1,k}}{\Delta z^P_{j,-}}; \quad \partial_{z^T,F} V_{i,j,k} = \frac{V_{i,J,k} - V_{i,j,k-1}}{\Delta z^T_{k,-}}$$

where $\Delta a_{i,+} \equiv a_{i+1} - a_i$ and $\Delta a_{i,-} \equiv a_i - a_{i-1}$, such that the upwind approximation is:

$$\begin{aligned}
\partial_a V_{i,j,k} &= \partial_{a,F} V_{i,j,k} \mathbb{1}_{\{s_{i,j,k} > 0\}} + \partial_{a,B} V_{i,j,k} \mathbb{1}_{\{s_{i,j,k} < 0\}} + \partial_a \bar{V}_{i,j,k} \mathbb{1}_{\{0 < s_{i,j,k} < 0\}} \\
\partial_{z^P} V_{i,j,k} &= \partial_{z^P,F} V_{i,j,k} \mathbb{1}_{\{\mu(z^P_j) > 0\}} + \partial_{z^P,B} V_{i,j,k} \mathbb{1}_{\{\mu(z^P_j) < 0\}} \\
\partial_{z^T} V_{i,j,k} &= \partial_{z^T,F} V_{i,j,k} \mathbb{1}_{\{\mu(z^T_k) > 0\}} + \partial_{z^T,B} V_{i,j,k} \mathbb{1}_{\{\mu(z^T_k) < 0\}}
\end{aligned} \tag{30}$$

where the term $\partial_a \bar{V}_{i,j,k} \mathbb{1}_{\{0<s_{i,j,k}<0\}} = u'(y_{j,k} + ra_i)$ captures the points where savings are equal to zero. Define the initial step[28] as the value of "staying put"

$$V^0_{i,j,k} = \frac{u(y_{j,k} + ra_i)}{\rho}$$

such that, given the iteration step Δ and the transition flows $\lambda_\varepsilon(j, j'), \lambda_\eta(k, k')$ calculated in the Income Process section, the Implicit method discretization of equation (28) is defined as:

$$\begin{aligned} \frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta} + \rho V^{n+1}_{i,j,k} = u(c^n_{i,j,k}) + \partial_a V^{n+1}_{i,j,k}(s^n_{i,j,k}) - \partial_{z^P} V^{n+1}_{i,j,k}(\beta_{z^P} z^P_j) - \partial_{z^T} V^{n+1}_{i,j,k}(\beta_{z^T} z^T_k) \\ + \sum_{k' \neq k} \lambda_\eta(k, k')(V^{n+1}_{i,j,k'} - V^{n+1}_{i,j,k}) + \sum_{j' \neq j} \lambda_\varepsilon(j, j')(V^{n+1}_{i,j',k} - V^{n+1}_{i,j,k}) \end{aligned} \tag{31}$$

where $n + 1$ is the next step of the iteration and n the current step, and the policy functions $c^n_{i,j,k}, s^n_{i,j,k}$ are derived from the current value function:

$$\begin{aligned} c^n_{i,j,k} &= (u')^{-1}[\partial_a V^n_{i,j,k}] \\ s^n_{i,j,k} &= y_{j,k} + ra_i - c^n_{i,j,k} \end{aligned} \tag{32}$$

Plugging the finite-difference upwind approximations in equation (30), we obtain the full form the discretized HJB equation:

$$\begin{aligned} \frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta} + \rho V^{n+1}_{i,j,k} = u(c^n_{i,j,k}) &+ \frac{V^{n+1}_{i+1,j,k} - V^{n+1}_{i,j,k}}{\Delta a_{i,+}}(s^{n,F}_{i,j,k})^+ + \frac{V^{n+1}_{i,j,k} - V^{n+1}_{i-1,j,k}}{\Delta a_{i,-}}(s^{n,B}_{i,j,k})^- \\ &+ \frac{V^{n+1}_{i,j+1,k} - V^{n+1}_{i,j,k}}{\Delta z^P_{j,+}}(-\beta_{z^P} z^P_j)^+ + \frac{V^n_{i,j,k} - V^{n+1}_{i,j-1,k}}{\Delta z^P_{j,-}}(-\beta_{z^P} z^P_j)^- \\ &+ \frac{V^n_{i,j,k+1} - V^n_{i,j,k}}{\Delta z^T_{k,+}}(-\beta_{z^T} z^T_k)^+ + \frac{V^n_{i,j,k} - V^{n+1}_{i,j,k-1}}{\Delta z^T_{k,-}}(-\beta_{z^T} z^T_k)^- \\ &+ \sum_{k' \neq k} \lambda_\eta(k, k')(V^{n+1}_{i,j,k'} - V^{n+1}_{i,j,k}) + \sum_{j' \neq j} \lambda_\varepsilon(j, j')(V^{n+1}_{i,j',k} - V^{n+1}_{i,j,k}) \end{aligned} \tag{33}$$

[28] Which is equivalent of a boundary condition for computing the PDE

Which, in matrix notation, is given by:

$$\frac{1}{\Delta}(V^{n+1} - V^n) + \rho V^{n+1} = u^n + \mathbf{A}^n V^{n+1} + \mathbf{\Lambda}_{z^P} V^{n+1} + \mathbf{\Lambda}_{z^T} V^{n+1} \tag{34}$$

where $\mathbf{\Lambda}_l = \mathbf{\Lambda}_l^D + \mathbf{\Lambda}_l^J$ is the sum of the transition matrices for the drift and jump components of each component of the income process, $l \in \{z^P, z^T\}$. The Vectors V, u^n are of length $I \times J \times K$, while the matrices $\mathbf{A}^n, \mathbf{\Lambda}_{z^P}, \mathbf{\Lambda}_{z^T}$ are of $(I \times J \times K) \times (I \times J \times K)$ dimension. We can simplify further (34) by taking a matrix Λ_z, which represents the sum of the persistent and transitory components of the process, and has the same dimension $(I \times J \times K) \times (I \times J \times K)$ but with different structure:

$$\frac{1}{\Delta}(V^{n+1} - V^n) + \rho V^{n+1} = u^n + \mathbf{A}^n V^{n+1} + \mathbf{\Lambda}_z V^{n+1} \tag{35}$$

Lastly, we compute the iteration by defining the infinitesimal generator matrix of the HJB equation (28) as $\mathbf{L}^n \equiv \mathbf{A}^n + \mathbf{\Lambda}_z$, such that the next step of the iteration of the value function is given by:

$$V^{n+1} = \left[(1 + \Delta\rho)\mathbf{I} - \Delta\mathbf{L}^n\right]^{-1} (\Delta u^n + V^n) \tag{36}$$

which is monotone given that the matrix $\mathbf{M}^n = [(1 + \Delta\rho)\mathbf{I} - \Delta\mathbf{L}^n]$ is a non-negative matrix. To obtain the stationary value function and policy functions, we follow the algorithm in Achdou et al. (2022): Guess an initial value for the value function $V^0_{i,j,k}$ for all the points in the grid and for $n = 0, 1, 2, \ldots$ follow

1. Compute the derivatives $\partial_a V_{i,j,k}, \partial_{z^P} V_{i,j,k}, \partial_{z^T} V_{i,j,k}$ using (30)

2. Compute c^n from (32)

3. Find V^{n+1} from (36)

4. If V^{n+1} is close enough to V^n, stop. otherwise, go to step 1

A.3.2 KF equation

The KF equation for the stationary distribution, dropping ex-ante heterogeneity $(i,j) \in \mathbb{I} \times \mathbb{J}$ subscripts in the productivity-wealth distribution, is given by:

$$\begin{aligned} 0 = &-\partial_a(s(a,z^P,z^T)g(a,z^P,z^T)) - \partial_{z^P}(-\beta_{z^P}z^P g(a,z^P,z^T)) \\ &- \partial_{z^T}(-\beta_{z^T}z^T g(a,z^P,z^T)) - (\lambda_\eta + \lambda_\varepsilon)g(a,z^P,z^T)) \\ &+ \lambda_\eta \phi_\eta(z^T) \int_{-\infty}^{\infty} g(a,z^P,s)ds + \lambda_\varepsilon \phi_\varepsilon(z^P) \int_{-\infty}^{\infty} g(a,u,z^T)du \end{aligned} \tag{37}$$

which can be reduced when using the infinitesimal generator notation:

$$0 = \mathcal{L}^*(a,z^P,z^T,c(a,z^P,z^T))[V]g(a,z^P,z^T)$$

where $\mathcal{L}^*$ is the adjoint operator of the infinitesimal generator. As we already have the matrix **L**, solving for the stationary distribution becomes straightforward as finding the kernel of the transpose of matrix **L**:

$$\mathbf{L}^T g = 0 \tag{38}$$

A.3.3 Income Process

A.3.4 Feynman-Kac formula

We use the Feynman-Kac formula to compute for the Quarterly MPC of a 500 US$, as in Kaplan et al. (2018). Let us define the consumption over a period τ as

$$\tilde{C}_{i,j,\tau}(a,z^P,z^T) = \mathbb{E}\left[\int_0^\tau c_{i,j}(a_t,z_t^P,z_t^T)dt \middle| a_0 = a, z_0^P = z^P, z_0^T = z^T\right] \tag{39}$$

such that the fraction consumed out of x additional units of liquid wealth over a period τ is given by:

$$MPC_\tau^x = \frac{\tilde{C}_{j,\tau}(a+x,z^P,z^T) - \tilde{C}_{j,\tau}(a,z^P,z^T)}{x} \tag{40}$$

The conditional expectation in (39) can be computed via the Feynman-Kac formula:

$$0 = c_{i,j}(a,z^P,z^T) + \mathcal{L}_{i,j}(a,z^P,z^T,0)[\Gamma] + \partial_t \Gamma_{i,j}(a,z^P,z^T,0) \tag{41}$$

where $\tilde{C}_{i,j,\tau}(a, z^P, z^T) = \Gamma_{i,j}(a, z^P, z^T, 0)$, and we set $\Gamma_{i,j}(a, z^P, z^T, \tau) = 0$ as the terminal condition. We solve this PDE numerically, with the matrix $\mathbf{L}$ and the consumption policy function $\mathbf{c}$obtained in the HJB equation, by iterating backwards

$$\begin{aligned} 0 &= c + \mathbf{L}\Gamma_n + \frac{\Gamma_{n+1} - \Gamma_n}{\Delta t} \\ \Gamma_n &= \left[\frac{1}{\Delta t}\mathbf{I} - \mathbf{L}\right]^{-1}\left(c + \frac{1}{\Delta t}\mathbf{\Gamma_{n+1}}\right) \end{aligned} \tag{42}$$

A.3.5 Stationary Equilibrium

A.3.6 Transition Dynamics

A.3.7 Consumption Decomposition

Let $\Theta = (r, w, \underline{a}, T)$. To compute the decomposition (10), I employ the numerical algorithm in Kaplan et al. (2018)

1. Compute the MIT-shock transition path of prices $\{r_t, w_t, \tau_t, T_t\}_{t\geq 0}$ and aggregate consumption $\{C_t\}_{t\geq 0}$ given a path of $\{\underline{a}_t\}_{t\geq 0}$ borrowing constraints or $\{\chi_t\}_{t\geq 0}$ borrowing wedges.

2. Given the path of prices and borrowing constraints $\{\Gamma_t\}_{t\geq 0} = \{r_t, w_t, \tau_t, T_t, \underline{a}_t\}_{t\geq 0}$, we compute the partial equilibrium response of consumption to a time-varying object: Take an input price/allocation path $\theta_t^k \in \Gamma_t$ while leaving the rest held constant at their steady-state values $\theta_t^{-k} = \overline{\theta^{-k}}$ and compute the consumption policy functions $\{c_{i,j}(a, \varphi, \xi, t)\}_{T\geq t\geq 0}$ and infinitesimal operators $\{\mathbf{A}_t\}_{T\geq t\geq 0}$ by solving the transition dynamics of the time-dependent HJB 4 with a terminal condition $\mathbf{V}_T^k = \lim_{t\to\infty} v_t(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$

3. Given the path of infinitesimal operators $\{\mathbf{A}_t\}_{T\geq t\geq 0}$, take the time dependent KF-operators $\{\mathbf{A}^*{}_t\}_{T\geq t\geq 0}$ and compute partial equilibrium distribution objects given $\{\Gamma_t\}_{t\geq 0} = \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0}$, $\mu_t^k = \mu_t(da, d\varphi, d\xi; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$.

4. The aggregate partial-equilibrium consumption response to object θ^k is given by:

$$C_t^{i,j}(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0}) = \int c_{i,j}(a, \varphi, \xi, t; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})\mu_t(da, d\varphi, d\xi; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$$

   Aggregate the function $C_t^{i,j}(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$ for fixed effects i and discount rate heterogeneity j to obtain aggregate consumption.

5. The object dC_t from proposition .... can be numerically computed as deviations from the steady-state

$$dC_t = \sum_{k=1}^{K} \int_{\tau=t}^{\infty} \frac{\partial C_t}{\partial \Theta_{k\tau}} d\Theta_{k\tau} d\tau \approx \sum_{k=1}^{K} \int_{\tau=t}^{\infty} \left(\frac{C_t - C^{SS}}{\Delta \Theta_{k\tau}} \right) \Delta \Theta_{k\tau} d\tau$$