COMP3121 21T2 Assignment 4 Q4

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Since we are determining largest number of black rooks on board, which can be treated as a max flow question with each unit of flow representing the possible position of black rook on board.

First, we need to create a bipartite graph where every row is a vertex on the left, and every column is a vertex on the right side of graph, making $2n$ vertices in total. Then convert bipartite graph into flow network by establishing links between rows and columns based on the bishop position on board. In where bishops locate as well as all its diagonal positions should not have links or have links with capacity of 0, which means black rook cannot be placed at any of these positions. Connect all other vertices with edge capacity of 1, which represents this position can be placed at most one black rook. Next, we add a source s and sink t to this graph, adding directed edges from s to each row with capacity of 1, since each row can only contains one black rook as specified in question. Similarly, we need to add directed edges from each column to sink t with capacity of 1, as each column could only fit one black rook.

We now run the Edmons-Karp algorithm to find the maximum flow through such network, the maximum number of flows which can reach the sink s in the end, is equivalent to the largest number of black rooks which can be placed on board with restriction as required.