COMP3121 21T2 Assignment 4 Q2

Written by Zheng Luo (z5206267)

This question can be solved by combining binary search with max flow. First, we need to create a bipartite graph with n vertices representing n warehouses on the left, and n vertices of shops on the right hand side of graph, making $2n$ vertices in total. Next, create a source s and a sink t, adding directed edges from source s to all warehouse vertices with capacity of 1, since each warehouse only has one truck with enough goods to supply only one shop. Similarly, adding directed edges from shops to sink t with capacity of 1, which represents the status of shops for receiving goods. The time cost for delivering goods through road i is d_i, which can be sorted in increasing order by merge sort with time complexity of $O(nlog(n))$. We can add the road i with minimum time cost into graph based on the sorted list, by adding link between corresponded warehouse and shop with capacity of 1. Then run the Edmons-Karp algorithm to find the maximum flow through such network, if the maximum flow is less to n, which means all shops do not have goods supplied from unique warehouse, then continue the road adding operation until maximum flow is equal to n. Which represents they are enough to obtain a matching of warehouse with shops which is of size n. Eventually, construct the last residual network, and record all occupied links, these are the best route option with minimum time cost, and the total time cost is equivalent to the route with maximum time cost among all occupied links. The overall time complexity is $O(m \times VE^2)$, where m is the total amount of roads we need to added into graph in worst case, V is total vertices, and E is the number of edges in the graph.