COMP3121 21T2 Assignment 3 Q1

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In order to count the number of ways to place brackets in the expression, so that the evaluation will return true, two sub-problems need to be considered. Assuming there is total n symbols and $n-1$ operations in the expression.

1. Number of way to place bracket from l_{th} symbol to r_{th} symbol, resulting the expression as true.
2. Number of way to place bracket from l_{th} symbol to r_{th} symbol, resulting the expression as false.

Hence $T(l_{th}, r_{th})$ can be defined as number of way to place bracket from l_{th} symbol to r_{th} symbol, resulting the expression as true, for example, $true$ AND $false$ OR $true$ will results $T(1,2)$ to $1(true) \times 0(false) = 0$ true expression by evaluating $true$ AND $false$. Therefore the problem can be solved by utilising the strategy of dividing large problem into smaller sets until it cannot be smaller. In the previous example, $true$ AND $false$ OR $true$ can be divided into ($true$ AND $false$) OR ($true$) by separating OR operator, then evaluate the smaller problem sets on each side, and eventually combine all the expressions. In general, this strategy can be standardised as:

$$T(l,r) = \sum_{m=1}^{r-1} TSplit(l,m,r) \tag{1}$$

$$T(l,r) = \sum_{m=1}^{r-1} FSplit(l,m,r) \tag{2}$$

Split functions perform differently based on different operators, the summations are concluded below:

Table 1: The relationship between operator and TSplit function

Opeator\Split function	$TSplit(l,m,r)$
AND	$T(l,m) \times T(m+1,r)$
OR	$T(l,m) \times F(m+1,r) + T(l,m) \times T(m+1,r) + F(1,m) \times T(m+1,r)$
NAND	$T(l,m) \times F(m+1,r) + F(l,m) \times F(m+1,r) + F(1,m) \times T(m+1,r)$
NOR	$F(l,m) \times F(m+1,r)$

Table 2: The relationship between operator and FSplit function

Opeator\Split function	$FSplit(l,m,r)$
AND	$T(l,m) \times F(m+1,r) + F(l,m) \times F(m+1,r) + F(1,m) \times T(m+1,r)$
OR	$F(l,m) \times F(m+1,r)$
NAND	$T(l,m) \times T(m+1,r)$
NOR	$T(l,m) \times F(m+1,r) + T(l,m) \times T(m+1,r) + F(1,m) \times T(m+1,r)$

Overall, the complexity is $O(n^3)$, since there are $O(n^2)$ ranges as well as the situation $O(n)$ in Tsplit and Fsplit need to be covered.