COMP3121 21T2 Assignment 1 Q5

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Part A

Answer:$f(n) = O(g(n))$
Let $f(n) = log_2(n)$ and $g(n) = \sqrt[10]{n}$.
Since L'Hôpital's rule states if $f(x)$, $g(x) \to \infty$ and they are differentiable, then:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)} = \lim_{n\to\infty} \frac{10}{\sqrt[10]{n}} = 0 \tag{1}$$

$f(n) = O(g(n))$ has be proven when there exist positive constants c and n_0 such that $0 \leq f(n) \leq c \times g(n)$ for all $n \geq n_0$, and $g(n)$ eventually dominates the $f(n)$ proven by L'Hôpital's rule.

Part B

Answer:$f(n) = O(g(n))$
Let $f(n) = n^n$ and $g(n) = 2^(n \log_2(n^2))$, and similarly $f(n) = O(g(n))$ can be proven when there exist positive constants c and n_0 such that $0 \leq f(n) \leq c \times g(n)$ for all $n \geq n_0$. Let

$$h(n) = \frac{f(n)}{g(n)} \tag{2}$$

L'Hôpital's rule states if $f(x)$, $g(x) \to \infty$ and they are differentiable, then:

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)} = L \tag{3}$$

Also, Epsilon-Delta Definition of Limit states that: if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that, for all $x \in D$, if $0 < |x - c| < \delta$, then

$$|f(x) - L| < \varepsilon \tag{4}$$

Both $f(n)$ and $g(n)$ are differentiable, and the fraction of $f'(n)$ and $g'(n)$ tends towards 0 when $n \to \infty$, hence,

$$L \to 0 \tag{5}$$

$$\varepsilon > 0 \tag{6}$$

The combination of equations (1),(2),(3), (4), and (5) will result:

$$|\frac{f(n)}{g(n)}| < \varepsilon$$

$$f(n) \leq \varepsilon \times g(n)$$

Since $\varepsilon > 0$ and $c = \varepsilon$, hence $f(n) \leq c \times g(n)$, and eventually $f(n) = O(g(n))$.

Part C

Answer:Neither
Let $f(n) = n^(1 + \sin \pi n)$, $g(n) = n$. Because the existence of sine function, the result can be obtained by considering both even and odd inputs.
When the input is an even number $2k$:

$$f(2k) = (2k)^(1 + \sin 2\pi k) = 2k = n = g(n)$$

When the input is an odd number $2k + 1$:

$$f(2k+1) = (2k+1)^(1 + \sin \pi) = n^2 = g(n)^2$$

However when the input is an odd number $2k - 1$:

$$f(2k-1) = (2k-1)^(1 + 0 - 1) = 1$$

The inconsistency between different inputs are resulting the functions are neither of the two.