Proceedings of SALT 33: 000–000, 2023

The morphosemantics of incremental plurality in Hualapai (Yuman)*

Robert Henderson
University of Arizona

Abstract Hualapai is a Yuman language with a verbal morphological system, seen in various languages of the region, which poses difficulties for a compositional analysis. In particular, Hualapai verb morphology exhibits *incrementality* (see Baerman 2016, 2019, 2024), where there is no one-to-one mapping between forms and meanings. Instead, forms are ordered on a scale tracking morphological complexity, and more complex morphological forms are mapped, all things being equal, to meanings that are higher on some semantic scale. In the case of Hualapai, the incremental system concerns a plurality, which in this case conflates plural argument marking and plural event marking, also known as pluractionality. This paper provides the first compositional morphosemantic treatment of Hualapai verbal plurality, which in addition, is used to think about how we might handle such incremental systems in general using standard morphological and semantic tools.

Keywords: plurality, pluractionality, morphology, incrementality, Yuman

1 Introduction

An areal feature of indigenous languages of the Southwest United States and Northwest Mexico are morphologically complex systems of plura(actiona)lity. In particular, there are multiple instances of unrelated languages implementing so-called 'incremental' or 'scale-based plura(actiona)lity'—that is, there is no one-to-one mapping between exponents and plural meanings (e.g., Seri (Iso) Baerman 2016, Hualapai (Yuman) Baerman 2019, Salinan (Iso) Baerman 2024, etc.). Instead, these languages have a list of plural meanings ordered by some notion of "more plural" (call it $<_s$ for semantic order), along with a list of exponents ordered by some morphosyntactically defined order (call it $<_m$ for morphological order). Paradigms are

* This paper would not be possible without the other members of the *Compositional Morphosemantics of Plurality Group, NSF SBS Grant #1945641*—Homar Aguilar, Matthew Baerman, Gabriela de la Cruz Sánchez, Heidi Harley, Megan Harvey, Jérémy Pasquereau, John W.W. Powell, and Lelia Rhodes. I would like to especially thank John and Jérémy for their expert knowledge of Yuman languages and incremental pluractionality as a areal feature of northwest Mexico and the American Southwest.

well-formed as long as these orders are in scale alignment. More precisely, $\alpha <_m \beta$ iff $[\![\alpha]\!] <_s [\![\beta]\!]$

Such systems immediately generate three questions, First, how do we define the morphological order ($<_m$)? It could be that some forms are longer than other forms, it could be that some forms are morphologically complex than other forms, it could be by fiat, etc. Second, how do we define the semantic order ($<_s$)? It could be by entailment, it could be complexity of features (defined in various ways), etc. Finally, and most challenging, what kind of theory of the morphosemantic interface could generate scale-based morphological systems? The lack of one-to-one mapping between forms and meanings makes meaning composition exceedingly difficult.

The primary goal of this paper is to present a complete account, addressing all three questions for the Hualapai language. The core idea is that Hualapai verbs will bear features that are ordered on a scale and which stand in one-to-one correspondence with the surface forms that we see. These scalar features, on the semantic side, are cashed out as presuppositions that constrain verb meanings, but which are consistent with the range of form-meaning variation that we see across paradigms. The result is locally, within a particular form, compositional proceeds as expected, but globally, across paradigms, the scale-based structure emerges.

In addition to Hualapai, we will also very quickly to the case of Seri, also know as *Cmiique Iitom*, and show what a simple extension from Hualapai is not possible, which we will take to be an important point about these kinds of systems. In particular, it plays a role in showing that we are dealing with systems that involve conventionalization. This is important because, at first pass, one might be tempted to avoid giving these systems a standard compositional account. Instead, one might wish to say that the meaning contribution of the morphology in question is determined iconically, or via some kind of manner implicature—i.e., bigger morphological forms go with bigger pluralities, or more complex forms have more complex (i.e., more plural) meanings. We show emphatically that this is not the case. Our major takeaway, then, beyond the particular analysis we give for Hualapai, is that these are real morphosemantic systems. We must have a compositional theory for them, even if they may make us rethink the nature of morphosemantic composition.

2 The basic morphosemantic system

We begin by looking at the case of Hualapai, a Yuman language spoken in Northwest Arizona along the Colorado River. Watahomigie, Bender, Watahomigie Sr & Yamamoto (2001a) describe verbs as coming in one of four forms.[1]

1 The situation is actually more complex, with verbs sometimes having more or less forms. For instance, this verb *dabil* also has a form *ʔdabiːlj* for a plural (non-paucal) subject acting on a singular

(1) a. ɖabil ‘one burns one’
b. ɖabilj ‘two/a few burn one’
c. ɖaɖbiːl ‘one burns many’
d. ɖaɖbiːlj ‘many burn many’

(2) a. ɖagwan ‘one beats up someone’
b. ɖagwanj ‘two/a few beat up someone’
c. ɖaɖgwan ‘many beat up someone’
d. ɖaɖgwanj ‘many beat up many’

There are three critical things to note: (i) the second forms involve a paucal subject reading, (ii) the fourth forms involve a ‘greater plural’ subject and object reading, and (iii) the third forms involve either a greater plural subject or greater plural object depending on verb class.

The choice to discuss Hualapai plural inflection generally—in terms of first form, second form, etc.—rather than referring to particular affixes, has not been an accident. The reason is that the mapping from specific exponents to meanings is not deterministic, though there are some sub-regularities.

Hualapai instead shows the two features that are definitive of incremental morphological systems (Baerman 2016). There is no one-to-one mapping between meanings and their exponents Meanings, which have shown can be ordered, are instead assigned to exponents to form paradigms somewhat arbitrarily, but in a ways that respect this order. The following figure, inspired by Baerman 2019: fig. 5.3, illustrates these two properties.

Form 1	**Form 2**	**Form 3**	**Form 4**	
gilgyo	gilgyo-j	gilgyo:	gilgyo:-j	‘tie’
hwal	hwa:l	hwa:l-j		‘dig’

Figure 1

First, note that we cannot say that vowel length marks the second form because it actually marks the third form for stem *gilgyo* ‘tie something large’. Similarly, we

object, e.g., *βaːjach iʼí ɖabiːljikwi* ‘People are burning a log’ (Watahomigie, Bender, Watahomigie Sr & Yamamoto 2001b: p. 299). This idea is that these four forms are paradigmatic in occurring across many verb stems. We return to some of these ‘extra’ readings below, but follow Watahomigie et al. on the centrality of these paradigms.

cannot say that *-j* marks the second form because it actually marks third and forth forms for stem *hwal* 'dig'. This is what we mean by a lack of one-to-one mapping between exponents and meanings.

The second property we are interested in is incremental marking, which we can see by considering those forms which seem to be marked by multiple exponents. In particular, note that while vowel length marks different meanings in these two paradigms—Form 3 vs. Form 2—in both cases, having vowel length in addition to the *-j* affix marks a meaning further up the semantic scale we have established.

Incrementality of this particular type is important because it vividly illustrates the idea behind the alignment of morphological and semantic scales absent of one-to-one marking. It is clear bearing both vowel length and an affix is more morphologically complex than only having one or the either—that is, $\alpha <_m \alpha \oplus \beta$. Critically, what we additionally find is that while we cannot predict what an $\alpha \oplus \beta$ form means—note in Figure 5 that it covers different cells in different paradigms—we do know that these double-marked $\alpha \oplus \beta$ forms always coincide with readings that are more plural compared to those that bear α or β alone. In sum, we may not know the meaning of particular forms but incrementality ensures scale alignment. We can be sure that $[\![\alpha]\!] <_s [\![\alpha \oplus \beta]\!]$ in virtue of the morphological order $\alpha <_m \alpha \oplus \beta$.

It is important to pause here and consider what a prima facie challenge data such as these pose for a compositional semantics of the morphology in question. It would be one thing if system were simply unpredictable, but here things are even worse because there is a tantalizing hint of compositionality. The forms with more exponents have meanings that involve greater pluralities.

At least in this restricted case, there is an iconic logic to this pattern—doing $\alpha \oplus \beta$ must yield a more plural meaning than either doing α or β on its own. Could a system like this be easily treated via an appeal to iconicity? We might be tempted to say that a greater number of exponents iconically yields a verbal predicate requiring a greater number of participants. The problem is that the generalization does not hold. While there are pockets of this pattern, it is not always the case that "more exponents" means "more plural". Consider the fact that prefixal plurals, like the *ji-* prefix, uniformly outrank other kinds vowel length, suffixation, or even, critically, both vowel length and suffixation.

Form 1	**Form 2**	**Form 3**	**Form 4**	
jigyo	jigyo-:-j	ji-jgyo	ji-jgyo-:-j	'bite'

Figure 2

The iconic logic falls apart here. There is no reason to think that prefixation, which we see in Form 2 in 2, should be iconically more plural than having two exponents,

which we see in Form 3. While fatal to iconic account, these kinds of data also cause problems for other kinds of accounts, for instance one based on a Manner implicature. Once again, in what sense would prefixation be more complex than suffixation such that it should yield more complex meanings?

Even if it were the case that an iconic account could work on morphological grounds, there would still be semantic hurdles. The simplest kind of iconic account would say that that big forms involve big plurality. The analysis would draw an analogy to how the size of a co-speech gesture can increase the cardinality of a plural participant.

(3) a. People were everywhere (sweeping gesture)

b. People everywhere (bigger sweeping gesture)

In the case of Hualapai, though, it is not at all clear that higher forms require bigger pluralities. This is a point that we will come back to when we give an analysis of the truth conditions of sentences with verbs of various forms, but the crux of the issue is that the Forms 2-4 involve vague notations of cardinality, namely *few* and *many*. The result is that it is hard to control the cardinalities at issue. Consider a scenario where one person burns 500 things. Compare that to a scenario in which 20 people burn 20 things. In the first case we could say that one person burned many things (i.e., Form 3), and in the second we could say that many people burned many things (Form 4), but it is not the case that we have higher cardinalities involved in the Form 4 scenario. Perhaps there is some iconic story you could tell about the meanings involved, but it must not be the simplest such account. Combined with the fact that the morphology is also not necessarily iconic, we have a strong language-internal argument against the iconic approach.

There are further crosslinguisic arguments that incremental plural systems must be dealt with using standard, compositional morphosemantic resources. In particular, due to the fact that iconicity is contrasted with arbitrariness, while we might find crosslinguistic variation in how some iconic sign is interpreted across speech communities, the variation should be rather constrained and transparent. Looking at other languages in the same areal region with similar systems, morphologically speaking (i.e., in terms of form), but rather dramatically different in terms of meaning. This is unexpected on an iconic account.

To make this argument, we will consider the language Seri, which is spoken in northwest Mexico, in two villages on the coast of the Gulf of California, Haxöl Iihom and Socaaix (Marlett 1981). Seri verb stems alternate cross-classifying two features, subject number and pluractionality Pasquereau 2019; Pasquereau & Cabredo Hofherr 2020. The SG vs PL is precisely what you would expect—the latter marks non-atomic subjects. The MULT forms indicates pluralities of events

that take place at different times. Finally, the DIST forms (canonically) indicate a plurality of events distributed over a plural object.

'wrap'		Pluractionality		
		neutral	multiple	dist
Number	singular	*iyacapnij*	*iyacapanl*	*iyacapnalca*
	plural	*iyacapnalcoj*	*iyacapzil*	*iyacapzilca*

Like Hualapai, Seri shows incremental morphological marking, but with respect to a different scale meanings (Baerman 2016; Pasquereau & Cabredo Hofherr 2020).

(4) **The Scale of Plurality**
SG NEUT « SG MULT « SG DIST « PL NEUT « PL MULT (« PL DIST)

First, note that the Seri systems seems to be trafficking in very different semantic notions, and unpredictably so—this is surprising on an iconicity account, and the heart of the crosslinguistic objection. There is nothing like the MULT/DIST contrast in the Hualapai paradigms where verbs have different pluractional meanings up the morphological scale. In the other direction, there is nothing in the Seri paradigms where we rank the cardinality of the subject (i.e., paucal vs many), as we have in Hualapi.

Additionally, note that the Seri examples look even worse on the idea that "big exponents" go with "big plurals" or that "complex exponents" go with "complex meanings". There is no reason that having a plural subject should be more plural / more complex than having a pluractional event with multiple themes, and yet, SG DIST « PL NEUT. Similarly, there is no reason that having a pluractional event with multiple themes should be more plural / more complex than having a plural event with multiple temporal trace, and yet, SG MULT « SG DIST. While we do not have space to develop an account for both Hualapai and Seri in this work, it is clear that meanings involved in the Seri paradigm must be rich in the way that we would not expect if they were transparently accessible via iconic form-meaning mapping.

The main takeaway is that we see these surface-similar incremental plural systems showing up in languages of this region. They are not all trafficking in the same semantic notions, which is important because it shows that this is not just a one off thing. It's not like these languages have a kind of specialized morphology for specialized semantic purposes, and we can chalk it up to iconicity or something. Additionally, iconicity accounts (or relatives that focus on form—i.e., Manner), fail on other morphological or semantic grounds. What we need is a real theory of how incremental systems work compositionally. In the next section we will do this for Hualapai. In particular, we will argue that the exponents we see on the surface,

while not one-to-one linked to truth conditions of verbs, are one-to-one linked to presuppositions which compositionally constrain possible meanings for verb-forms bearing those exponents. Moreover, they constrain them in such a way that the patterns we see across paradigms emerge.

3 A deeper dive into the meaning of Hualapai plural verbs

In the previous subsection, we have ruled out an null hypothesis. We now have to deal with the serious problems at hand: (i) What should the truth conditions of the forms in (5-`[forms-dagwan]`) be? (ii) How do we, based on those truth conditions, place those forms into a semantic order? (iii) How does the morphology we see in the surface contribute to the meanings of these forms, given the lack of one-to-one correspondence between exponents and meanings? (iii) How the morphology we see on the surface come to respect the semantic order given the lack of one-to-one correspondece between exponents and meanings?

(5) a. ɖabil ‘one burns one’
b. ɖabilj ‘two/a few burn one’
c. ɖaɖbiːl ‘one burns many’
d. ɖaɖbiːlj ‘many burn many’

(6) a. ɖagwan ‘one beats up someone’
b. ɖagwanj ‘two/a few beat up someone’
c. ɖaɖgwan ‘many beat up someone’
d. ɖaɖgwanj ‘many beat up many’

We will begin with fleshing out the meanings of these forms before turning to morphology and questions of composition. Our guiding intuition is that we should be thinking about the entire system of verbal plural marking in Hualapai in terms of vague notions of high/low cardinality. First, this is reflected in the translations given by Watahomigie et al. (2001b) throughout the text. In no case do we have a simple singular vs. plural contrast. Instead, all "plurals" are translated with vague cardinality quantifiers: *two/a few*, *many*, *a lot*.

We should, of course, be skeptical of translations, but we see attested examples that are consistent with the meanings as described. For instance, consider the fact that across the paradigms there are no paucal object forms. This suggests that low cardinality objects should not require plural verb forms, which is attested. Consider object number in the following example.

(7) *Nya đálach qwa:q nye:đik qwa:q hwak'm*
nya đal-a-ch qwa:q nye:-đi-k qwa:q hwak-m
I 1.father-DEF-SUBJ deer 3/3.hunt-TEMP/then-SS deer 3.be.two-DS
'u:k gae:kwiny.
u:-k gae:-k-wi-ny
3/3.see-SS 3/3.shoot-SS-AUX/do-PAST

'When he was hunting, my father saw two deer and shot them.' (Watahomigie et al. 2001b: p. 335-6)

The first conjunct makes it clear that there are two deer, but neither the verb *'u:k* "see" or *gae:* "shoot" bears third category plural forms, which should indicate object plurality. The following examples show that such forms indicating object plurality do exist for these verbs, which only emphasizes that non-atomic reference is not what is at issue for this species of "plural" marking.

(8) *bà'gweg'u:ja*
ba'-gwe-g-'u:-**j**-a
person-thing-NOM-see-**PL**-Agent

'person who sees things/researcher' (Watahomigie et al. 2001b: p. 202)

(9) *gáe:jthik jìjiyámk vawímwiny*
gae:-**j**-th-k ji-jiyam-k va-wim-wi-ny
1/3.shoot-**PL**-really-SS **1/3.PL**-miss-SS 1/3.EMPH-intensely.do-AUX/do-PAST

'I really shot at them but I missed them all.' (Watahomigie et al. 2001b: p. 69)

Taking vague cardinality route has an additional virtue in that it connects with the idea, first proposed by Baerman (2019), that third and fourth category forms should be thought of in terms of pluractionality. One piece of evidence is that with intransitives, plural marking of the kind seen in on transitive verbs can clearly indicated repeated actions. For instance, we have contrast like in (10).

(10) a. *điv'ik* 'one person kneels down once'

b. *điv'i:j'k* 'one person kneels down multiple times'

The generalization is that with intransitives, we can mark a paucal subject, but when we add more plural morphology, we get a clear multiple event reading. This has a logic to it. In the case of transitives, where there are more core arguments, these

plural forms can implicate these extra arguments rather than the temporal trace, as we see for intransitives.

If third and fourth forms of the Hualapai verbal number paradigm involve distributive pluractionality, the requirement that the subject / object have a large, unspecified cardinality, would follow from the standard behavior of pluractionals. It is well known that pluractionality, while involving pluralities of events, only rarely demand mere non-atomicity (e.g., Henderson 2012, 2017; Lasersohn 1995; Wood 2007; Hofherr & Laca 2012). It is one of the ways in which pluractionality is not like the most common type of nominal plurality crosslinguistically. Instead, pluractional predicates are often only satisfied by plural events of a sufficiently large, though vague, cardinality. We can turn this unspecified, though large cardinality of events into an equal number of participants, through distributive entailments on an argument. This plurality of events requirement would be paired with a distributive requrement, namely each event has a distinct participant, resulting in plurality of participants whose cardinality is large, though not directly specified.

Once again, there is evidence that we do, in fact, have a requirement for a distributive interpretation for these high cardinality third and fourth forms. Note in (11-12) that collective predicates like *ʠagávk/ʠigávk* 'gather' in Hualapai tolerate singular agreement in the clear presence of a plural subject. In both cases we have a singular subject verb, even paired with explicit plural marking of the subject as in in (12).

(11) *Waksích isavgó búkal ʠigávkyu.*
waksi-ch isavgo buk(a)-l ʠigá-v-k-yu
cow-SUBJ corral foot-at 3.gather-STATE-SS-AUX/be

'The cattle gathered at the corner of the corral (or close to the fence of the corral).' (Watahomigie et al. 2001b: p. 52)

(12) *Ba:jach we ʠagávikyu.*
ba:-j-ch we ʠagáv-k-yu
person-PL-SUBJ DEM/over.that.place-around 3.gather.around-SS-AUX/be

'People are gathering around over there.' (Watahomigie et al. 2001b: p. 294)

This strongly suggests that we don't need plural marking when we don't have a pluractional event, for instance, when we have a large cardinality plural subject participating in a single event collectively.

Summarizing, we want an account in which the following holds: (i) the so-called singular forms are consistent with plural arguments, so long as their cardinality is small, (ii) forms with positive cardinality requirements state them in a vague,

context-dependent way—e.g., like *a few*, *many*, *a lot*, and (iii) third and forth forms require are pluractionals, and in particular, distributive pluractionals. What we do in the next section is provide just such a model-theoretic treatment of verbal number in Hualapai, which, once specified, induces a semantic order $<_s$ between forms that we can correctly align with the morphological order $<_m$.

4 A model-theoretic account of Hualapai verbal number and the semantic scale

Before we can provide an account of Hualapai verbal number, we must first lay out a few assumptions of the analysis. We follow Lasersohn (1995); Link (1983/2002) in taking natural language predicates to denote in a structured domain of individuals D_e and a structured domain of events D_ε. The domain of individuals D_e is the powerset of a designated set of individuals IN minus the empty set $\wp^+(\mathsf{IN}) = \wp(\mathsf{IN}) \setminus \emptyset$. We assume a denumerable set of variables of type e: $x, x', y, y' \ldots$. Similarly, the domain of events D_ε is the powerset of a designated set of events EV minus the empty set $\wp^+(\mathsf{EV}) = \wp(\mathsf{EV}) \setminus \emptyset$. Variables of type ε: $\mathbf{e}, \mathbf{e}' \ldots$. Atomic individuals and atomic events are the singleton sets in $\wp^+(\mathsf{IN})$ and $\wp^+(\mathsf{EV})$, respectively, identified by the predicates $\mathbf{atom}_{et}$ and $\mathbf{atom}_{\varepsilon t}$. A_ε is the set of all atomic events and A_e is the set of all atomic individuals. The 'part of' relation $\leq$ over individuals or events is set inclusion over $\wp^+(\mathsf{IN})$ or $\wp^+(\mathsf{EV})$. The sum operation $\oplus$ is set union over $\wp^+(\mathsf{IN})$ or $\wp^+(\mathsf{EV})$.

We assume in the style of Davidson (1967) that Hualapai verbs are predicates of events, and for simplicity's sake that, that verb stems denote cumulatively closed predicates of atomic events.

(13) Cumulative Closure (following Krifka 1989).
The cumulative closure of P is the smallest predicate $*P$ such that:

a. $P \subseteq *P$

b. if $a \in *P$ and $b \in *P$, then $a \oplus b \in *P$

We additionally make the standard neo-Davidsonian assumption that events are mapped to their participants by some finite set of thematic roles: **ag**, **th**, etc., which are functions of type εe from events (type ε) to individuals (type e) . We use theta-role functions for their argument indexing ability alone. That is, we do not assume that they generate the traditional entailments about their arguments, e.g., Dowty 1991. We also make the standard assumption that theta-role functions are themselves cumulatively closed, but suppress $*$-notation to keep formulas simpler.[2]

2 Relation Closure based on Krifka 1986: For any n-place relation R, $**R$ is the smallest relation such

The result is that a simple transitive sentence in Hualapai like (14), would have the translation in (15) after existential closure, with the specified truth conditions.

(14) *Ólohch* *John* *gaɬóhkwìny*
olo-h-ch John gaɬoh-k-wi-ny
horse-Dem/that-Subj John 3/3.kick-SS-Aux/do-Past

'The horse kicked John.' (Watahomigie et al. 2001b: p. 177)

(15) $\exists e[*\text{KICK}(e) \wedge \mathbf{ag}(e) = \iota x.*\text{HORSE}(x) \wedge \mathbf{th}(e) = \text{JOHN}]$

With (15) we not only have an illustration of our basic semantic assumptions, but also the account of first category verbs in Hualapai, namely those that bear no plural morphology. Essentially, they are unmarked and unconstrained, which is good because, as we have seen, such verbs are consistent with plural arguments. We saw this, for example, with *qwa:q hwak'm* 'two deer' in (7). We take plural morphology then to add constraints that actively force certain arguments to have particular kinds of plural interpretations.

We can see how this strategy works by considering the second category forms which are associated with a paucal interpretation of the subject. A virtue of our neo-Davidsonian account is that we can actually place constraints on the subject through constraints on the event argument, which is related to entities denoted by the subject through the appropriate theta-role function. Assuming CARD is a measure function, we can treat second form paucals as placing a vague, low, cardinality condition on the agent of the event as in (17).

(16) *Merich* *joq* *gwájik* *Bobm* *hwákak* *gige:vkyu.*
Meri-ch joq gwajik Bob-m hwak-(a)k gigev-k-yu
Mary-SUBJ juniper near Bob-COM 3.two.together-SS 3.stand.PAUC-SS-AUX

'Mary and Bob are standing beside the juniper tree.' (Watahomigie et al. 2001b: p. 51)

(17) *gige:vkyu* $\rightsquigarrow \lambda x \lambda e[*\text{STAND}(e) \wedge \mathbf{ag}(e) = x$
$\wedge \exists n[1 < n < \text{FEW}_{\text{STD}} \wedge \text{CARD}(\mathbf{ag}(e)) = n]]$

that (i) $R \subseteq **R$ and, (ii) if $\langle a_1, \ldots, a_n \rangle \in **R$ and $\langle b_1, \ldots, b_n \rangle \in **R$, then $\langle a_1 \oplus b_1, \ldots, a_n \oplus b_n \rangle \in **R$.

This verb phrase can then felicitously combine with the subject *Merich Bobm* 'Mary and Bob (lit. with Bob)', because its cardinality is greater than one and less than the contextually specified standard for FEW.

When we move to third category forms we have analytical options. For instance, we could provide a uniform semantics for such forms and let morphosyntactic considerations generate the difference between those that require a large cardinality subject and those that require a large cardinality object. In the absence of detailed syntactic work on Hualapai, we will instead focus on the truth conditions and treat the two subclasses of Hualapai transitive verbs as having the same semantic template, but targeting different thematic roles. Recall, though, that we want to treat third and fourth forms as pluractionals, in particular, as distributive pluractionals. The large cardinality of the subject/object is due to requiring a large cardinality of events be distributed across some argument.

To implement distributive pluractionality, we follow a long literature that notes that event pluralities must often be distinguished along some trace that provides the counting criterion (e.g., Henderson 2012; Lasersohn 1995; Pasquereau 2019 among others). To help with this notion, we will introduce a bit of notation to return the set of atomic subevents of an event that differ on some trace γ.

(18) $\mathbf{E}_e(\gamma) = \{e' | e' \leq e \wedge \forall e'' \leq e'[\mathbf{atom}(e'') \rightarrow \gamma(e') = \gamma(e'')] \wedge \neg\exists e'''[\gamma(e''') = \gamma(e') \wedge e''' \not\leq e']\}$
'The set of events e' in e that share an image under γ'

We can now handle forms like ɖaɖbiːl 'one burns many' in (19b) which computes event pluralities with respect to the theme, as well as forms like ɖaɖgwan 'many beat up someone' in (19a), which event pluralities determined via the agent.

(19) a. ɖaɖbiːl $\rightsquigarrow \lambda y \lambda x \lambda e[*\text{BURN}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists m[\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = m]]$

b. ɖaɖgwan $\rightsquigarrow \lambda y \lambda x \lambda e[*\text{BEAT}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y \wedge \exists m[\text{MANY}_{\text{STD}} < m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m]]$

Note that that condition $\text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m$ in b will require that m-many beating events with distinct agents. If, in addition, we know that $\text{STD} < m$, then we can conclude that the subject high as high, though unspecified cardinality, as required by such forms.

A consequence of this semantic analysis is that third and fourth forms must be interpreted distributively. We think there is good evidence for this, and for the notion that these verb forms involve meaning and not syntactic agreement, from the fact that explicitly collective predicates like *ɖagávk*/*ɖigávk* 'gather' in Hualapai

tolerate singular agreement in the clear presence of a plural subject. Note the lack of a distinction in plural agreement between gathering cattle in (20) and gathering people in (21). In both cases we have a singular subject verb despite, at least (21), explicit plural marking.

(20) *Waksích isavgó búkal đigávkyu.*
waksi-ch isavgo buk(a)-l đigá-v-k-yu
cow-SUBJ corral foot-at 3.GATHER-STATE-SS-AUX/be

'The cattle gathered at the corner of the corral (or close to the fence of the corral).' (Watahomigie et al. 2001b: p. 52)

(21) *Ba:jach we đagávikyu.*
ba:-j-ch we đagáv-k-yu
person-PL-SUBJ DEM/over.that.place-around 3.gather.around-SS-AUX/be

'People are gathering around over there.' (Watahomigie et al. 2001b: p. 294)

This suggests that third form plural subjects are semantically plural in some sense, and do not merely reflect the features of some argument. In particular, they at least require a large cardinality and an event that is not clearly collectively interpreted. We would predict, for instance, that if there were multiple gatherings of people or cattle in a location, we could use a plural form of *đagávk*/*đigávk* 'gather'. This would reflect the pluractional interpretation we posit is required.

The natural extension then to fourth forms like đađbiːlj 'many burn many', as seen in (22), is that we target both core argument thematic roles.

(22) *Ba:jach gwèjaláy nyuwí đađbiːljkwi.*
ba:-j-ch gwèjaláy nyu-wí đađbiːlj-k-wi
person-PL-SUBJ trash 3/3.SUB.do burn.PL-SS-AUX/do

'People are burning a lot of trash.' (Watahomigie et al. 2001b: p. 247)

(23) $\text{đađbiːlj} \rightsquigarrow \lambda y \lambda x \lambda e[*\text{BURN}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m,n[\text{MANY}_{\text{STD}} < m,n \wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = n]]$

There is evidence for treating the agent and theme cardinality conditions as two separate constraints. The reason is that they are separable. Watahomigie et al. 2001b notes that there are a small number of verbs that actually have fifth forms, which even further increases the object cardinality. The verb *sahák* 'to hang' is one

such verb. Note that in addition to the fourth forms where 'many hang many', we have a fifth form that requires many hang even more things.

(24) Watahomigie et al. 2001b: p. 254

a. sahák (wi) 'to hang'

b. sahájk '(a few/many) to hang one thing'

c. đis'hák '(one) to hang many things'

d. đis'hájk '(many) to hang many things'

e. điđs'hájk '(many) to hang a lot of things'

These forms argue for treating the two cardinality constraints separately, as in (25), but also reinforces again the idea that verbal plurality in Hualapai involves gradable cardinality, ranging from few, to many, to a lot.

(25) điđs'hájk $\rightsquigarrow \lambda y \lambda x \lambda e[*\text{BURN}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m,n[\text{MANY}_{\text{STD}} < m \wedge \text{ALOT}_{\text{STD}} < m$
$\wedge \text{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \text{CARD}(\mathbf{E}_e(\mathbf{th})) = n]]$

With this analysis we have a comprehensive treatment of the formal semantics of the Hualapai plural verb paradigm, but we still do not have an account of the semantic scale. As noted previously, we cannot reduce the notion of the semantic scale to that of enailment. It's just not the case that a sentences built on third form verbs with a high cardinality object entail minimal pairs with a second form verb, which merely requires a paucal subject. Thus, having a proposal for the truth conditions of such verbs cannot, on its own, give us an account of the semantic scale. That said, this account does allow us to define a plurality scale based on properties of the semantic objects that satisfy the verbs in the paradigm as defined. We give an informal definition in (26), which we make precise in the appendix.

(26) **Semantic Scale $<_{\mathbf{s}}$ (Informal):** Given V and V' of type $\langle e,\langle \varepsilon,t\rangle\rangle$—think subject and event arguments, the two arguments Hualapai verb paradigms concern—$V <_s V'$ just in case the smallest cardinality i+e for an individual/event-pair satisfying V has a lower cardinality than the smallest among such pairs satisfying V'

Let's consider how this works. We know that first form verbs in Hualapai can have a singular subject participating in a non-pluractional event. Thus, the smallest individual/event pairs have cardinality 2. Moving to the second form paucal, the smallest individual/event pairs that satisfy such verbs must have a cardinality

greater than 2 because the subject alone, before even considering the event, must have a cardinality of at least 2. The smallest such pairs would have cardinality 3 involving a dual subject participating in an atomic event. Moving on to the third forms, these we have argued involve pluractionality. Such forms always involve an event argument with cardinality greater than MANY_{STD}, which must be greater than 3, and so the smallest individual/event pairs satisfying second form verbs have a lower cardinality than those satisfying third form verbs. Moving to the fourth forms, we have a case where both the subject and event arguments must have a cardinality exceeding MANY_{STD}, which must result in a larger cardinality than the smallest such pairs satisfying third form verbs, which only require the event argument to have cardinality. Finally, we have seen that fifth forms involve standards even greater than many, i.e., ALOT_{STD}. Critically, these forms, in every paradigm given by (Watahomigie et al. 2001b), have another argument that must exceed MANY_{STD}. Clearly then smallest pairs satisfying MANY_{STD} will be smaller than the smallest pairs satisfying MANY_{STD} and ALOT_{STD}, respectively, given that $\text{MANY}_{\text{STD}} < \text{ALOT}_{\text{STD}}$. Thus, fifth forms also behave according to the semantic scale in (26).

We now have an analysis, both of the truth conditions of the Hualapai verb forms, but also an analysis of the semantic order $<_s$. Forms are ranked higher, or we might say, "more plural", if they involve more agents and more events. Critically, though, the distribution of those agents and events is such that forms do not entail each other. We thus have need of a semantic scale which exists alongside the truth-conditional content of the the various forms. In the next section we turn to the morphological scale and show that, just as with the meaning of the forms, we can order the forms themselves along a scale of morphological complexity such that all things being equal, higher ranked forms should having meanings that are higher on the semantic scale.

5 Hualapai verbal number and the morphological scale

The choice to discuss Hualapai plural inflection vaguely—in terms of first form, second form, etc.—has not been an accident. The reason that mapping from meanings to specific exponents is not deterministic. Hualapai shows two features that we will also see when we consider plural verbal morphology in Seri. In particular, (i) there is no one-to-one mapping between meanings and their exponents, and (ii) meanings, which have shown can be ordered, are instead assigned to exponents to form paradigms somewhat arbitrarily, but in a ways that respect this order. This is what has been called *incremental* morphology (e.g., Baerman 2016).

We see an example of these properties in the figure below. First, not that we cannot say that vowel length marks the second form because it actually marks the third form for stem *gilgyo* 'tie something large'. Similarly, we cannot say that *-j*

marks the second form because it actually marks third and forth forms for stem *hwal* ‘dig’. This is what we mean by a lack of one-to-one mapping between exponents and meanings.

Form 1	**Form 2**	**Form 3**	**Form 4**	
gilgyo	gilgyo-j	gilgyo:	gilgyo:-j	‘tie something large’
hwal	hwa:l	hwa:l-j		‘dig’

At the same time, we see in this example an illustration of incrementality. Note that while vowel length marks different meanings in these two paradigms, in both cases having both vowel length and the *-j* affix marks a meaning further up the semantic scale we have established. There is an iconic logic to this pattern. Doing $\alpha \oplus \beta$ must yield a more plural meaning than either doing α or β on its own. This is a pattern that holds across all verb paradigms. This is precisely what is meant by incremental morphology. Knowing that $\alpha <_m \alpha \oplus \beta$ means that while we may not know $[\![\alpha]\!]$ or $[\![\alpha \oplus \beta]\!]$, due to lack of one-to-one mapping, we do know that $[\![\alpha]\!] <_s [\![\alpha \oplus \beta]\!]$. The assignment of meanings to these exponents in this morphological order is in alignment with the semantic scale.

We use $\oplus$ in this example because the main result of Baerman 2019 is that the Hualapai morphological order should have the structure of addition. That is, it should form a commutative monoid with its standard algebraic preordering. Baerman 2019 does not use these terms, but it is clearly the intent. When we try to make good on this intent what we will see is that his account as presented fails to order all exponents correctly under the preorder induced by addition. But, when make adjustments to ensure the ordering, we also resolve a morphological puzzle in Hualapai involving string equivalent forms with a reduplicated numeral prefix versus those with a sequence of numeral prefix and homophonous causative. We take this to be strong argument for Baerman’s account, which captures the algebraic structure of Hualapi number agreement, but also clarifies not immediately related morphological facts.

A commutative monoid is a set P closed under a binary operation that has a identity element and satisfies associativity and commutativity, which induces an algebraic preordering on P such that $x \leq y$ iff $\exists z[y = x + z]$. This is the familiar structure of the positive integers under addition. We have already seen examples from Hualapai which suggest a similar structure. Consider the paradigm for ‘tie something large’ from Figure 5, specifically *gilgyo-j* $\leq$ *gilgyo:* $\leq$ *gilgyo:-j*. We can say that forms with both length and the *-j* suffix are greater than those with just length because there is something we could add to length, namely the *-j* suffix, which would equal that higher form. The same reasoning works to show that forms with the *-j* suffix rank below those with both *-j* and length. This is perfectly parallel

to the fact that $2 \leq 3$ and $1 \leq 3$ because there are integers, 1 and 2 respectively, such that $1+2 \leq 3$.

The question now is can we find this structure throughout the verbal morphology. Baerman 2019 proposes that this is the case, though as we will see, there are two wrinkles. To begin, let's consider all of the exponents we see for their forms at issue. Baerman provides the following hierarchy based on the empirical fact that should some meaning be assigned one of these exponents, those meaning higher on the semantic scale must not use exponents to the left of where we started.

(27) suff $\leq$ length $\leq$ suff $\oplus$ length $\leq$ prefix $\leq$ prefix $\oplus$ suff $\leq$ prefix $\oplus$ length $\leq$ prefix $\oplus$ length $\oplus$ suffix $\leq$ prefix $\oplus$ prefix $\oplus$ suffix

Baerman tries to capture the additive structure of this morphological hierarchy by saying that these exponents expone numerical features. Assume the following correspondences.

(28) a. suff $\leftrightarrow$ 1

b. length $\leftrightarrow$ 2

c. prefix $\leftrightarrow$ 4

Then, the hierarchy in (27) looks like (29).

(29) $1 \leq 2 \leq 1 \oplus 2 \leq 4 \leq 4 \oplus 1 \leq 4 \oplus 2 \leq 4 \oplus 2 \oplus 1 \leq 4 \oplus 4 \oplus 1$

This system captures structural properties of the system familiar from addition. For instance, we must assert length is greater than suffixation, but with this fact establish we immediately capture the fact that prefixation along with suffixation must be less than prefixation with length. It follows from the fact that $a \leq b$ ensures $a+x \leq b+x$.

There are critical features, though, that this system lacks, which is present in commutative monoids. In particular, we require addition to be a total function. Here, though, note that there combinations not attested. While we can have double prefixation with suffixation, there is no double prefixation with length. That is, *prefix* $\oplus$ *prefix* $\oplus$ *length* is undefined. This is tells us that we are actually dealing with a partial commutative monoid, defined in (30) following Wehrung 2017, but more importantly, the fact that we have missing values in the system means that the ordering relation as given does not hold.

(30) A *partial commutative monoid* is a structure $(P, \oplus, 0)$, where P is a set, $0 \in P$, and $\oplus$ is a partial binary operation on P satisfying the following properties, for all $x, y, z \in P$:

a. *Associativity*: $x \oplus (y \oplus z)$ is defined iff $(x \oplus y) \oplus z$ is defined, and then the two values are equal.

b. *Commutativity*: $x \oplus y$ is defined iff $y \oplus x$ is defined, and then the two values are equal.

c. *Zero element*: $x \oplus 0$ is defined with value x.

The algebraic preordering on P is defined by

$$x \leq^{\oplus} y \text{ if } (\exists z)(y = x \oplus z), \quad \text{for all } x, y \in P.$$

Note the that algebraic preorder requires that a is less than b just in case we can find the difference between them, i.e., an object c such that $a + c = b$. The justifying relationship holds for all pairs in the feature preorder in (29) except for one, namely $1 \leq 4 \oplus 4 \oplus 1$. The issue is that we do not have number 8. There is no x feature hierarchy such that $x \oplus 1 = 4 \oplus 4 \oplus 1$. This should give us pause. Baerman shows that Hualapai plural verb morphology could almost have structure of a partial commutative monoid, but there is something funny going on with *prefix* $\oplus$ *prefix* $\oplus$ *suffix* forms. Actually, when we look at these forms there are other issues. Note, for instance, we also have a failure of associativity. We don't precisely know the bracketing, but *(prefix* $\oplus$ *(prefix* $\oplus$ *suffix))* should be defined if and only if *((prefix* $\oplus$ *prefix)* $\oplus$ *suffix)*. This hold for our other triple-feature forms, namely *prefix* $\oplus$ *length* $\oplus$ *suffix*, where both *prefix* $\oplus$ *length* and *length* $\oplus$ *suffix* are both defined. In contrast, *prefix* $\oplus$ *prefix* is not defined, and so *((prefix* $\oplus$ *prefix)* $\oplus$ *suffix)* must be undefined, which by associativity requires that *(prefix* $\oplus$ *(prefix* $\oplus$ *suffix))* be undefined apparently counter to fact.

This is all to say that if we want to treat Hualapai plural verb morphology as having an additive algebraic structure, which we can almost do, there is a problem to resolve with the double prefix forms. Interesting, there are independent reasons to think that these forms require an alternative analysis. As Baerman 2019 notes, there has been a question in the literature about whether to analyze certain cases of double prefixations as exactly that or as reduplication. The issue is that, as discussed in Watahomigie et al. 2001b, the most common causative forms, when they take a plural prefix, the prefix is a copy of the form of the causative. We get paradigms that look like the following, where I have highlighted the doubled prefix.

(31) a. jithul ∼ **jij**thuːl ∼ **jij**thuːl ∼ **jij**thuːlj 'wash'

b. điboq ∼ **điđ**boqj ∼ **điđ**boːq ∼ **điđ**boːqj 'spill'

In earlier work, Redden 1966 and Watahomigie, Bender & Yamamoto 1982 treat these as reduplication, but in Watahomigie et al. 2001b, the authors of the latter work have revised their view in favor of the idea that we have homophony of two prefixes in this case. That is, in *jithul* we have a causative prefix *ji-*, which in the plural is preceded by a plural prefix that happens to have the same form.

We bring up these forms because we can contrast them another class of stems which we believe do involve reduplication. These are verbs that begin with an s-/th-causative.

(32) a. sqwaːn ∼ sqwaːnj ∼ **ɖi**sqwaːnj ∼ **ɖiɖ**sqwaːnj ‘peel’

b. thigóm ∼ thigómj ∼ **ɖi**thgómj ∼ **ɖiɖ**thgómj ‘break’

Baerman 2019 analyzes these as involving a causative *ɖi-* prefix, which only appears in higher ranking forms for these verbs, and which can then take a homophonous plural *ɖi-* in the highest ranking forms. This allows him to maintain a formal similarity with the causatives in (31).

This analysis has a major drawback, though, which is that we do not lose the causative *s-/th-* in these higher number forms, which makes us skeptical that *ɖi-* is a causative in these cases. We want to propose a counteranalysis where all the stems in (32) and (31) have a uniform causative prefix across all forms, whether *ji*, *ɖi-*, *s-*, or *th-*. For the *ji* and *ɖi-* casusatives, there is matching plural prefix. Critically, *s-* and *th-* causatives take the *ɖi-* plural prefix (as Watahomigie et al. 1982: p. 254 says), which we see in forms like **ɖi***sqwaːnj*.

When we move to forms like **ɖiɖ***sqwaːnj*, in virtue of treating *s-* as the causative and *ɖi-* as a plural prefix, we have the option of treating the second *ɖi-* as bona fide reduplication rather than a second plural prefix. This allows us to preserve that analysis in Watahomigie et al. 2001b for the *ji* and *ɖi-* causatives where we do not have reduplication, merely the addition of a homophonous plural prefix. This we distinguish from true reduplication of plural prefixes for high number forms of *s-*, *th-* and other causatives.[3]

We now have a solution to the problem of the morphological order. If the relevant double-prefixed forms are not two sequences of prefixes, but reduplication, a different exponent, then we can treat is as exponing a different feature as in (33).

(33) a. suff ↔ 1

b. length ↔ 2

3 It is perhaps not surprising that we do not see reduplication as an option in the *ji*/*ɖi-* causatives. The reason is that for these stems it would involve a tripling of identical elements (the causative, it’s matching plural marker, and the reduplicant), which is perhaps a bridge too far.

c. prefix $\leftrightarrow 4$

d. reduplication $\leftrightarrow 7$

We can combine these features in the attested ways which now correctly gives us a partial commutative monoid. Every relation in the order is supported by the addition operation and we no longer have the associativity problem. The sequence of two prefixes, which was undefined outside of the context of a suffix, is no longer treated as a sum.

(34) $\text{suff} \leq \text{len} \leq \text{suff} \oplus \text{len} \leq \text{pref} \leq \text{pref} \oplus \text{suff} \leq \text{pref} \oplus \text{len} \leq \text{pref} \oplus \text{len} \oplus \text{suff} \leq \text{redup} \oplus \text{suff}$

(35) $1 \leq 2 \leq 1 \oplus 2 \leq 4 \leq 4 \oplus 1 \leq 4 \oplus 2 \leq 4 \oplus 2 \oplus 1 \leq 7 \oplus 1$

Though we depart from Baerman's analysis of the double-prefix forms, our reanalysis is a vindication of his approach. In particular, we it to be strong argument for the algebraic analysis. Assuming it led us precisely to an area of morphological analysis that was independently contentious (reduplicaton vs. double prefixation), and then pointed us to the correct analysis. With it, we now have a well-define morphological order in terms of a partial commutative monoid which stands alongside the semantic order we developed in the previous section based on the sum cardinality of verb stem arguments. In the next section we bring semantics and morphology together.

6 Hualapai scale alignment via degree semantics

We are now in a position to think about compositional morphosemantics of Hualapai plural verb morphology. We will work in a roughly Distributed Morphology framework Halle & Marantz (1993), acknowledging that there are likely theoretical issue the Hualapai facts raise for the framework that we are not fully exploring. The intuition behind the analysis is that these numerical features we have fleshed out in the previous section should be interpreted as presuppositions on the forms in question. The presuppositional contribution of features is analagous to how features are interpreted in other domains. Recall how pronouns in English are often analyzed as denoting a variable x paired with a presupposition inherited from the gender features on that pronoun (e.g., Sudo 2012).

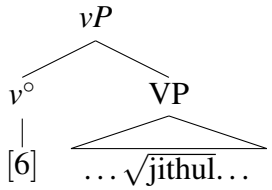
(36) a. $pro_{[+nom,+fem]} \leftrightarrow$ she $\leftrightarrow \text{FEMALE}(x) : x$

b. $pro_{[+nom,+masc]} \leftrightarrow$ he $\leftrightarrow \text{MALE}(x) : x$

The exponents we see in Hualapai correspond to numerical features, which I propose are interpreted as presuppositions on the *vP* denotations, just like the gender features on pronouns. The presuppositions introduced by these numeral features will involve ordered cardinality standards—e.g., std_1, std_2, std_3, etc.—which we can read of the numerical feature in question. Critically, I am not assuming that, for instance, the std_2 equals cardinality 2. These behave like other standards, in that the precise degree can vary, but the order between them must be respected. A particular form, corresponding particular features, will introduce a specific presupposition that will be compatible with various meanings for the *vP* in which it occurs—this is good, because we do not, in fact, have one-to-one mapping. The presuppositions in question, though, will constrain the meanings of forms across a paradigm such that they are well-formed, i.e., they track the semantic order.

To begin, Hualapai plural verb morphology concerns both the event and external argument. For this reason we take it implicate the v° head that introduces introduces the external argument (and in event-semantic frameworks, relates it to the event argument). In particular, we take the v° head to host the ordered numerical features introduced in the previous section.

(37)



We know how these features will be dealt with at the morphophonological interface—due to uniqueness of featural decomposition in algebraic structure, we will insert a prefix and a suffix. The question is how to get the correct semantic interpretation. Critically, we want to understand how the semantic scale, repeated below from (26), is respected.

(38) **Semantic Scale $<_s$ (Informal, repeated from 26):**
Given V and V' of type $\langle e, \langle \varepsilon, t \rangle \rangle$, $V <_s V'$ just in case the smallest cardinality i+e for an individual/event-pair satisfying V has a lower cardinality than the smallest among such pairs satisfying V'

We propose that two things happen at the interpretation of this v° head. First, we insert some truth conditional meaning **M** in the context of the root in question—recall we have one of the six possibilities in (39).

(39) a. Neutral
$\mathbf{M_1} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = x]$

b. Paucal
$\mathbf{M_2} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists n[1 < n < \mathrm{FEW}_{\mathrm{STD}} \wedge \mathrm{CARD}(\mathbf{ag}(e)) = n]]$

c. Many Objects
$\mathbf{M_{3.1}} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m[\mathrm{MANY}_{\mathrm{STD}} < m \wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{th})) = m]]$

d. Many Subjects
$\mathbf{M_{3.2}} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m[\mathrm{MANY}_{\mathrm{STD}} < m \wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{ag})) = m]]$

e. Many Subjects and Many Objects
$\mathbf{M_4} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m,n[\mathrm{MANY}_{\mathrm{STD}} < m,n \wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{th})) = n]]$

f. Many Subjects and Even More Objects
$\mathbf{M_5} = \lambda V \lambda x \lambda e[*\mathrm{V}(e) \wedge \mathbf{ag}(e) = x \wedge \mathbf{th}(e) = y$
$\wedge \exists m,n[\mathrm{MANY}_{\mathrm{STD}} < m \wedge \mathrm{ALOT}_{\mathrm{STD}} < m$
$\wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{ag})) = m \wedge \mathrm{CARD}(\mathbf{E}_e(\mathbf{th})) = n]]$

For this toy example, we will act as if we don't know what meaning is inserted. It serves two purposes. First, we know that the particular exponents we see do not map one-to-one onto meanings. There is undoubtedly some amount of memorization here, but but suppressing the meanings we can get insights into learnability—that is, we can see how much information account could a learner extract about meanings of forms from what is transparently marked by surface forms. Second, and relatedly, this will allow us to highlight the effect of the presupposition, which we take to be the reflex of the features which also control the surface forms that we see.

Turning to said presuppositions, in addition to inserting a meaning from (39), we insert a presupposition directly read off of the feature in question. We think of this like the presuppositional features of pronouns which accompany their truth conditions—i.e., a variable. In this case, the presupposition for a feature N has the following form.

(40) $\mathrm{MIN\text{-}CARD}(V_{\langle e,\langle \varepsilon,t\rangle\rangle}) \geq std_{\mathrm{N}}$, where

a. $\mathrm{MIN\text{-}CARD}(\alpha_{\langle \beta_1,\ldots,\langle \beta_n,t\rangle\rangle})$ is the smallest cardinality of any sequence $b_1,\ldots,b_n$ satisfying α.

We make this concrete by continuing example (37) above. The meaning of the *vP* containing *jithul* 'wash' and feature 6 must be as in (41).

(41) $\lambda x \lambda e.\text{MIN-CARD}(\mathbf{M}_?([\![\text{jithul}]\!])) \geq std_6 : \mathbf{M}_?([\![\text{jithul}]\!])(x)(e)$

As mentioned above, we are obscuring the truth-conditions we are assigning to the verb with the 6 feature—i.e., using $\mathbf{M}_?$ in place of a meaning in (39), but according to the presupposition, whatever meaning we assign, the smallest pairs that satisfy the result must have a cardinality greater than std_6. This is not so informative on it's own, but when we compare to other features, we see that we now have a degree-based account of incremental morphology. Consider an alternative possibility in which we instead used feature 7 with *jithul*. In this case, we would have the presupposition in (42).

(42) $\lambda x \lambda e.\text{MIN-CARD}(\mathbf{M}'_?([\![\text{jithul}]\!])) \geq std_7 : \mathbf{M}'_?([\![\text{jithul}]\!])(x)(e)$

While we may not know what meanings to assign to $\mathbf{M}_?$ and $\mathbf{M}'_?$, the presupposition ensures that we must pick a meaning for $\mathbf{M}_?$ whose smallest satisfying pairs is smaller than those of $\mathbf{M}'_?$. For instance, if we say $\mathbf{M}_?$ is the paucal, then $\mathbf{M}'_?$ must not be the neutral. Or, if we say $\mathbf{M}_?$ has many objects, then $\mathbf{M}'_?$ must not be the paucal. Critically, this is exactly the core generalization we started with! Incremental plural systems are characterized by the fact that while we may not know the meanings of any forms α and β, the fact that $\alpha <_m \beta$ is enough to ensure that $[\![\alpha]\!] <_s [\![\beta]\!]$.

Incremental plurality in Hualapai thus falls out from from two features of the system. First, there is an algebra of features which gives us an order of morphological complexity. Second, we have a presupposition based on those features that arranges possible *vP* meanings on a semantic scale—here the MIN-CARD scale. The result is that paradigms are well-formed just in case they exhibit the scale alignment induced by the way features are turned into presuppositions.

This analysis not only makes sense of a system that is otherwise quite challenging for compositionality, it also makes predictions that are borne out. In particular, we predict syncretisms up the scale of meaning. For instance, if we pick the paucal meaning for *jithul*+6, committing to the minimal elements satisfying it meaning because greater that std_6, then we could also use that same form for the "many object" meaning because it would also satisfy that same presupposition. This is because the presupposition only sets a lower bound on the cardinality of participants through the MIN-CARD relation. In fact, this is the case for *jithul*+6, which has the form *jijthu:l*, and which is use for the paucal and the many object readings. There is actually pervasive strategy in the Hualapai system to make just two distinctions. We have neutral form which contrasts with another that includes the paucal and all stronger meanings Watahomigie et al. 1982: pg. 223-274. That this strategy is possible is predicted by our account.

7 Conclusions

What can we take away from this work so far, beyond an analysis of Hualapi verbal plural marking? First, we should recognize that incremental systems exist and need a compositional semantic analysis. They are not simply iconic. Making this recognition, though, results in analytical difficulties. The lack of one-to-one mapping between forms and meanings make is hard to do composition by assigning each form a meaning. What we have argued for here in Hualapai (and what we would argue for other languages in the region like Seri), is that the exponents we see on the surface are not exactly telling us about the meanings of verbs. Instead, they reflect presuppositions on the range of possible meanings for those verbs.

In the case of Hualapai, choosing some exponent α over β is about signaling how big the arguments of whatever function we end up with tend to be. There is thus a kind of lossy relationship between the forms and the meanings. The forms we have, especially in relation to other forms, give us through their associated presuppositions a range of possible meanings they are compatible with. This range of meanings, while difficult to resolve compositionality, can have meaning-based structure to it (i.e., the semantic order). We would argue that this is what makes such a system learnable, and ultimately stable, such as it is. On first hearing a form, the learner may not know exactly what it means, but can use the semantic and morphological scales, along with other known forms, to reduce the space of possible meanings.

References

Baerman, Matthew. 2016. Seri verb classes: Morphosyntactic motivation and morphological autonomy. *Language* 792–823.

Baerman, Matthew. 2019. Feature duality. In *Morphological Perspectives: Papers in Honour of Greville G. Corbett*, 124. Edinburgh University Press.

Baerman, Matthew. 2024. Verbal number and argument marking in salinan. Talk presented SSILA workshop "An areal exploration of plural systems of the indigenous languages of northwest Mexico and the southwest United States".

Davidson, Donald. 1967. The logical form of action sentences. In Nicholas Resher (ed.), *The Logic of Decision and Action*, 81–95. Pittsburgh: University of Pittsburgh Press.

Dowty, David. 1991. Thematic proto-roles and argument selection. *Language* 67(3). 547–619.

Halle, Morris & Alec Marantz. 1993. Distributed morphology and the pieces of inflection. In Kenneth Hale & Samuel Jay Keyser (eds.), *The View from Building 20*, 111–176. Cambridge, Massachusetts: MIT Press.

Henderson, Robert. 2012. *Ways of pluralizing events*: University of California, Santa Cruz PhD dissertation.

Henderson, Robert. 2017. Swarms: Spatiotemporal grouping across domains. *Natural Language & Linguistic Theory* 35. 161–203.

Hofherr, Patricia Cabredo & Brenda Laca. 2012. *Verbal plurality and distributivity*, vol. 546. Walter de Gruyter.

Krifka, Manfred. 1986. *Nominalreferenz und Zeitkonstitution. Zur Semantik von Massentermen, Pluraltermen und Aspektklassen.*: Universität München PhD dissertation.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Theo Vennemann & Johan van Bentham (eds.), *Semantics and Contextual Expression*, 75–115. Cinnaminson, NJ: Foris Pubns USA.

Lasersohn, Peter. 1995. *Plurality, Conjunction and Events*. Dordrecht: Kluwer.

Link, Godehard. 1983/2002. The logical analysis of plurals and mass terms: A lattice-theoretical approach. In Paul Portner & Barbara Partee (eds.), *Formal Semantics: The Essential Readings*, 127–146. Malden, MA: Blackwell. doi:10.1002/9780470758335.ch4.

Marlett, Stephen Alan. 1981. *The structure of Seri*: University of California, San Diego PhD dissertation.

Pasquereau, Jérémy. 2019. Pluractional numerals in Seri are distributive. Handout of a talk at LAGB 2019.

Pasquereau, Jérémy & Patricia Cabredo Hofherr. 2020. Two types of pluractionality in Seri. Handout of a talk presented at Semantics of Under-represented Languages in the Americas 11.

Redden, James E. 1966. Walapai i: phonology. *International Journal of American Linguistics* 32(1). 1–16.

Sudo, Yasutada. 2012. *On the semantics of phi features on pronouns*: Massachusetts Institute of Technology PhD dissertation.

Watahomigie, Lucille J, Jorigine Bender, Philbert Watahomigie Sr & Akira Y Yamamoto. 2001a. Hualapai reference grammar (revised and expanded edition). *Endangered Languages of the Pacific Rim, A2–003) Kyoto: Nakanishi Printing Co* .

Watahomigie, Lucille J, Jorigine Bender, Philbert Watahomigie Sr & Akira Y Yamamoto. 2001b. Hualapai reference grammar (revised and expanded edition). *Endangered Languages of the Pacific Rim, A2–003) Kyoto: Nakanishi Printing Co* .

Watahomigie, Lucille J, Jorigine Bender & Akira Y Yamamoto. 1982. *Hualapai reference grammar*. American Indian Studies Center, UCLA.

Wehrung, Friedrich. 2017. *Refinement monoids, equidecomposability types, and*

Boolean inverse semigroups, vol. 2188. Springer.
Wood, Esther Jane. 2007. *The semantic typology of pluractionality*: University of California Berkeley PhD dissertation.

Robert Henderson
Communications 306
University of Arizona
Tucson, AZ 85721
rhenderson@arizona.edu