Graph Coloring Dataset Metadata and Chromatic Numbers

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1 Chromatic Numbers of Graph Instances

1.1 Dataset and Sources

The graph instances used in our experiments were taken from the **DIMACS / COLOR benchmark dataset** hosted at Carnegie Mellon University [1]. This collection contains a total of **79 graphs** across various families such as DSJC, DSJR, *school*, and *queen*. While the dataset provides vertex and edge counts for each graph, many optimal chromatic numbers $\chi(G)$ were listed as unknown ("?").

To fill in the missing chromatic numbers, we referred to the following sources:

1. The **Graph Coloring Benchmarks** curated by Daniel Porumbel [2], which summarize best-known and proven colorings for DIMACS instances.

2. Additional research papers and benchmark collections, including Moalic & Gondran [3], Hertz *et al.* [4], and Loudni [5], which provide updated or best-known colorings obtained by metaheuristic or hybrid algorithms.

When an exact $\chi(G)$ was known, we report it directly. Otherwise, we include the best-known upper bound k^*—the smallest number of colors achieved by any published algorithm to date.

2 Graph Descriptions

All graphs used in this project are taken from the **DIMACS benchmark dataset** [1]. These instances are standard benchmarks used to evaluate graph coloring algorithms. The dataset contains 79 graphs belonging to several well-known families, each generated in a specific way as described below.

2.1 DSJC Graphs

The `dsjcX.Y` graphs were generated by Johnson *et al.* [6]. Here, `X` represents the number of vertices, and `Y` indicates the edge density (the probability that any two vertices are connected). For example, `dsjc1000.5` has 1000 vertices and a density of 0.5. These graphs are widely used to test and compare graph coloring algorithms due to their controlled random structure.

2.2 Flat Graphs

The `flatX_K` family, created by Culberson, divides the vertex set into `K` nearly equal-sized groups and places edges only between different groups. Finding an optimal `K`-coloring reconstructs the original partitioning. Here, `X` is the total number of vertices and `K` equals the known chromatic number.

2.3 Leighton Graphs

The `le450_K` graphs, introduced by Leighton [7], consist of 450 vertices and have a known chromatic number `K`. Each of these graphs includes a clique (a fully connected subgraph) of size `K`, guaranteeing that $\chi(G) = K$.

2.4 Geometric Graphs

The `dsjrX.Y` and `rX.Y` families are random geometric graphs. The `dsjrX.Y` graphs were introduced by Johnson *et al.* [6] and are generated by placing vertices randomly in a unit square, connecting those that fall within a given distance. The `rX.Y` graphs, later produced by Trick and Morgenstern, follow a similar approach. The suffix "`c`" (as in `dsjr500.1c`) denotes the complement of a graph. Chromatic numbers for these graphs are summarized in Trick's repository [8].

2.5 Large Random Graphs

The `C2000.5` and `C4000.5` graphs are large-scale random graphs containing up to four million edges. They are mainly used to benchmark the scalability and performance of coloring algorithms.

2.6 Special Graphs

The `latin_square_10` and `school*` graphs were generated by Gary Lewandowski during the Second DIMACS Challenge. These represent Latin square and class scheduling problems, respectively, offering real-world connections between graph coloring and constraint satisfaction.

3 Chromatic Numbers / Best-Known Colorings

Table 1: Chromatic numbers and best-known colorings for selected graphs in our dataset. In cases where the optimal $\chi(G)$ was not provided in the DIMACS dataset, the best-known upper bound k^* is reported.

Graph Instance	**(—V—, —E—)**	**Chromatic / Best k^***	**Source**
DSJC1000.1.col.b	(1000, 99258)	20	[3]
DSJC1000.5.col.b	(1000, 499652)	83	[4]
DSJC1000.9.col.b	(1000, 898898)	224	[3]
DSJC125.1.col.b	(125, 1472)	5	[2]
DSJC125.5.col.b	(125, 7782)	17	[2]
DSJC125.9.col.b	(125, 13922)	44	[2]
DSJC250.1.col.b	(250, 6436)	8	[2]
DSJC250.5.col.b	(250, 31366)	28	[3]
DSJC250.9.col.b	(250, 55794)	72	[5]
DSJC500.1.col.b	(500, 24916)	12	[4]
DSJC500.5.col.b	(500, 125249)	48	[3]
DSJC500.9.col.b	(500, 224874)	126	[3]
DSJR500.1.col.b	(500, 7110)	12	[4]
DSJR500.1c.col.b	(500, 242550)	85	[5]
DSJR500.5.col.b	(500, 117724)	122	[4]
school1_nsh.col	(352, 14612)	14	[2]
queen10_10.col	(100, 2940)	11	[9]
queen11_11.col	(121, 3960)	11	[9]
queen12_12.col	(144, 5192)	12	[9]
queen13_13.col	(169, 6656)	13	[9]
queen14_14.col	(196, 8372)	14	[9]
queen15_15.col	(225, 10360)	15	[9]
queen16_16.col	(256, 12640)	16	[9]

3.1 Remarks

For DSJC and DSJR graphs, the reported values represent the best-known colorings obtained by meta-heuristic algorithms (such as hybrid genetic, VNS, and simulated annealing methods). Exact proofs of optimality remain unknown for most large random instances. For structured graphs like the *queen* family, the chromatic number is mathematically proven to equal the board size ($\chi(Q_n) = n$).

References

[1] M. Trick, *COLOR/Graph Coloring Instances*. Available at: https://mat.tepper.cmu.edu/COLOR/instances.html

[2] D. C. Porumbel, *Graph Coloring Benchmarks and Best Algorithms*. Available at: https://cedric.cnam.fr/~porumbed/graphs

[3] L. Moalic and P. Gondran, "Variations on memetic algorithms for graph coloring problems," *arXiv preprint arXiv:1401.2184*, 2014.

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[5] S. Loudni, "Graph Coloring Results," *GREYC Laboratory Benchmark Page*. Available at: https://loudni.users.greyc.fr/Coloring.html

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[8] M. Trick, "Graph Coloring Benchmarks and Data Sets." Available at: https://cedric.cnam.fr/~porumbed/graphs, accessed 2025.

[9] J. Mitchell, "On the Chromatic Number of Queen Graphs," *Journal of Combinatorial Theory, Series B*, 1990.