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Simulaciones: https://github.com/Rmejiaz/ModeladoSimulacion/blob/main/Cuadernos/Cap6.ipynb

Capitulo 6 - Phase Plane

6.4. Rabbit versus Sheep

Consider the following "Rabbit vs Sheep" Problems, where $x, y \geq 0$. Find the fixed points, investigate their stability, draw the nullclines, and sketch plausible phase portraits. Indicate the basins of attraction of any stable fixed points.

6.4.1

$$\dot{x} = x(3-x-y) = 3x - x^2 - xy$$
$$\dot{y} = y(2-x-y) = 2y - xy - y^2$$

Puntos de equilibrio:

$$x(3-x-y) = 0$$
$$y(2-x-y) = 0$$

$x=0, y=0$
$x=3, y=0$
$x=0, y=2$

Estabilidad:

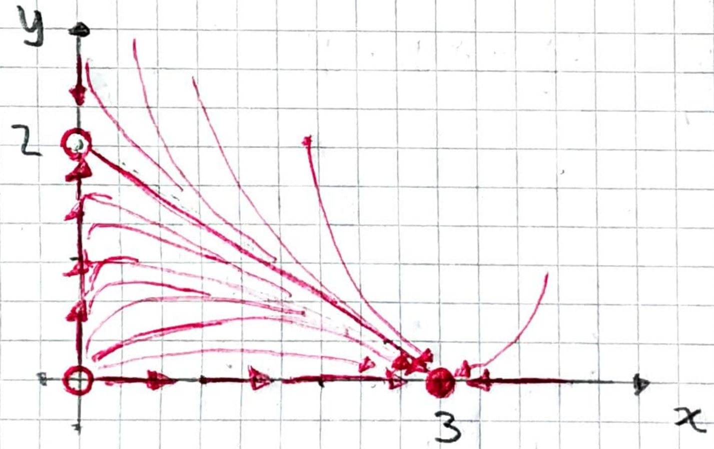
Jacobiano

$$F_x = \begin{bmatrix} 3-2x-y & -x \\ -y & 2-x-2y \end{bmatrix}$$

$$F_x\Big|_{X=(0,0)} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow \begin{matrix} \lambda_1 = 3 & V_1 = (1 \ \ 0) \\ \lambda_2 = 2 & V_2 \ (0 \ \ 1) \end{matrix}$$

$$F_x\Big|_{X=(3,0)} = \begin{bmatrix} -3 & -3 \\ 0 & -1 \end{bmatrix} \quad \begin{matrix} \lambda_1 = -1 & V_1 = (-1{,}5 \ , \ 1) \\ \lambda_2 = -3 & V_2 = (1 \ , \ 0) \end{matrix}$$

$$F_x\Big|_{X=(0,2)} = \begin{bmatrix} 1 & 0 \\ -2 & -2 \end{bmatrix} \quad \begin{matrix} \lambda_1 = 1 & V_1 = (-1.5 \ , \ 1) \\ \lambda_2 = -2 & V_2 = (0 \ , \ 1) \end{matrix}$$



6.4.2

$$\dot{x} = x(3-2x-y) = 3x - 2x^2 - xy$$
$$\dot{y} = y(2-x-y) = 2y - xy - y^2$$

Equilibrios:

$(0,0)$; $(1,1)$; $(\frac{3}{2},0)$; $(0,2)$

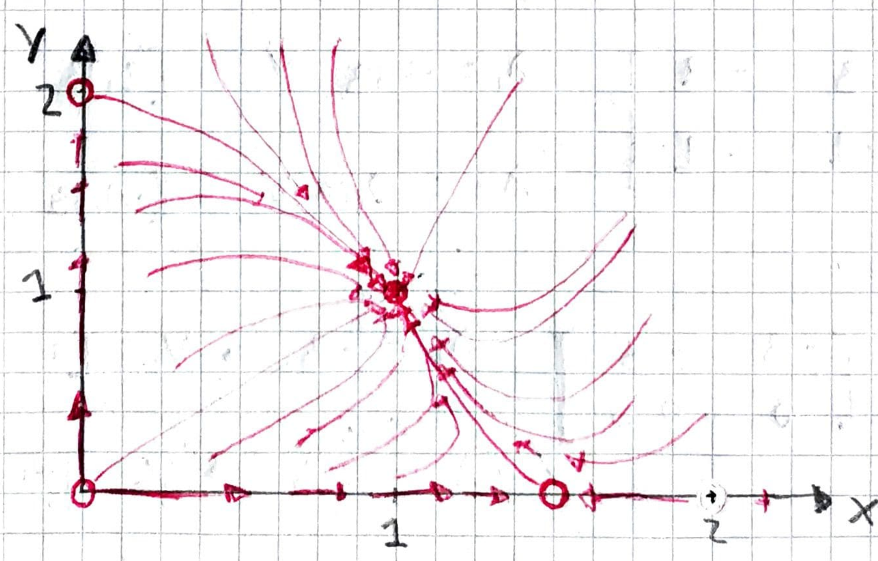
$$F_x = \begin{bmatrix} 3-4x-y & -x \\ -y & 2-x-2y \end{bmatrix}$$

$$F_x\Big|_{x=(0,0)} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= 3 \rightarrow V_1 = [1 \quad 0] \\ \lambda_2 &= 2 \rightarrow V_2 = [0 \quad 1] \end{aligned}$$

$$F_x\Big|_{x=(1,1)} = \begin{bmatrix} -2 & -1 \\ -1 & -1 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= -2{,}618 \rightarrow V_1 = [1{,}61 \quad 1] \\ \lambda_2 &= -0{,}38 \rightarrow V_2 = [-0{,}61 \quad 1] \end{aligned}$$

$$F_x\Big|_{x=(\frac{3}{2}\ \ 0)} = \begin{bmatrix} -3 & -1.5 \\ 0 & 0.5 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= -3 \rightarrow V_1 = [1 \quad 0] \\ \lambda_2 &= 0.5 \rightarrow V_2 = [-0.42 \quad 1] \end{aligned}$$

$$F_x\Big|_{x=(0,2)} = \begin{bmatrix} 1 & 0 \\ -2 & -2 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= -2 \rightarrow V_1 = [0 \quad 1] \\ \lambda_2 &= 1 \rightarrow V_2 = [-3,\ 2] \end{aligned}$$



6.4.3

$$\dot{x} = x(3-2x-2y) = 3x - 2x^2 - 2xy$$
$$\dot{y} = y(2-x-y) = 2y - xy - y^2$$

Puntos de equilibrio:

$$(0,2) \; ; \; \left(\tfrac{3}{2}, 0\right) \; ; \; (0,0)$$

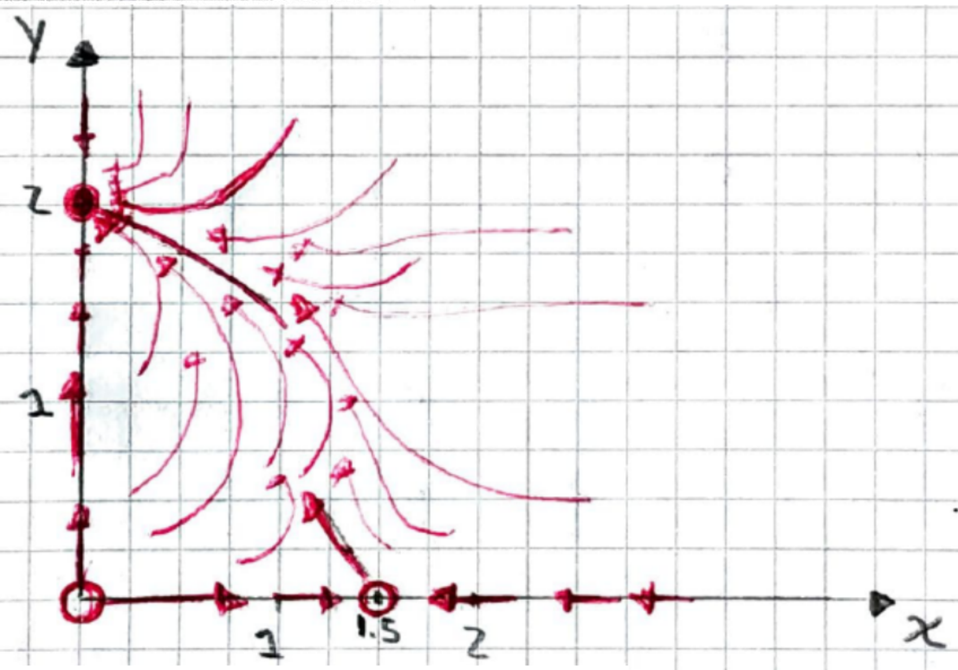
Jacobiano:

$$F_x = \begin{bmatrix} 3-4x-2y & -2x \\ -y & 2-x-2y \end{bmatrix}$$

$$F_x\Big|_{x=(0,2)} = \begin{bmatrix} -1 & 0 \\ -2 & -2 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= -2 \rightarrow V_1 = [0 \;\; 1] \\ \lambda_2 &= -1 \rightarrow V_2 = [-1 \;\; 2] \end{aligned}$$

$$F_x\Big|_{x=(1.5,0)} = \begin{bmatrix} -3 & -3 \\ 0 & 0.5 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= -3 \rightarrow V_1 = [1 \;\; 0] \\ \lambda_2 &= 0.5 \rightarrow V_2 = [-0.8 \;\; 1] \end{aligned}$$

$$F_x\Big|_{x=(0,0)} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \qquad \begin{aligned} \lambda_1 &= 3 \rightarrow V_1 = [1 \;\; 0] \\ \lambda_2 &= 2 \rightarrow V_2 = [0 \;\; 1] \end{aligned}$$



y
2
1
1
1.5
2
x