PROJECT 1.1
LONGITUDINAL DYNAMICS

Index

1. Power needed for motion

This section is devoted to compute the total resistance to motion and the power needed for motion of a vehicle moving in straight running at constant speed V with a longitudinal slope tan(α), see Fig. 1.

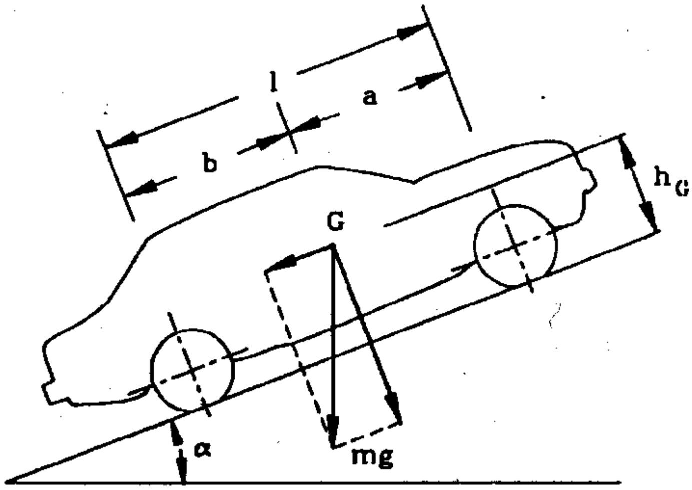

Figure 1: Longitudinal forces on the vehicle.

The external forces acting on the vehicle when it is running at constant speed V on a road having a certain longitudinal slope tan(α) are:

- the aerodynamic drag,
- the rolling resistance,
- the component of the vehicle weight that is parallel to the direction of motion (this is due to the longitudinal slope of the road),
- the resistance due to the aerodynamic lift or negative lift.

The total resistance to motion is therefore given by the following expression:

$$R = \left(mg\,cos(\alpha) - \frac{1}{2}\rho V^2 SC_z\right)(f_0 + kV^2) + \frac{1}{2}\rho V^2 SC_x + mg\,sin(\alpha), \tag{1}$$

where the meaning of symbols is listed below,

- m: total mass of the vehicle, in kg
- g: acceleration of gravity, 9.81 m/s^2
- ρ: air density, 1.3 kg/m^3
- α: longitudinal slope angle, in rad
- V: longitudinal speed of the vehicle, in m/s
- S: front area of the vehicle, in m^2
- $f = f_0 + KV^2$ rolling resistance coefficient, $f_0 = 0.013$, $K = 6.51 \times 10^{-6}$, s^2/m^2
- C_z: aerodynamic lift coefficient
- C_x: aerodynamic drag coefficient.

If the contribution of the aerodynamic lift is neglected (it is realistic in commercial vehicles, not for sport and F_1 cars), collecting the terms in V and V^2, the Eq.1 becomes

$$R = A + BV^2, \tag{2}$$

with

$$A = mg\big(f_0 cos(\alpha) + sin(\alpha)\big),$$
$$B = mgKcos(\alpha) + \frac{1}{2}\rho S C_x. \qquad (3)$$

The knowledge of the resistance R allows to compute the power needed for motion P_n (at constant speed)

$$P_n = VR = AV + BV^3 \qquad (4)$$

It is worth noting that in logarithmic scale terms AV and BV^3 are two straight lines with slopes 1 and 3. When the vehicle is moving with slope $\alpha = 0$, the abscissa of their intersection represents the characteristic speed V_{car} while the ordinate is the half of the characteristics power P_{car}

$$V_{car} = \sqrt{\frac{A}{B}} \qquad (5)$$

$$P_{car} = 2A\sqrt{\frac{A}{B}} \qquad (6)$$

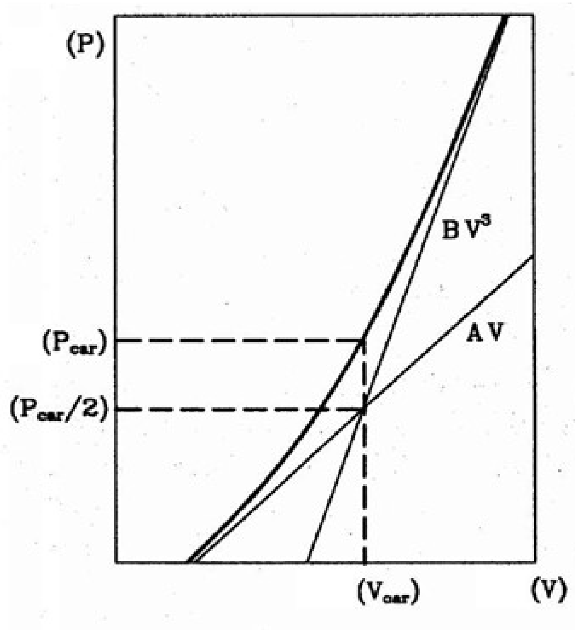

Figure 2: Power needed for motion.

Using data collected in annex 1, it is required to

1. compute and plot the total resistance R, the terms A and BV^2 as function of the speed V for two different slopes, $\tan(\alpha) = 0$ and $\tan(\alpha)=0.1$.
2. Compute and plot in logarithmic scale the power needed for motion P_n and the single terms composing the power AV and BV^3.
3. Compute the characteristic power P_{car} and the characteristic velocity V_{car}.
4. Compute and plot in a logarithmic scale the normalized power needed for motion P_n/P_{car} as function of the normalized velocity V/V_{car}, V_{car} being the one determined at $\tan(\alpha) = 0$. Plot the diagram for five different longitudinal slopes ranging from $\tan(\alpha) = 0 \div 0.4$.

2. Computation of the maximum power available at the wheels

The power needed for motion P_n is supplied by the power unit installed on the vehicle. In our case each vehicle is equipped with an internal combustion engine. The aim of the present section is that of computing the maximum power available at the wheels. The internal combustion engine known parameters are

- V_{cil}, size of the engine, cm^3
- ω_{min}, minimum rotational speed of the engine,
- map of the engine, namely the specific fuel consumption *q* (given in gr/Hph) as function of the rotational speed for different mean effective pressure *mep* (given in bar). These data can be found in the file *pln_team(nteam).txt*.

The power available at the engine shaft can be computed according to the following equation if *mep* values are known at different rotational speeds of the engine ie,

$$P_e = \frac{k\, mep V_{cil}\, \omega_e}{2\pi} \tag{7}$$

The constant k indicates the number of the thermodynamic cycles occurring at each revolution of the engine (k = 1 for a two stroke engine and k = 1/2 for a four stroke engine).

The maximum power exploited by an internal combustion engine at maximum throttle can be approximated by the following expression [3],

$$P_e = \sum_{i=0}^{3} P_i \omega_e^i \tag{8}$$

where the terms P_i are

$$P_0 = 0 \qquad P_1 = P_{MAX}/\omega_{max} \qquad P_2 = P_{max}/\omega^2_{max} \qquad P_3 = P_{max}/\omega^3_{max}. \tag{9}$$

The term ω_{max} is the rotational speed at which the maximum power occurs. The driving torque M_e can therefore be computed as

$$M_e = P_e/\omega_e \tag{10}$$

The knowledge of the power available at the engine shaft allows to obtain the power at the driving wheels

$$P_a = \eta_t P_e \tag{11}$$

where η_t indicates the efficiency of the whole power train. An exhaustive description of the law governing the variability of η_t with the operating conditions is reported in [1] and [2].

Following the considerations reported in this section, using the experimental engine map reported in *pln_team(nteam).txt* and the data reported in Annex 1, it is asked to

1. compute and plot the experimental curve of the engine power and the maximum torque (under maximum throttle conditions),
2. approximate the maximum power curve with the empirical model of Artamonov (extend the curve well beyond the curve peak),
3. plot the curve of the power available at the wheels.

3. Gradeability and initial choice of the transmission ratios

The knowledge of the power needed for motion P_n and the power supplied by the internal combustion engine P_e in the speed range of interest allows to design the gear ratios τ_{gi} to maximize the performance of the vehicle in the whole operating range (in the present exercise it is assumed that the final transmission ratio τ_f is known).

The gear ratio that allows to reach the maximum speed is obtained imposing that $P_n = P_a$ when the engine can supply in absolute the maximum power (see Fig. 3 a).

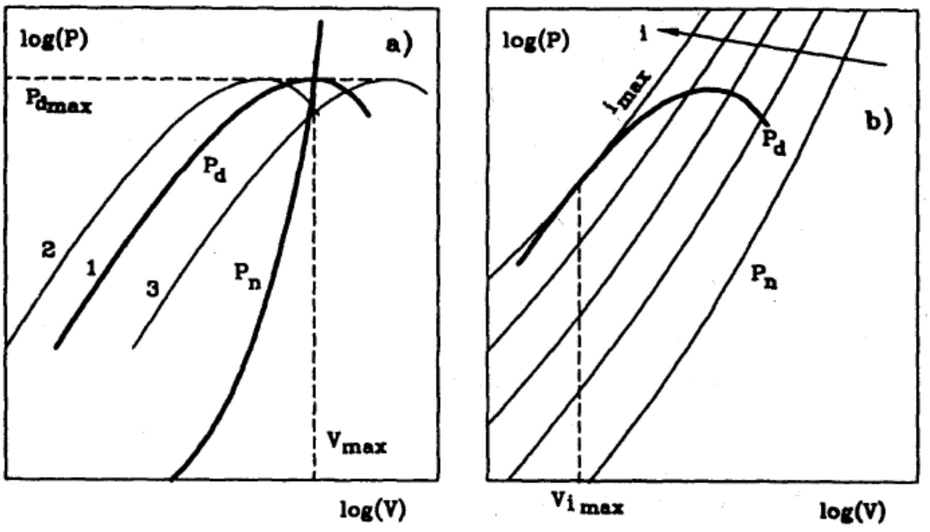

Figure 3: Maximum speed on flat suface (a) and maximum slope (b).

The condition that $P_n = P_{aMAX}$ at V_{MAX} leads to define the maximum speed that the vehicle can reach:

$$V_{\max} = A^*\left(\sqrt[3]{B^*+1} - \sqrt[3]{B^*-1}\right) \tag{12}$$

where

$$\begin{aligned} A^* &= \sqrt[3]{\frac{P_{a,\max}}{2m\,g\,K + \rho\,S\,C_x}} = \sqrt[3]{\frac{P_{a,\max}}{2B}} \\ B^* &= \sqrt{1 + \frac{8m^3\,g^3\,f_0^3}{27P_{a,\max}^2\,(2m\,g\,K + \rho\,S\,C_x)}} = \sqrt{1 + \frac{4A^3}{27P_{a,\max}^2\,B}} \end{aligned} \tag{13}$$

The equations 10 and 11 are referred to the so called Cardano's formula that holds if the aerodynamic lift is neglected.

The maximum speed V_{max} leads to compute the related gear ratio that in general is the top gear ratio:

$$\tau_{g\,top} = \frac{V_{\max}}{\tau_f\,R_e\omega_{\max}} \tag{14}$$

where ω_{max} is the rotational speed at which occurs the maximum power P_{max}. The term R_e is the effective rolling radius that is lower than the undeformed tire radius. For a radial tire, it can be assumed R_e = 0.98R: Refer to the text book for more detailed analysis. The top gear ratio can be reduced (undergeared) to improve the performance in acceleration or increase (overgeared) to minimize the fuel consumption. It is worth noting that the degree of undegearing is defined as:

$$\lambda_u = \frac{\omega_{\max}}{\omega_{P,\max}} \tag{15}$$

At this stage, being the top gear ratio already defined, it is important to compute the bottom one. It is important that the bottom gear allows the vehicle to move in a certain speed range at a defined maximum slope. To this end, to enable the engine to start from rest, it is important that the minimum speed is included in this speed range (Fig. 3b). The assumption that the maximum slope is covered at reduced speed (this means that the aerodynamic drag can be neglected) lead to the following expression that allows to compute the bottom gear:

$$\tau_{g\,bottom} = \frac{M_e \eta_t}{\tau_f R_e\, m g (f_0 \cos(\alpha_{\max}) + \sin(\alpha_{\max}))} \tag{16}$$

The engine torque M_e to be introduced in Eq. 14 could be the maximum torque at the minimum engine speed while the maximum slope to be chosen is tan(α_{max}) = 0,25 or eventually tan(α_{max})= 0.33. The bottom gear ratio should guarantee that at an engine rotational speed ω_e = 1000 rpm, the vehicle speed is not higher than 8 km/h (ω_e = 1000 rpm) → V ≤ 8kmh). Such a specification is important for driving in congested traffic as a high bottom ratio leads the frequent use of the clutch.

The top and bottom gear ratios allow to define the sequence of the intermediate ratios. A geometric sequence lead to define that

$$\tau_{g\,(i)} = \tau_{g\,(i-1)} \sqrt[N-1]{\frac{\tau_{g\,top}}{\tau_{g\,bottom}}} \tag{17}$$

where N represents the number of gear ratios.

Following the above considerations, it is required to

1. compute the gear ratios $\tau_{g\,bottom} \div \tau_{g\,top}$
2. plot the curves of P_a as function of the vehicle speed V for the different gear ratios. Plot in the same graph the P_n with null and maximum slope

References

[1] G. Genta, “Meccanica dell’Autoveicolo”, V edizione, Levrotto & Bella, Febbraio, 2000.

[2] G. Lechner, H. Naunheimer, “Automotive Transmissions-Fundamentals, Selection, Design and Application”, I edizione, Springer, Febbraio, 1999.

[3] M. D. Artamonov, “Motor Vehicle, fundamentals and design”, Mir, Mosca, 1976.