

AUTOMOTIVE ENGINEERING  
POLITECNICO DI TORINO  
MOTOR VEHICLE DESIGN  
(02LNLLI)

PROJECT 1\_2

A. Y. 2017-2018

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## 1 Maximum power that can be transferred by the tires to the ground

The power needed for motion increases with speed and slope. This power is supplied by the power unit installed on the vehicle. Apart from the power unit performance, some limitations related to the maximum power that can be transferred by the tires to the road can occur. The maximum power exploited at the tire - ground contact point is given by the following expression:

$$P_{\max} = V \sum_{\forall i} F_{zi} \mu_{ip}, \quad (1)$$

where the summation is extended to all the driving wheels while  $F_{zi}$  represents the vertical load acting on the wheel hub of each driving wheel. Considering the longitudinal dynamics only, it is realistic to assume that there is symmetry plane and that all the forces act in that plane. The equations of equilibrium in that plane ( in  $x$  and  $z$  direction and the rotation about the point  $O$ ) ([1], pp.157-160) lead to obtain the expressions of the vertical forces  $F_{zi}$  acting on the front and rear axle (we are assuming a 2 axles vehicle). Such expressions, considering  $V = \text{const.}$  and neglecting the aerodynamic lift are

$$\begin{cases} F_{z1} = mg \frac{(b - \Delta x_2) \cos(\alpha) - h_G \sin(\alpha) - K_1 V^2}{l + \Delta x_1 - \Delta x_2} \\ F_{z2} = mg \frac{(a + \Delta x_1) \cos(\alpha) + h_G \sin(\alpha) - K_2 V^2}{l + \Delta x_1 - \Delta x_2} \end{cases} \quad (2)$$

where

$$K_1 = \frac{\rho S}{2mg} C_x h_G, \quad K_2 = -K_1, \quad (3)$$

$$\Delta x_i = R_{li}(f_0 + KV^2),$$

while the undefined terms adopted in 2 and in 3 are explained in Figure 2. The term  $R_l$  is referred to the loaded radius of each wheel that for the radial tires is  $R_l = 0.92R$ .  $R$  is referred to the undeformed radius that can be computed according to the normed designation. Remember that a tire designation for examples as 155 / 65 SR 13 is referred to a tire having a total width (W) of 155 mm and an aspect ratio H/W=65% of W. The first letter (S) indicates the maximum speed that can be reached by the tire (ex.: S  $\Rightarrow V_{\max} = 180 \text{ km/h}$ ) while the second one is referred to the tire plies (R=radial). The last number indicates the rim diameter in inches.

It follows that for a tire designed as 155 / 65 SR 13 the loaded radius is:

$$R_l = 0.92 \left( \frac{13 \cdot 25.4 \cdot 10^{-3}}{2} + 155 \cdot 10^{-3} \cdot 0.65 \right) = 0.245 \text{ m.}$$

The longitudinal force coefficient  $\mu_{ip}$  varies with the longitudinal speed  $V$  according to the law

$$\mu_{ip} = c_1 - c_2 V,$$

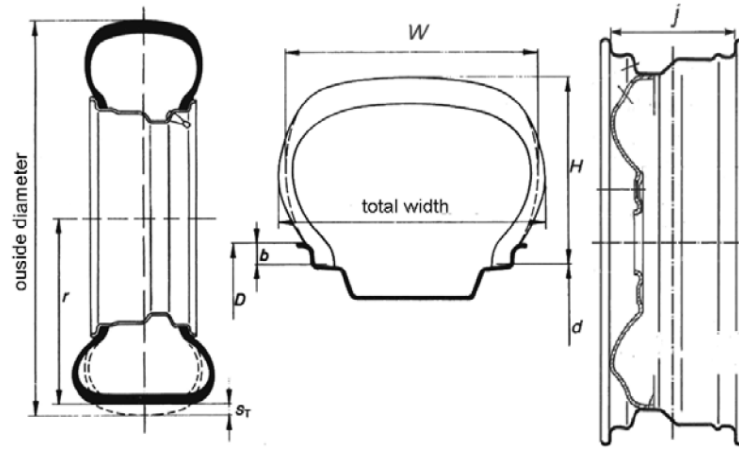


Figure 1: Tire characteristic

where the coefficients  $c_1$  and  $c_2$  are realistically:

	$c_1$ [-]	$c_2$ [s/m]
dry road	1.1	$6 \cdot 10^{-3}$
wet road	0.8	$8 \cdot 10^{-3}$

According to above mentioned considerations it is asked to:

1. compute and plot the maximum power that can be transmitted at the wheel ground contact point as function of speed  $V$ . This analysis should be computed considering the longitudinal slope null and the dry and wet road. This means that are required two plots, one for dry road and another for wet road.
2. Plot on the same diagrams the power needed for motion  $P_n$  considering that with a front/rear driving wheels the vertical load  $F_{z2}/F_{z1}$  should be considered instead of  $mg \cos a$ .
3. Compute the maximum speed  $V$  related to the maximum power that can be transmitted by the wheels. This is given by the intersection of the curves  $P_{\max}$  and  $P_n$ .

## 2 Acceleration performance

Acceleration performance

The difference between the available power  $P_a$  and the needed power  $P_n$  ( $P_a - P_n$ ) is available to accelerate the vehicle. This surplus of power is equal to the variation of the kinetic energy:

$$P_a - P_n = \frac{d\mathcal{T}}{dt}, \quad (4)$$

where  $\mathcal{T}$ , represents the kinetic energy of all vehicle moving parts:

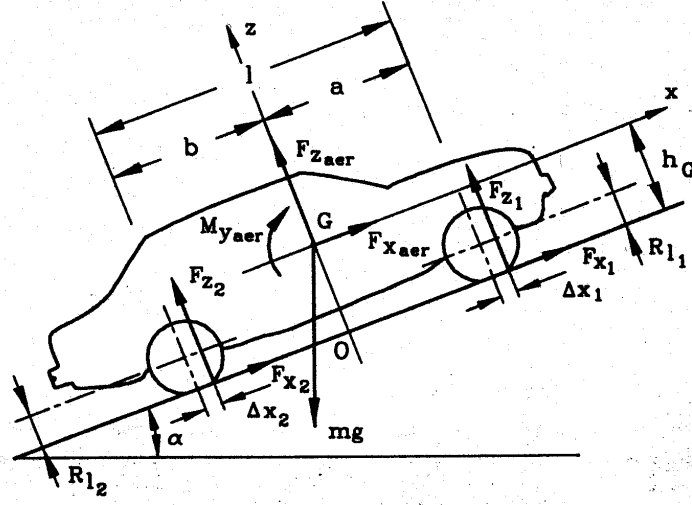


Figure 2: Forces acting on the vehicle.

$$\mathcal{T} = \frac{1}{2} m_e V^2. \quad (5)$$

The term  $m_e$  represents the equivalent mass and is given by the following expression:

$$m_e = m + \frac{J_w}{R_e^2} + \frac{J_t}{R_e^2 \tau_f^2} + \frac{J_e}{R_e^2 \tau_f^2 \tau_g^2}, \quad (6)$$

where  $J_w$ ,  $J_t$ ,  $J_e$  are the moment of inertia of the wheels, the moment of inertia of the power train and of the engine.

The equivalent mass represents the mass of the vehicle that moving at a certain speed  $V$  is storing the same energy of the actual system. It is worth to note that  $m_e$  depends on the gear ratio  $\tau_g$ . Substituting eq. 5 into eq. 4 and derivating the speed  $V$  with respect to the time, the maximum acceleration of the vehicle at maximum throttle is given by the following expression:

$$a_{\max} = \left( \frac{dV}{dt} \right)_{\max} = \frac{P_a - P_n}{m_e V}. \quad (7)$$

In Fig. 3 is reported an example of the maximum acceleration curve as function of the speed at the different gear ratios.

Separating the variables and integrating the equation 7 with respect to  $V$ , the minimum time ( $T_{V_1 \rightarrow V_2}$ ) needed to pass from speed  $V_1$  to speed  $V_2$  is obtained:

$$T_{V_1 \rightarrow V_2} = \int_{V_1}^{V_2} \frac{m_e}{P_a - P_n} V dV = \int_{V_1}^{V_2} \left( \frac{1}{a_{\max}} \right) dV. \quad (8)$$

The expression 8 must be computed in the speed range in which the equivalent mass is constant. The integral can be solved numerically. To this end, the time  $t_s$  for shifting from one gear to

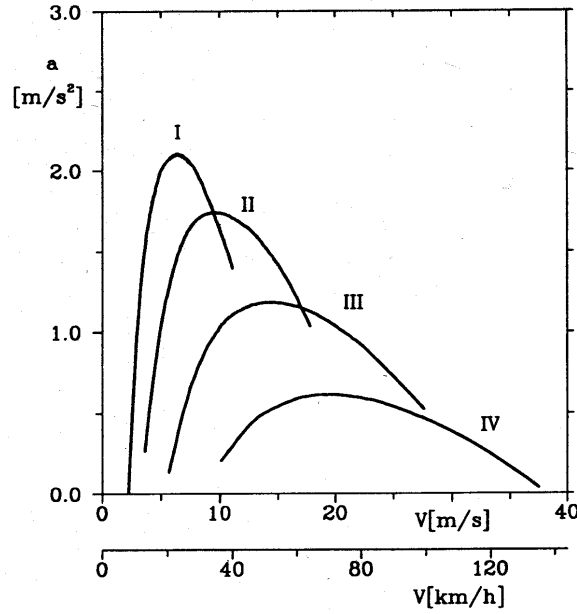


Figure 3: Maximum acceleration as function of the speed at the different gear ratios.

another must be taken into account. In this time interval it is realistic to assume that the speed is constant.

To this end, it is required to

1. compute the equivalent mass for each gear ratio,
2. compute and plot the maximum acceleration  $a_{\max} = \left(\frac{dV}{dt}\right)_{\max}$  as function of  $\tau_g$ .
3. Compute and plot the theoretical and actual time required to reach a speed  $V = 100$  km/h starting from the minimum vehicle speed.

### 3 Computation of the fuel consumption for the NEDC driving cycle

The fuel consumption can be easily computed if it is known the map reporting the specific fuel consumption. Such information are available for each team in the data file `pln_team#.txt`. In fact, the  $pme$  [bar] and the specific consumption  $q$  [g/hph] are available at each rotational speed  $\omega_e$  of the engine for different configurations of the throttle.

The process allowing to compute the fuel consumption starts from the acquisition of vehicle speed  $V$ , the gear engaged  $\tau_g$  and the computation of the power needed for motion  $P_n$  at each operating condition. Being  $P_n$  equal to the power available at the wheels  $P_a$ , the power exploited by the engine at a certain rotational speed can be easily computed as  $P_e = P_n/\eta_t$ . Therefore, at each operating condition, the  $pme$  can be computed using equation of the Power at the engine shaft as function of the  $pme$  and  $\omega_e$  while  $\omega_e$  can be computed as:

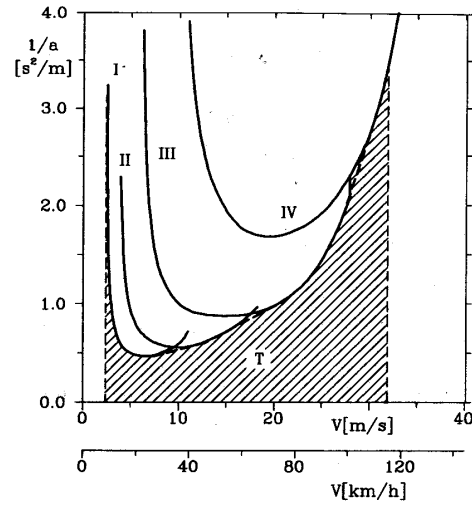


Figure 4: Inverse of the maximum acceleration as function of the maximum speed. The dashed area represents the theoretical time required to pass from speed  $V_1$  to speed  $V_2$ .

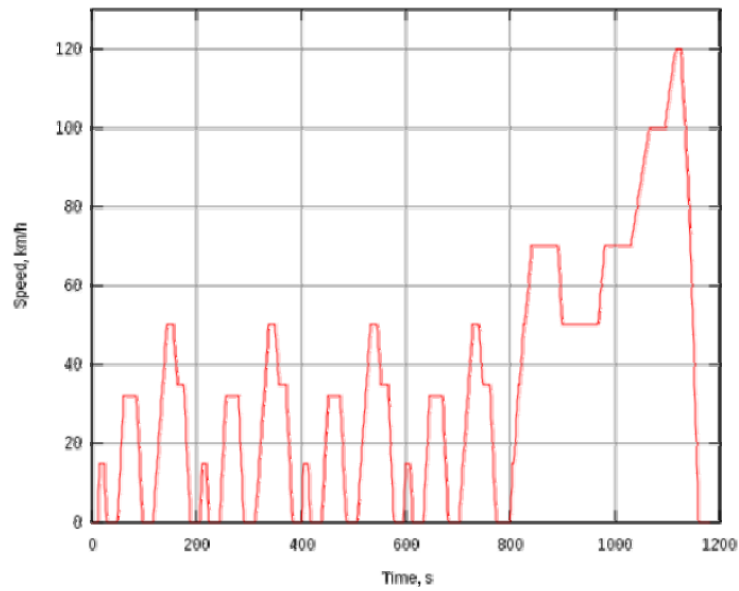


Figure 5: NEDC Driving Cycle.

$$\omega_e = \frac{V}{\tau_g \tau_f R_e}. \quad (9)$$

At each rotational speed of the engine  $\omega_e$ , the knowledge of the *pme* leads to identify the corresponding specific consumption  $q$ . A double linear interpolation is frequently needed as both the engine rotational speed  $\omega_e$  and the *pme* coming from the operating conditions doesn't match the data of the engine map. It is possible to realize a linear interpolation also between the minimum value of *pme* at a certain rotational speed and the value with the curve at null engine efficiency ( $q = 1/H\eta_e$ ;  $H = 4.4 \cdot 10^7$  J/kg).

It is therefore possible to compute the fuel consumption  $Q$  [l/100km] in the different operating conditions. It is given by the following expression:

$$Q_i = \frac{1}{\eta_t} \left( (AV_{mi} + BV_{mi}^3) + m_e V_{mi} \frac{V_{i+1} - V_i}{t_{i+1} - t_i} \right) (t_{i+1} - t_i) \frac{q}{\rho_f}, \quad (10)$$

where

$$V_{mi} = \frac{V_{i+1} + V_i}{2},$$

while  $\rho_f$  is the fuel density ( $\rho_f = 739$  kg/m<sup>3</sup>).

Expression 10 is used only if  $P_n$  is positive. If  $P_n$  is negative (the engine is driven by the external load)  $Q_i = 0$  as the fuel flow is locked. If the engine is running at idle ( $V_i = 0$ ), the fuel consumption is computed as

$$Q_i = Q_m (t_{i+1} - t_i),$$

where  $Q_m$  is the volumetric flow at idle (consider  $Q_m = 0.62$  l/h). The fuel consumption can therefore be computed as

$$Q = \sum_{i=1}^{i=n} Q_i.$$

Following the above considerations, it is required to:

- compute the volumetric fuel consumption  $Q$  [l/100km] for the driving cycle NEDC (Figure 5).

## References

- [1] G. Genta, "Meccanica dell'Autoveicolo", V edizione, *Levrotto & Bella*, Febbraio, 2000.
- [2] G. Lechner, H. Naunheimer, "Automotive Transmissions-Fundamentals, Selection, Design and Application", I edizione, *Springer*, Febbraio, 1999.
- [3] M. D. Artamonov, "Motor Vehicle, fundamentals and design ", *Mir*, Mosca, 1976.