PROJECT 2

TIRES

Index

Evaluation of the slope of the F_x-σ curve in the origin, of the side slip stiffness and of the self-alignment torque stiffness.

This section is devoted to compute the slope of F_x-σ curve, F_y-α and M_z-α. Data are relative to the Pacejka's model for different tires.

Table 1 Tire data for each team

Tire model	Team number										
Tire 1 - 155/65 R13 (automotive)	1	8	15	22	29	36	43	50	57	64	71
Tire 2 - 185/60 R14 (automotive)	2	9	16	23	30	37	44	51	58	65	72
Tire 3 - 205/55 R16 (automotive)	3	10	17	24	31	38	45	52	59	66	73
Tire 4 - 150/60 R17 (motorbike)	4	11	18	25	32	39	46	53	60	67	74
Tire 5 - 170/60 R17 (motorbike)	5	12	19	26	33	40	47	54	61	68	75
Tire 6 - 245/35 R19 (sport car)	6	13	20	27	34	41	48	55	62	59	76
Tire 7 - 255/35 R20 (sport car)	7	14	21	28	35	42	49	56	63	70	77

a. Slope in the origin of F_x-σ curve

The magic formula of the Pacejka's model provides the longitudinal (forward) force F_x as function of slip ratio σ (slip percentage)

$$F_x = D\sin\{C\,atan(B(1-E)(\sigma+S_h)+E\,atan[B\,(\sigma+S_h)])\}+S_v \tag{1}$$

with

$$C = b_0 \tag{2.1}$$

$$D = \mu_{xp}F_z = (b_1F_z+b_2)F_z \tag{2.2}$$

$$BCD = (b_3F_z^2+b_4\,F_z)e^{-b_5\,F_z} \tag{2.3}$$

$$E = b_6F_z^2+b_7F_z+b_8 \tag{2.4}$$

$$S_h = b_9F_z+b_{10} \tag{2.5}$$

$$S_v = 0. \tag{2.6}$$

The magic formula coefficients and the output are not consistent from a dimensional point of view: the vertical force F_z unit is kN, the slip ratio σ is percentage and the longitudinal force F_x unit is N.

Evaluate and plot

- the maximum longitudinal force coefficient μ_{xp} with different forces F_z [2 4 6 8 10] kN;
- the slope in the origin (*BCD*) of the F_x-σ curve with different forces F_z [2 4 6 8 10] kN;
- comments on the maximum value of the longitudinal force coefficient μ_{xp} and the slope in the origin *BDC*.

b. Side slip stiffness

The magic formula of the Pacejka's model provides the lateral force F_y as function of slip angle α and camber angle γ is:

$$F_y = D\sin\{C\,atan(B(1-E)(\alpha+S_h)+E\,atan[B\,(\alpha+S_h)])\}+S_v \tag{3}$$

with

$$C = a_0, \tag{4.1}$$

$$D = \mu_{yp}F_z = (a_1F_z + a_2)F_z\ , \tag{4.2}$$

$$BCD = a_3\sin\left(2\,\mathrm{atan}\left(\frac{F_z}{a_4}\right)\right)(1-a_5|\gamma|)\ , \tag{4.3}$$

$$E = a_6F_z + a_7, \tag{4.4}$$

$$S_h = a_8\gamma + a_9F_z + a_{10}, \tag{4.5}$$

$$S_v = a_{11}\gamma F_z + a_{12}F_z + a_{13}. \tag{4.6}$$

The a_{11} coefficient is equivalent to the following linear equation:

$$a_{11} = a_{111}F_z + a_{112} \tag{5}$$

Also in this case the magic formula coefficients and the output results are not consistent from the dimensional point of view. The lateral force F_y unit is N, the vertical force F_z unit is kN, the angles α and γ units are degrees.

Evaluate and plot

- The side slip stiffness *BCD* with different vertical forces F_z [2 4 6 8 10] kN.
- The maximum lateral force coefficient μ_{yp} with different vertical forces F_z [2 4 6 8 10] kN.
- The effect of the camber angle on the side slip stiffness with a vertical force F_z = 5 kN (consider camber angle γ in the range ±20°).
- Comment on the obtained results.

c. Self-alignment torque stiffness

The Magic Formula for the self-alignment torque is

$$M_z = D\sin\{C\ atan(B(1-E)(\alpha+S_h) + E\ atan[B\ (\alpha+S_h)])\} + S_v\,, \quad (6)$$

with

$$C = c_0, \quad (7.1)$$

$$D = c_1F_z^2 + c_2F_z, \quad (7.2)$$

$$E = (c_7F_z^2 + c_8F_z + c_9)(1 - c_{10}\gamma), \quad (7.3)$$

$$BCD = (c_3F_z^2 + c_4F_z)(1 - c_6|\gamma|)e^{-c_5F_z}, \quad (7.4)$$

$$S_h = c_{11}\gamma + c_{12}F_z + c_{13}, \quad (7.5)$$

$$S_v = (c_{14}F_z^2 + c_{15}F_z)\gamma + c_{16}F_z + c_{17}. \quad (7.6)$$

The vertical load F_z unit is kN, angles α and γ units are degrees and the self-alignment torque unit is Nm.

Evaluate and plot

- The stiffness *BCD* of the self-alignment torque with different vertical forces F_z [2 4 6 8 10] kN;
- The effect of camber angle on the stiffness *BCD* with a vertical load F_z = 5 kN (consider camber angle γ in the range ±20°).
- Comment the results obtained.

1. Tire nonlinear model

The aim of this section is to estimate the nonlinear tire behaviour in different operation conditions. In appendix 1 are reported the generalized forces developed by the tire at different vertical loads in a wide range of operation. Based on that data it is required to:

- evaluate and plot the longitudinal force coefficient μ_x as function of slip ratio with different forces F_z = [2 4 6 8 10] kN;
- evaluate and plot the lateral force coefficient μ_y as function of slip angle with different forces F_z =[2 4 6 8 10] kN *(γ=0)*;
- evaluate and plot the lever arm *t* of the force F_y which creates the self-alignment torque M_z *(M_z=$F_y t$)* as function of the slip angle with a vertical force F_z = 4 kN, *(γ=0)*;
- plot the Gough diagram;
- comment the results.

2. Evaluation of the interaction of the tire-road forces in longitudinal and lateral direction

Pacejkca's Magic Formula gives the behaviour of a tire in longitudinal or lateral forces separately. If the tire produces simultaneously forces along X' and Y' the situation can be different as the force along one axis limits the maximum force available along the other axis.

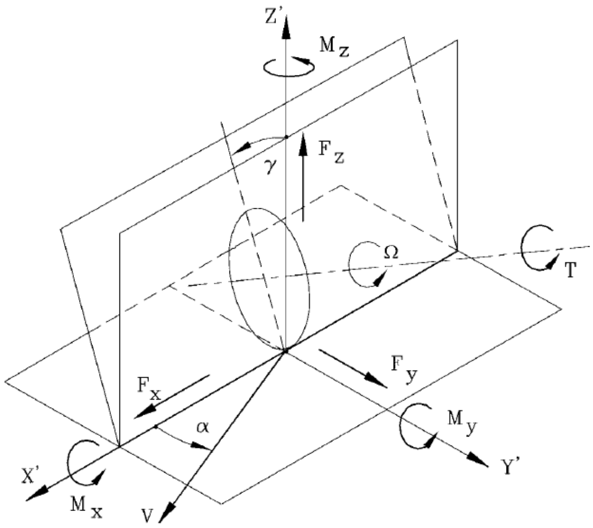

Figure 1 Reference system used to study forces exchanged between tire and ground. Definition of positive direction of forces, moments and side slip angle

A model to approximate the curves $F_y(F_x)$ is the elliptical model

$$\left(\frac{F_y}{F_{y0}}\right)^2 + \left(\frac{F_x}{F_{x0}}\right)^2 = 1 \qquad 8$$

where forces F_{y0} and F_{x0} are the force F_y exerted at the given side slip angle when no force F_x is exerted, and the maximum longitudinal force exerted at zero side slip angle respectively. The elliptical model gives

$$F_y = F_{y0}\sqrt{1-\left(\frac{F_x}{F_{x0}}\right)^2}, \qquad 9$$

and consequently

$$\mu_y = \mu_{y0}\sqrt{1-\left(\frac{\mu_x}{\mu_p}\right)^2} \qquad 10$$

$$C = C_0\sqrt{1-\left(\frac{F_x}{\mu_p F_z}\right)^2} \qquad 11$$

where μ_y and μ_{y0} indicate the friction coefficient in side direction with and without longitudinal force. The term C_0 indicated the cornering stiffness when no longitudinal force is produced.

It is required to:

- evaluate the lateral force F_y as function of the longitudinal one F_x for different slip angles $\alpha = [2\ 4\ 6\ 8\ 10]$ *deg* considering a vertical load $F_z = 4$ *kN* and camber angle γ equal to *0 deg*. (plot F_y for all α)
- evaluate and plot the side slip stiffness C as a function of the longitudinal force F_x considering a vertical load F_z=[2, 4, 6, 8, 10] kN.
- evaluate and plot the friction coefficient μ_y in side direction considering as reference the data corresponding to F_z=4 kN.

3. References

[1] G. Genta, L Morello, "The Automotive chassis", Sprinter Verlag, December 2008.

[2] H.B. Pacejka, Tire models for vehicle dynamics analysis, Vehicle Systems Dynamics, Vol. 21, October 1991