AUTOMOTIVE ENGINEERING
POLITECNICO DI TORINO
MOTOR VEHICLE DESIGN (02LNLLI)

PROJECT 4

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Contents

1 Ideal braking⇔ μ_x is the same for all the wheels

The total braking force acting on a vehicle can be written as

$$F_x = \sum_{\forall i} \mu_{x_i} F_{z_i}$$

where the summation is extended to all the braking wheels. Assuming that the rolling resistance and the aerodynamic drag are negligible with respect to the braking force, the equation of motion in the longitudinal direction is:

$$m\frac{dV}{dt} = \sum_{\forall i} \mu_{x_i} F_{z_i} - mg\sin(\alpha). \tag{1}$$

If it is assumed that μ_{x_i} is the same for all the wheels, equation 1 simplifies as:

$$\frac{dV}{dt} = \mu_x g\cos(\alpha) - g\sin(\alpha). \tag{2}$$

In the case of null longitudinal slope, equation 2 simplifies as:

$$\frac{dV}{dt} = \mu_x g.$$

At this stage it is therefore important to determine the forces acting on the front and on the rear axle during the braking action. Remember that we have assumed a symmetry plane and that for the longitudinal dynamics we can collapse the front and the rear wheels to that plane. The computation of the longitudinal forces exerted by the wheels for braking needs the estimation of the vertical force F_{z1} and F_{z2} acting on each axle. If the aerodynamic and rolling resistance forces are neglected, the vertical load acting on each axle is:

$$\begin{aligned} F_{z_1} &= \frac{m}{l}\left[gb\cos(\alpha) - gh_G\sin(\alpha) - h_G\frac{dV}{dt}\right] \text{ front axle,} \\ F_{z_2} &= \frac{m}{l}\left[ga\cos(\alpha) + gh_G\sin(\alpha) + h_G\frac{dV}{dt}\right] \text{ rear axle.} \end{aligned} \tag{3}$$

The substitution of expression F_{x1} and F_{x2} (eq.3) into eq. 1 ($F_{x_1} = \mu_x F_{z_1}$, $F_{x_2} = \mu_x F_{z2}$,) and solving eq. 1 eliminating μ_x leads to the following expression:

$$(F_{x_1} + F_{x_2})^2 + mg\cos(\alpha)^2\left(F_{x_1}\frac{a}{h_G} - F_{x2}\frac{b}{h_G}\right) = 0 \tag{4}$$

that relates the braking force F_{x_2} with respect to F_{x1}. The full line curve in Fig. 1 evidences how F_{x2} must vary with respect to F_{x1} to ensure the ideal braking. If μ_{x_1} or μ_{x_2} are considered constant, F_{x2} is related to F_{x1} according to the following expression:

$$\begin{aligned} F_{x2} &= mg\frac{b}{h_G}\cos(\alpha) - F_{x_1}\left(\frac{l}{\mu_{x_1}h_G} + 1\right), \\ F_{x2} &= \frac{\mu_{x2} mga\cos(\alpha)}{l - \mu_{x2}h_G} + F_{x_1}\left(\frac{\mu_{x_2}h_G}{l - \mu_{x2}h_G}\right). \end{aligned} \tag{5}$$

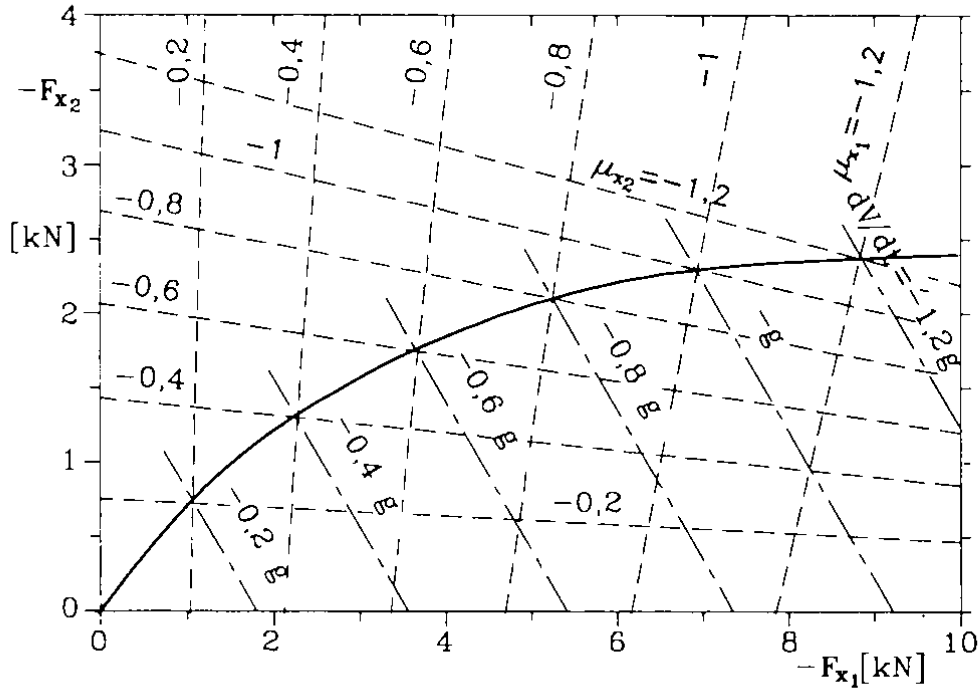

Figure 1: The full line reports the braking force F_{x_2} as function of F_{x_1} in condition of ideal braking. The dashed lines are referred to the braking force F_{x2} with respect to F_{x1} in the condition of constant μ_{x1}, constant μ_{x2} and constant dV/dt.

If the deceleration is constant, the relation between F_{x2} and F_{x_1} is:

$$F_{x_2} = mg\sin(\alpha) + m\frac{dV}{dt} - F_{x_1}. \quad (6)$$

According to the above considerations it is required to:

- plot the curve of the braking force at the rear axle as function of the braking force on the front axle in the following operating condition:
  - ideal braking,
  - μ_{x_1} = constant (consider μ_{x_1} in the range $-0.2 \div -1.2$ with step 0.2)
  - μ_{x_2} = constant (consider μ_{x_2} in the range $-0.2 \div -1.2$ with step 0.2)
  - constant deceleration minimum $\frac{dV}{dt} = -0.2$g, maximum $\frac{dV}{dt} = -1.2$g con with step 0.2g.

Consider the mass standard B (vehicle mass + driver mass = 75 kg and baggage mass =10 kg) and the mass standard F (St B + 4 passengers=280 kg and 1 baggage for each passenger = 40 kg totally). See Fig. 1 as reference. Make reference to the data adopted in Project 2

2 Braking in actual conditions

An actual braking system cannot follow the conditions of ideal braking. In general, the ratio between the braking torque on the rear and front axle is constant and can be defined as:

$$K_b = \frac{M_{b_1}}{M_{b_2}} \tag{7}$$

where M_{b1} and M_{b2} represent the braking torque on the rear and on the front axle.

If the radius of the rear and front wheels is the same, K_b can be expressed as

$$K_b = \frac{F_{x_1}}{F_{x_2}}. \tag{8}$$

The aim of this part is to design the coefficient K_b. To this end it is needed to introduce the concept of braking efficiency, defined as the ratio between the deceleration obtained in actual braking conditions and the deceleration obtained in the condition of ideal braking. It is worth to note that the computation must be undertaken considering the same value of maximum longitudinal adherence μ_x at the wheels whose longitudinal force coefficient is higher. Considering the condition of null longitudinal slope, the braking efficiency can be expressed as:

$$\eta_b = \frac{(dV/dt)_{actual}}{(dV/dt)_{ideal}} = \frac{(dV/dt)_{actual}}{\mu_x g}. \tag{9}$$

Assuming to neglect the aerodynamic drag, the rolling resistance, $(dV/dt)_{actual}$ can be computed following the steps reported here below:

$$\frac{dV}{dt} = \frac{F_{x1} + F_{x2}}{m} = \frac{F_{x1}\left(1 + 1/K_b\right)}{m} = \frac{F_{x2}\left(1 + K_b\right)}{m}, \tag{10}$$

where

$$\begin{aligned} F_{x1} &= \mu_{x1}\frac{mg}{l}\left(b - \frac{h_G}{g}\frac{dV}{dt}\right), \\ F_{x2} &= \mu_{x2}\frac{mg}{l}\left(a + \frac{h_G}{g}\frac{dV}{dt}\right). \end{aligned} \tag{11}$$

Substituting eqs. 11 in eq. 10, the braking efficiency η_b assumes the following expression:

$$\eta_b = \min\left\{\eta_{bf},\ \eta_{br}\right\} = \min\left\{\frac{b(K_b+1)}{lK_b + \mu_x h_G(K_b+1)},\ \frac{a(K_b+1)}{l - \mu_x h_G(K_b+1)}\right\}. \tag{12}$$

At this stage, K_b must be designed.

It is needed to define the value of μ_x at which the actual braking curve intersects the ideal braking curve. This allows to compute

$$K_b = \frac{b - \mu_x h_G}{l + \mu_x h_G - b} = \frac{l - \mu_x h_G - a}{\mu_x h_G + a}, \tag{13}$$

and therefore the relation between F_{x2} and F_{x1}.

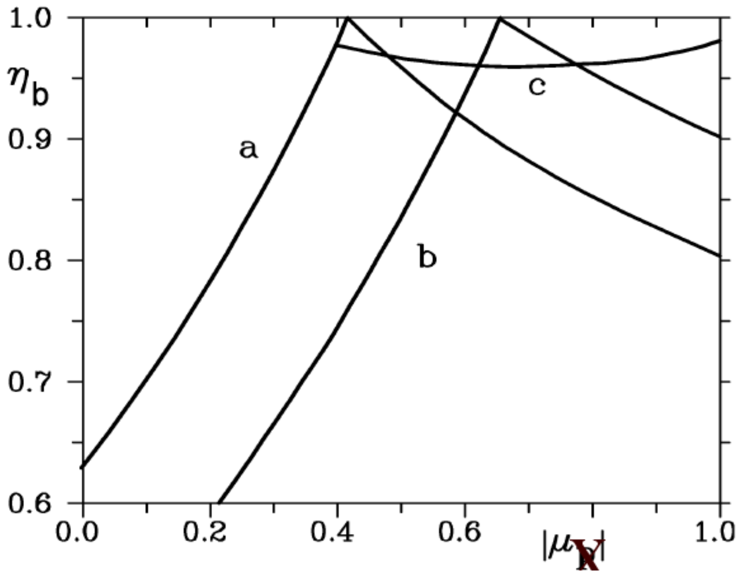

Figure 2: Possible curve of the braking efficiency.

Curves a and b in figure 2 reports the curves of the braking efficiency, selecting for μ_x a value of -0.4 and -0.7 at the intersection of the actual braking curve with the ideal one. They both present some drawbacks. The first one is characterized by the fact that considering an adherence higher than -0.4 (higher is referred to the absolute value of μ_x), the rear wheel brakes more that the front ones. When the limit conditions are reached, the rear wheels locks first triggering problems of stability. On the contrary, if curve b is followed, it happens that for a large range of μ_x the efficiency of the braking is low.

The drawbacks evidenced using the configurations a or b can be bypassed using a pressure proportioning valve that reduces the pressure to the rear brakes when a certain value of pressure has been reached in the circuit. The result is that F_{x2} versus F_{x1} plot becomes bilinear. Consequently the curve of ideal braking in Figure 3 can be better followed and the efficiency of the braking system is improved. With reference to the Figure 2, if the possible intersections between the ideal braking curve and the actual braking curve is set at $\mu_x = -0.4$ and the proportional valve starts operating at the 90 % of this max value for safety, the efficiency of the braking system in the range $-1 \le \mu_x \le 0$ is given by curve a for $-0.4 \lesssim \mu_x \le 0$ and by curve c for $-1 \lesssim \mu_x \lesssim -0.4$.

The design of the braking circuit must therefore follow the below described steps:

- design K_b using a value of $\mu_x = -0.4$ for which the actual braking intersects the ideal braking. At this stage the relation between F_{x1} and F_{x2} is fixed.
- Identify the braking condition for which the proportional valve starts working (point A in Figure 3). It is suggested to set this operating point when the 90 % of the actual braking force occurs (with respect to the rear one).

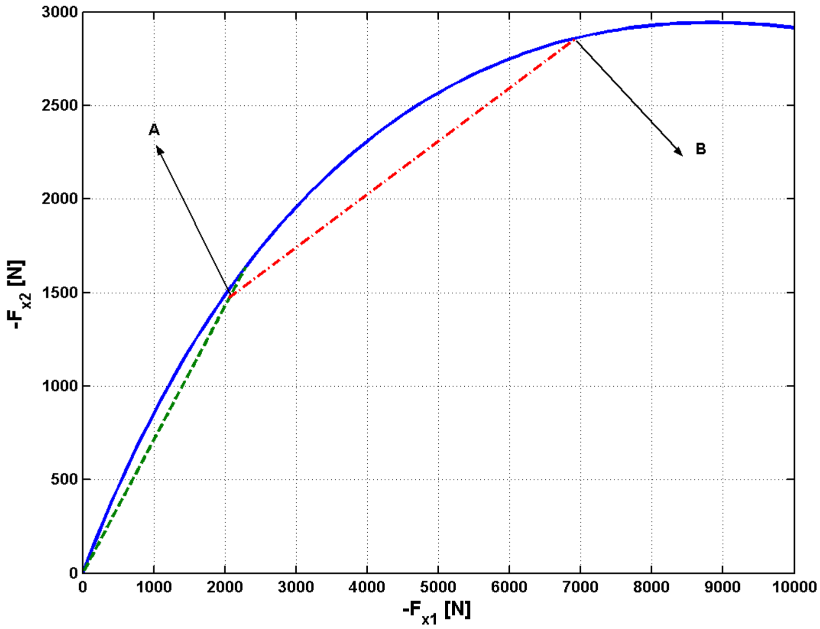

Figure 3: Actual braking curve with proportional valve.

- Design the proportional valve as $K_b' = F_{x1}/F_{x2}$ for $-1 \lesssim \mu_x \lesssim -0.4$. It is suggested to set the new intersection (point B in Figure 3) of the actual braking with the ideal braking at $\mu_x = -1$. The values of F_{x1} and F_{x2} at point B can be computed using equation 11 ($\mu_x = -1$, $dV/dt = -\mu_x g$).
- Compute the efficiency of the braking system in the range in which the proportional valve is working. Take into account that as well as the term K_b' has been computed, defined a certain value of F_{x1}, F_{x2} is determined and

$$\left(\frac{dV}{dt}\right)_{actual} = \frac{F_{x1} + F_{x2}}{m}.$$

This value of deceleration must be compared to the deceleration in ideal conditions

$$\left(\frac{dV}{dt}\right)_{ideal} = \mu_x g$$

where

$$\mu_x = \frac{l}{\dfrac{mgb}{F_{x1}} - h_G\left(1 + \dfrac{F_{x2}}{F_{x1}}\right)}$$

According to the above considerations, it is required to:

- design K_b,
- plot η_b as a function of μ_x,
- design $K_b^{'}$,
- plot F_{x_2} as a function of F_{x_1} taking into account the effect of the proportional valve. Report in the same graph the plot of the ideal braking.
- plot η_b as a function of μ_x taking into account the role of the proportional valve,

  The load standard F must be used.