MSc. Automotive Engineering

Car Body Design and Aerodynamics

Exercise 3 - Quarter-Car NVH Analysis

Ragheed Huneineh (S265776)

 5/21/2020

Abstract

In this work, NVH analysis on the quarter-car model has been performed on two different vehicles. The data was taken from the input and the results of a previous exercise. First, the state space representation of the quarter-car model was developed. Then, it was implemented in the MATLAB® GUI Application developed in a previous exercise to obtain the frequency response of the sprung mass acceleration within a chosen interval of frequency. The output was filtered according to the ISO 2631 standard to analyze the human sensitivity to mechanical vibration. After that, the power spectral densities (PSD) of the road profile, the sprung mass acceleration, and the filtered acceleration were calculated and plotted relative to different road grades and different vehicle speed. Eventually, an optimization analysis on the damping coefficient value based on the mean squared value (MSV) of the filtered acceleration has been performed. Results show that the frequency response and the optimal damping coefficient depend only on the vehicle's characteristics; however, the PSD and the MSV depend also on the road profile and the vehicle's speed.

Contents

Motivation

The main purpose of this exercise is to analyze the dynamic behavior of the quarter-car model. The idea is to implement the model to obtain the frequency response of the sprung mass acceleration within a given range of frequency values so we can filter it using the ISO 2631 standard for human sensitivity to mechanical vibration to obtain the frequency response of the acceleration perceived by the occupants. Consequently, the PSD of different road profiles, which can be obtained from an empirical formula, can be used as an input to the model to evaluate the PSD of the sprung mass acceleration and the filtered acceleration to have a better insight on the contribution of each frequency value. Eventually, the MSV can be calculated from the PSD to optimize the damping coefficient of the damper present in the model.

Objectives

The main objectives to be accomplished during this exercise are the following:

1. Collect the data of the vehicles.
2. Derive the state space representation of the quarter-car model.
3. Represent the frequency response of the accelerations using the bode diagram.
4. Represent the PSD of the road profile, sprung mass acceleration, and filtered acceleration for different road grades and different vehicle speed.
5. Calculate the MSV for different values of the damping coefficient to obtain the optimal value.

Methodology

1. Vehicle Choice

Two vehicles have been chosen to fulfill the aim of this exercise, the Fiat 500 and the BMW X5 xdrive40i shown in Figure 1 and Figure 2:

Figure 1: Fiat 500

Figure 2: BMW X5 xDrive40i

The relevant data are represented in Table 1:

Property	BMW X5 xDrive40i	Fiat 500
Wheelbase [mm]	2975	2300
Track [mm]	1666	1409
Tire Diameter [mm]	737	584
Tire Width [mm]	255	185
Steer Radius [mm]	6300	9300
Vehicle Mass [kg]	2060	932
CG [mm]	1801	1150
FR [mm]	1636	1500
RR [mm]	2529	1900
LUGGAGE [mm]	3511	2300
N. Front Row [-]	2	2
N. Rear Row [-]	3	2
GC Front [mm]	180	130
GC Rear [mm]	200	130
Drive	AWD	FWD

Table 1: Vehicles Data

2. Quarter-Car Model

2.1 State Space Representation

The state space representation of the model can be derived from the physical model which can be defined by analyzing Figure 3.

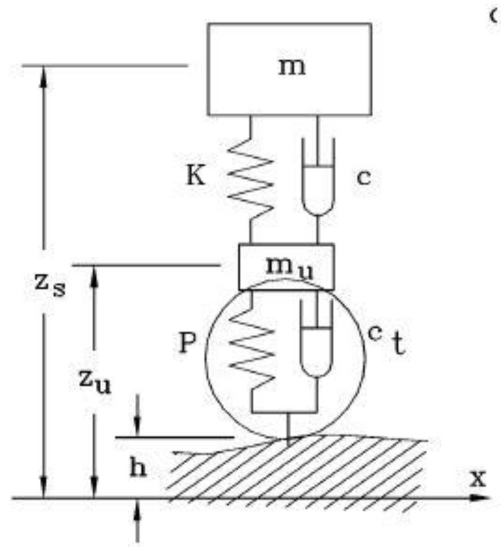


Figure 3: 2 Degrees of Freedom Quarter-Car Model

By neglecting the damper "c_t", the equations defining the physical model are the following:

$$m\ddot{z}_s + c(\dot{z}_s - \dot{z}_u) + K_s(z_s - z_u) = 0 \quad \text{.... (1)}$$

$$m_u\ddot{z}_u - c(\dot{z}_s - \dot{z}_u) + K_t(z_u - h) - K_s(z_s - z_u) = 0 \quad \text{...(2)}$$

In matrix form:

$$\begin{bmatrix} m & 0 \\ 0 & m_u \end{bmatrix}\begin{pmatrix} \ddot{z}_s \\ \ddot{z}_u \end{pmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{pmatrix} \dot{z}_s \\ \dot{z}_u \end{pmatrix} + \begin{bmatrix} K_s & -K_s \\ -K_s & K_t + K_s \end{bmatrix}\begin{pmatrix} z_s \\ z_u \end{pmatrix} = \begin{bmatrix} 0 \\ K_t \end{bmatrix} h$$

Which then can be written as:

$$\{\ddot{z}\} = -[M]^{-1}[K]\{z\} - [M]^{-1}[C]\{\dot{z}\} + [M]^{-1}[In]\{e\}$$

Finally:

$$\begin{pmatrix} \{\dot{z}\} \\ \{\ddot{z}\} \end{pmatrix}_{4x1} = \begin{bmatrix} [0]_{2x2} & I_{2x2} \\ [-M^{-1}K]_{2x2} & [-M^{-1}C]_{2x2} \end{bmatrix}\begin{pmatrix} \{z\} \\ \{\dot{z}\} \end{pmatrix}_{4x1} + \begin{bmatrix} [0] \\ [M^{-1}In] \end{bmatrix}_{4x1} \{e\}_{1x1}$$

The output required is the acceleration of the sprung mass. Although it's not a state variable, it's simply represented by the equation from the physical model. Hence, we can say:

$$Y = \ddot{z}_s = [1,0]\begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_u \end{Bmatrix} = [T_O]\{\ddot{z}\}$$

$$Y = [T_O](-[M]^{-1}[K]\{z\} - [M]^{-1}[C]\{\dot{z}\} + [M]^{-1}[In]\{e\})$$

In state space representation:

$$Y = [-T_O M^{-1}K]\begin{pmatrix} \{z\} \\ \{\dot{z}\} \end{pmatrix} + [T_O M^{-1} In]\{e\}$$

2.2 Frequency Response

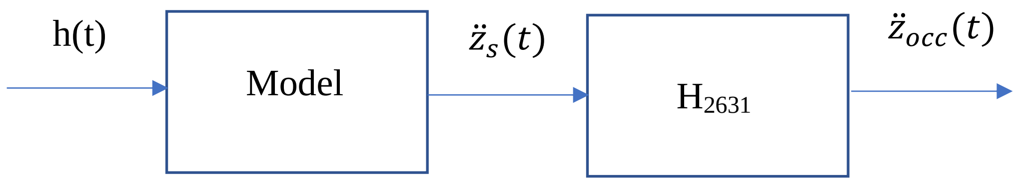
After finding the state space representation of the model, it can be easily implemented in MATLAB® from which we can find the frequency response in the frequency range of interest for vibration comfort which is (0.01, 80) [Hz]. We can also consider a filter that takes into account human sensitivity to vibrations as indicated by ISO2631:

$$H_{2631}(s) = \frac{80.03s^2+989s+0.02108}{s^3+78.92s^2+2412s+5614} \quad \ldots(4)$$

Note that the initial choice of the damping coefficient is the one considered optimal for the sprung mass according to:

$$c_{opt} \cong \frac{\sqrt{mK_s}}{2}$$

Therefore, the block diagram below represents how the model works:



The results are shown in Figure 4 and Figure 5:

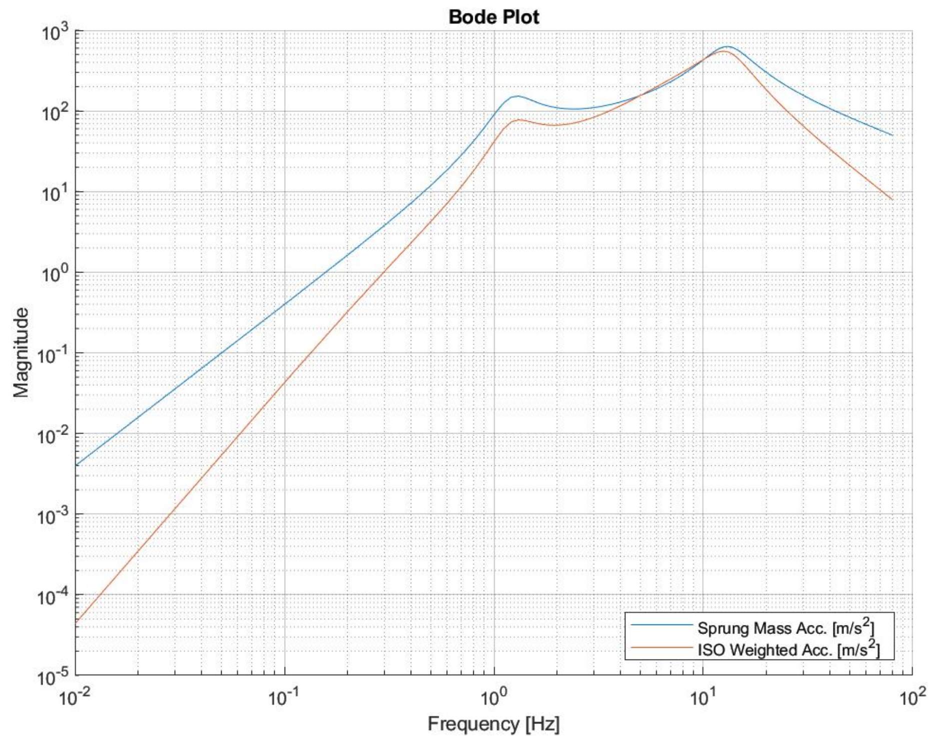


Figure 4: Bode Plot - Fiat 500

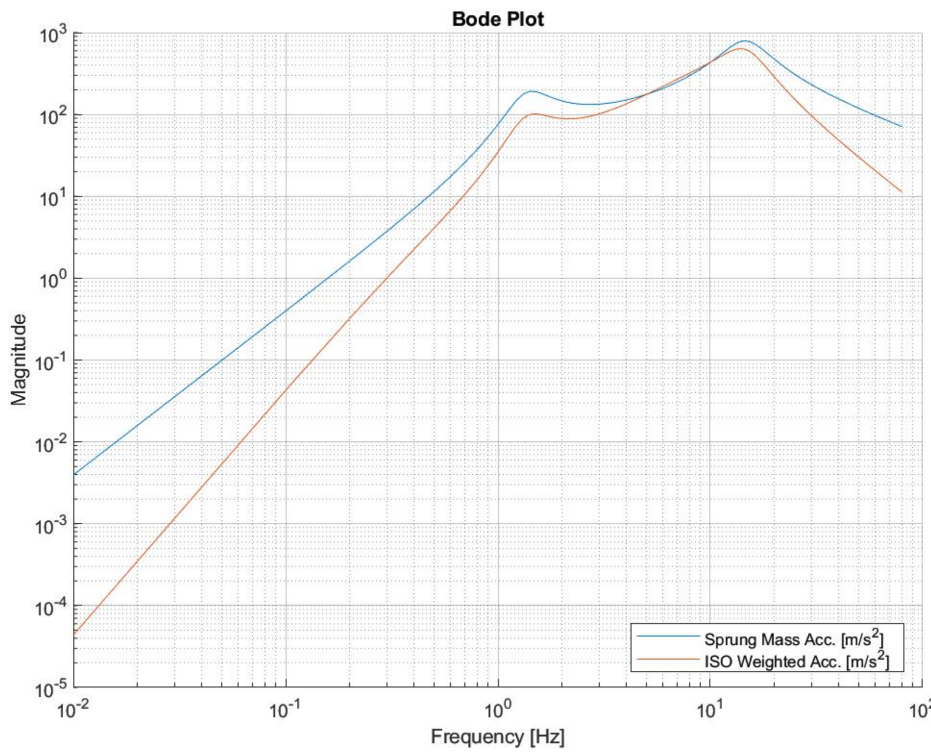


Figure 5: Bode Plot - BMW X5 xDrive40i

2.3 Power Spectral Density

2.3.1 Definition

By measuring a road profile on a sufficiently indicative range, we will find that it is a random signal whose mean is null. Therefore, the mean value can't be used to indicate the irregularity of a road profile. Another option is the mean squared value defined as:

$$E[h^2] = \frac{1}{L}\int_0^L h^2(x)dx \quad \ldots(5)$$

Where "L" is the distance along which the measurement is taken, and "x" is the instantaneous abscissa at which we have a single value of "h". By defining the autocorrelation function:

$$R(\lambda) = \lim_{L\to\infty} \frac{1}{L}\int_{-L/2}^{L/2} h_i(x)h_i(x+\lambda)dx \qquad \ldots(6)$$

We can directly see that the autocorrelation function and the mean squared value are identical when "$\lambda = 0$". The idea behind using the MSV is that its power spectral density can be obtained using direct Fourier transform indicating the contribution of each frequency value to the MSV, thus to the road profile irregularity. The PSD is thus defined as:

$$S(\nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\lambda)e^{-i\nu\lambda}d\lambda \quad \ldots(7)$$

Where "$f_l = \frac{\nu}{2\pi}$" is the distance domain frequency. Hence:

$$E[h^2] = R(0) = 4\pi\int_{-\infty}^{\infty} S'(f_l)df_l \quad \ldots(8)$$

Measurements in the past have been thoroughly analyzed indicating that the typical trend of the power spectral density of a road profile as a function of the distance domain frequency can be approximated by a straight line in logarithmic scale[1]. Hence, the ISO8608 states an empirical formula expressing the power spectral density of a road profile as a function of the distance domain frequency:

$$S = c\nu^{-N} \quad \ldots(9)$$

Where "c" is the road grade and "$N \cong 2$" is constant.

The accuracy of this empirical formula can be evidenced in Figure 6:

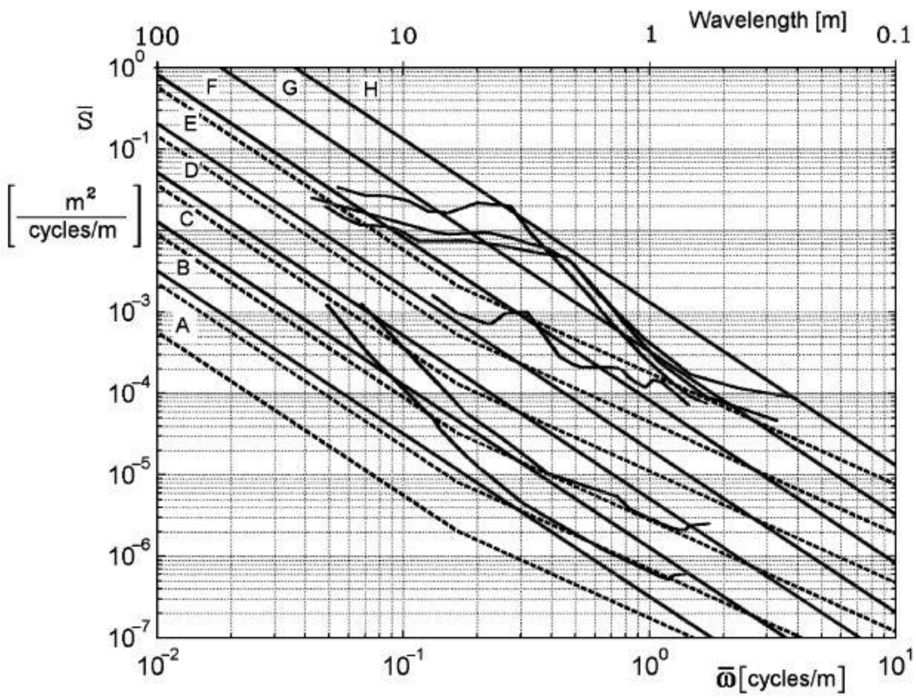


Figure 6: Power Spectral Density - Different Road Grades

Where the different values of the road grade coefficient are reported in Table 2:

Class	c_{min} [m²cycles/m]	c_{avg} [m²cycles/m]	c_{max} [m²cycles/m]
A	-	$1.6*10^{-7}$	$3.2*10^{-7}$
B	$3.2*10^{-7}$	$6.4*10^{-7}$	$1.28*10^{-6}$
C	$1.28*10^{-6}$	$2.56*10^{-6}$	$5.12*10^{-6}$
D	$5.12*10^{-6}$	$1.024*10^{-5}$	$2.048*10^{-5}$
E	$2.048*10^{-5}$	$4.096*10^{-5}$	$8.192*10^{-5}$
F	$8.192*10^{-5}$	$1.6384*10^{-4}$	$3.2768*10^{-4}$
G	$3.2768*10^{-4}$	$6.5536*10^{-4}$	$1.31081*10^{-3}$
H	$1.31081*10^{-3}$	$2.62144*10^{-3}$	-

Table 2: ISO8608 Road Description

Up to this point, the PSD has been defined using the distance domain frequency; however, we can switch to time domain frequency by considering the velocity of the vehicle, "V":

$$f = V * \nu \quad \ldots(10)$$

Therefore, we can rewrite equation (8):

$$E[h^2] = \int_0^\infty S(\nu)d\nu = \int_0^\infty S\left(\frac{f}{V}\right)\frac{1}{V}df = \int_0^\infty c\left(\frac{f}{V}\right)^{-N}\frac{1}{V}df = \int_0^\infty S(f)df \qquad \ldots(11)$$

And equation (9):

$$S_{in}(f) = cV^{N-1}f^{-N} \qquad \text{...(12)}$$

At this point, it's evident that by choosing the road grade of interest, a certain velocity of the vehicle, and the frequency range (0.01, 80) [Hz], we can represent the PSD of the road profile. Moreover, we can use the transfer function of the model, "G(s)", to obtain the PSD of the sprung mass acceleration:

$$S_{out}(f) = |G(s)|^2 * S_{in}(f) \qquad \text{...(13)}$$

And we can use the transfer function of the human sensitivity to mechanical vibration to obtain the PSD of the acceleration perceived by the occupants:

$$S_{occ}(f) = |H_{2631}(s)|^2 S_{out}(f) = |G(s) * H_{2631}(s)|^2 S_{in}(f) \qquad \text{...(14)}$$

2.3.2 Application

The discussed concepts concerning the PSD have been applied on the two vehicles of choice for two different road grades, "A" and "B", and two different vehicle speeds, "120 [km/h]" and "70 [km/h]", respectively. The results are presented in Figure 7, Figure 8, Figure 9, and Figure 10:

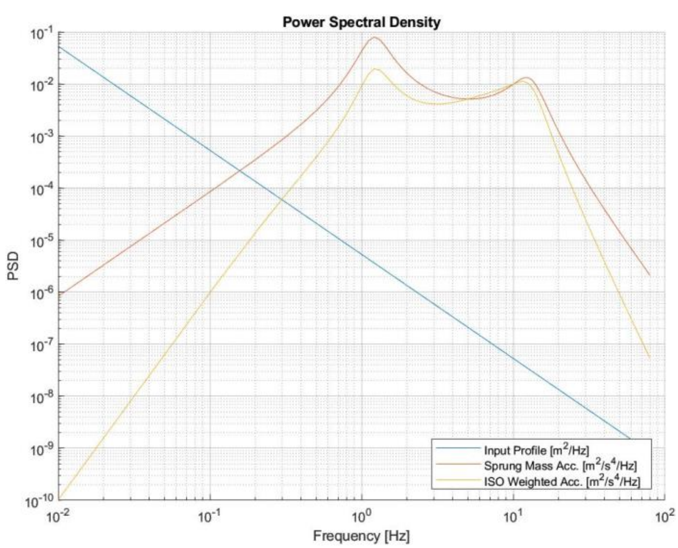


Figure 8: PSD - Road Grade A, 120 [km/h] - Fiat 500

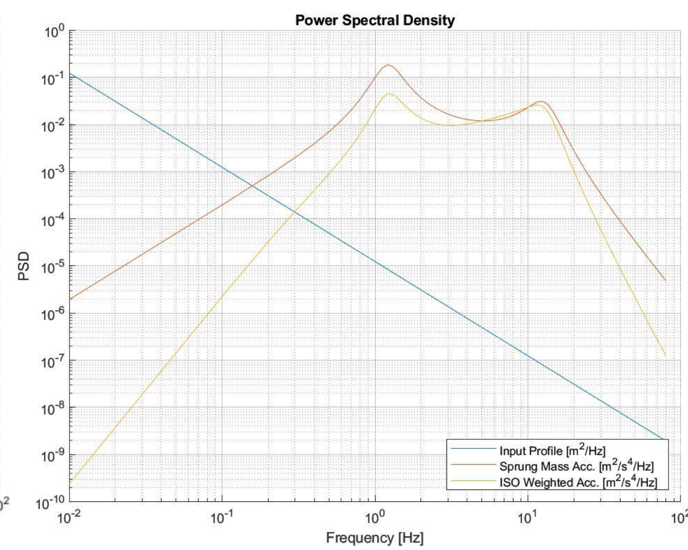


Figure 7: PSD - Road Grade B, 70 [km/h] - Fiat 500

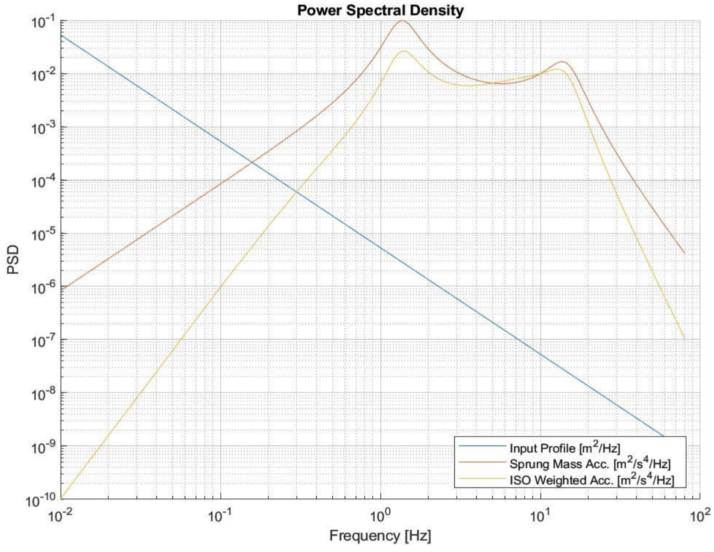


Figure 9: PSD - Road Grade A, 120 [km/h] - BMW X5

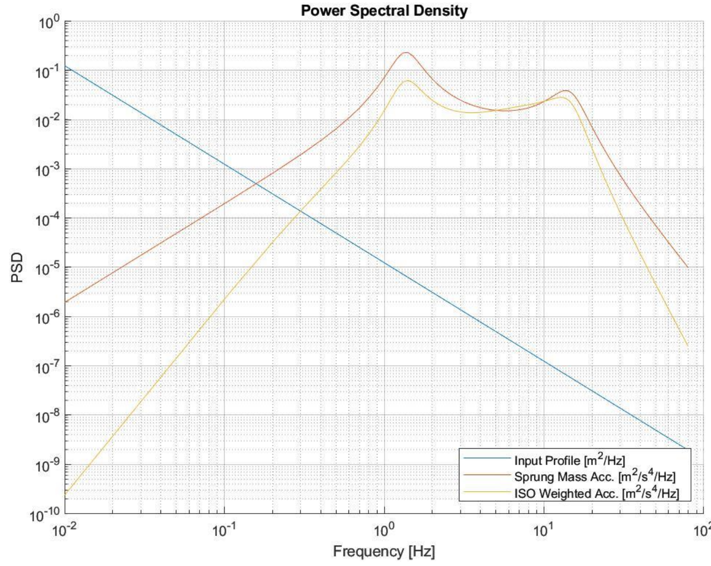


Figure 10: PSD - Road Grade B, 70 [km/h] - BMW X5

2.4 Damping Coefficient Optimization

As mentioned earlier, the initial damping coefficient value used is the optimal value considering the acceleration of the sprung mass. However, the optimal value relevant for this case of study should consider the filtered acceleration perceived by the occupants. This can be done by calculating the MSV from equation (11) for different values of the damping coefficient between (100, 10,000) [Ns/m] and then choosing the damping coefficient value corresponding to the minimum value of MSV. The results are presented in Figure 11, Figure 12, Figure 13, Figure 14, and Table 3:

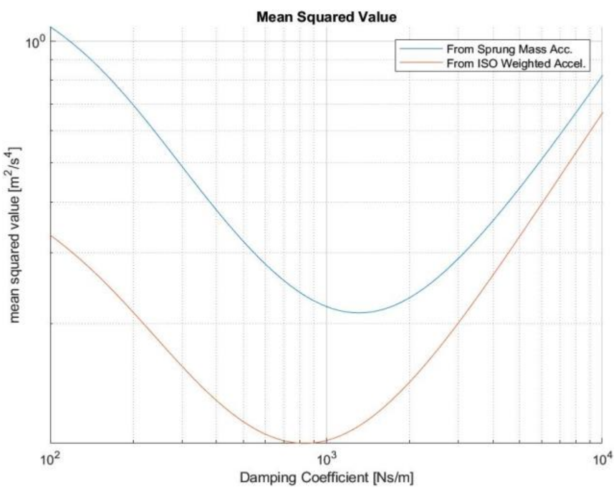


Figure 12: MSV - Road Grade A, 120 [km/h] – Fiat 500

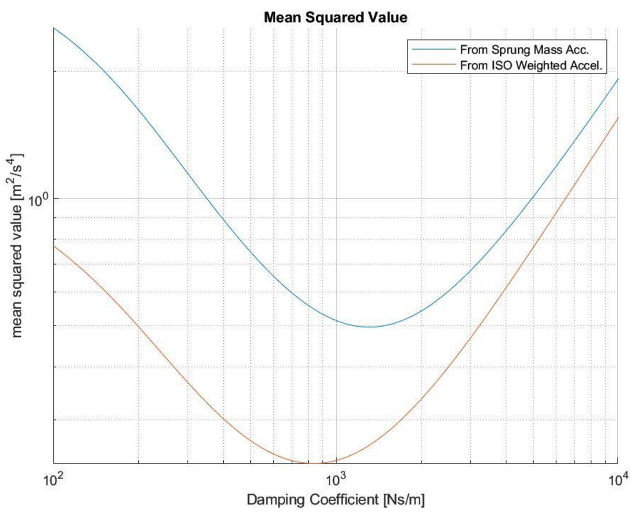


Figure 11: MSV - Road Grade B, 70 [km/h] - Fiat 500

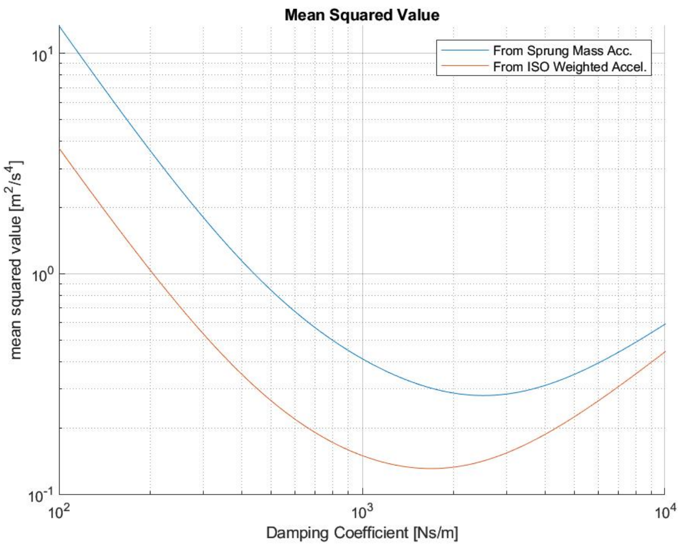


Figure 13: MSV - Road Grade A, 120 [km/h] - BMW X5

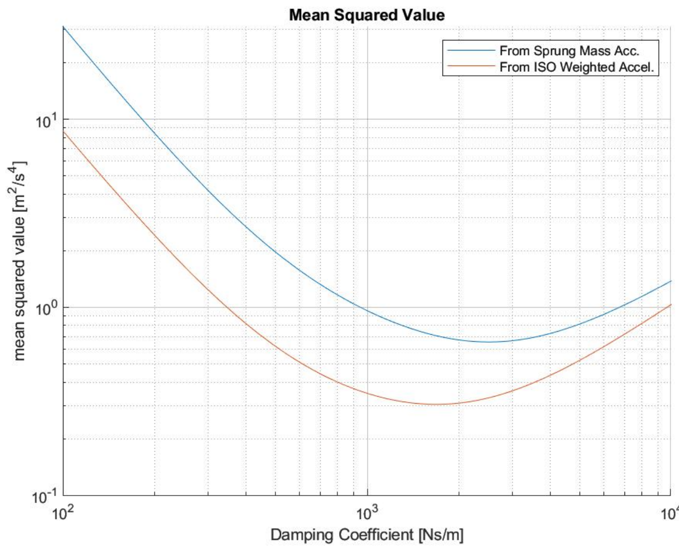


Figure 14: MSV - Road Grade B, 70 [km/h] - BMW X5

Vehicle	$C_{s,opt}$ [Ns/m]	$C_{occ,opt}$ [Ns/m]
BMW X5	3700	1690
Fiat 500	3300	840

Table 3: Damping Coefficient Optimal Values

Results Analysis

Starting from the bode diagrams, the results show two peaks representing the modes of resonance for the sprung mass and the unsprung mass in increasing order, respectively. Moreover, we can also see that due to the filter used for human sensitivity to mechanical vibration, the magnitude of these two peaks as perceived by the occupants is lower.

Shifting our analysis to the PSD, the first thing to notice is the PSD of the road profile which increases as the road grade degrades from “A” to “B”. It also shows that the lower frequencies corresponding to the largest wavelengths contribute more to the road irregularity. Moreover, we can clearly see that the magnitude of the accelerations increases with the road degradation. This result can be anticipated since we all experience worse vibrations as the road quality degrades. In fact, that’s why decreasing the velocity as the road degrades is a good idea to decrease the magnitude of the PSD. However, we can see that to maintain the same magnitude of the PSD by going from road grade “A” at “120 [km/h]” to road grade “B”, the velocity must decrease by more than “50 [km/h]”!

Finally, we can analyze the results obtained from the damping factor optimization analysis. The anticipated results were that the MSV of the acceleration perceived by the occupants should be lower than that of the acceleration of the sprung and this is clearly evident. However, the minimum values of these two curves occur at different values of the damping coefficient. Specifically, the corresponding damping coefficient value for the minimum MSV of the acceleration perceived by the occupants is lower than that of the acceleration of the sprung mass. Moreover, although the MSV changes with the road profile and vehicle speed, the occurrence of the minimum value doesn’t change, indicating that it depends solely on the vehicle’s characteristics, opening an opportunity for developing an empirical formula that can calculate it directly.

Conclusion

In this project, NVH analysis on the quarter-car model has been performed. First, the state space representation of the model was derived allowing us to obtain the frequency response of the acceleration of the sprung mass and the filtered acceleration according to human sensitivity to mechanical vibration. Second, the power spectral density of the road profile based on an empirical formula was presented from which the power spectral densities of the sprung mass acceleration and the filtered acceleration have been obtained. Finally, an optimization analysis on the damping coefficient value was performed based on the mean squared value of the filtered acceleration. Results have shown that the frequency response has two peaks representing the resonating modes of the sprung mass and the unsprung mass, and that the magnitude of the filtered signal is lower than that of the sprung mass. The power spectral densities have shown that the lowest frequencies have the highest impact on the road irregularity and that they depend on the road grade and the vehicle speed. Finally, the optimization analysis has shown that the optimal damping coefficient value from the occupant's perspective is lower than that of the sprung mass and that this value depends solely on the vehicle's characteristics.

References

1. Morello, Lorenzo, Lorenzo Rosti Rossini, Giuseppe Pia, and Andrea Tonoli. n.d. *The Automotive Body Volume II: System Design.*