MSc. Automotive Engineering

Car Body Design and Aerodynamics

Exercise 2
Evaluation of Torsional Stiffness of a Frame

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Abstract

In this work, the evaluation of the torsional stiffness of a frame composed of four thin-walled beams is presented. The longitudinal beams have a circular cross-section while the cross beams have a rectangular cross-section. The evaluation is performed analytically and numerically on a set of combinations between open and closed cross-section beams using two different load configurations and two types of meshing elements. Results have shown that a configuration with all closed cross-section beams has the highest torsional stiffness while the one with all open cross-section beams has a drastically lower value. The other two combinations have a torsional stiffness that lies somewhere between these two extremes with the one composed of longitudinal beams with closed cross-section having a higher value due to its circular cross-section.

Contents

Motivation

The main purpose of this exercise is to understand the effect of open/closed cross-sections on the evaluation of the torsional stiffness. The idea is to analyze the potential behind the use of open cross-section beams for material saving. Another purpose is to understand the limits behind the assumptions in the analytical approach and test its validity. This is simply done by comparing the results with those of the finite element analysis. Finally, the last purpose is to understand the effect of different load configurations and different meshing elements on the evaluation of the torsional stiffness.

Objectives

The main objectives to be accomplished during this exercise are the following:

1. Calculate, analytically, the torsional stiffness of the frame considering the different configurations.
2. Design the different configurations on a CAD software and use FEM to compute the corresponding torsional stiffness values.
3. Evaluate the effect of open/closed cross-sections, different load configurations, and different meshing elements on the result.

Methodology

1. Setup

1.1 Geometry

The analyzed frame is a ladder frame with 2 longitudinal beams and 2 cross beams. For closed cross-sections, the geometry is represented in ***Figure 1*** and ***Figure 2***:

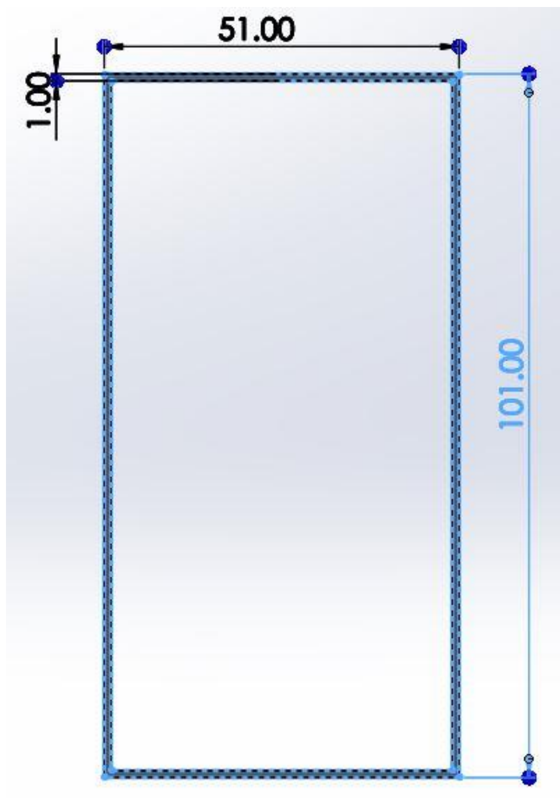

Figure 1: Cross Beam Closed Cross-Section

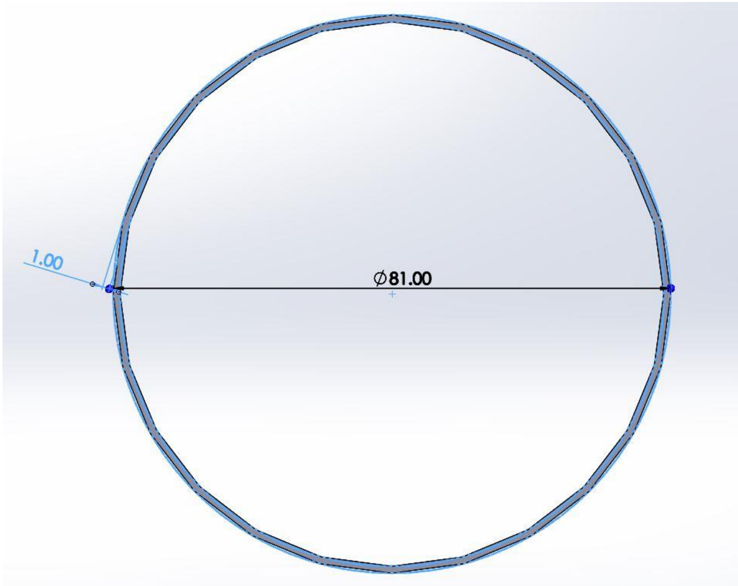

Figure 2: Longitudinal Beam Closed Cross-Section

For open cross-section beams, the geometry is represented in ***Figure 3*** and ***Figure 4***:

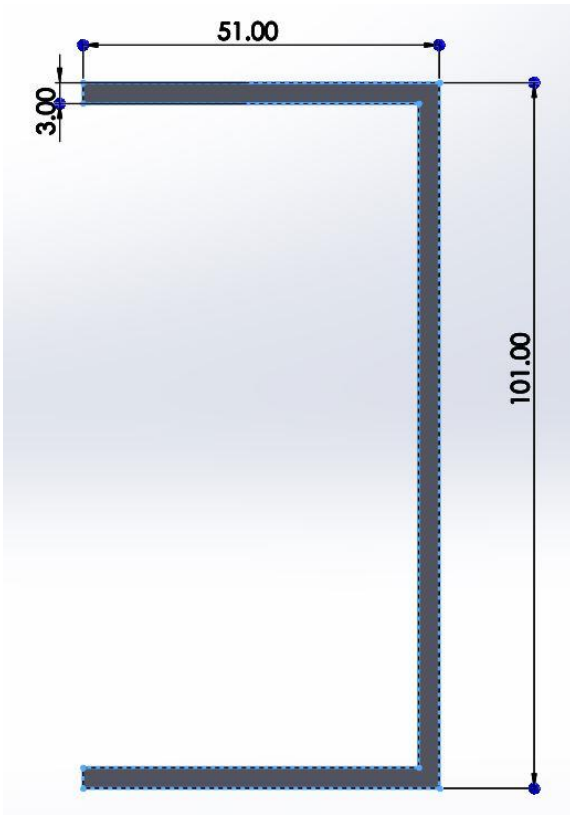

Figure 3: Cross Beam Open Cross-Section

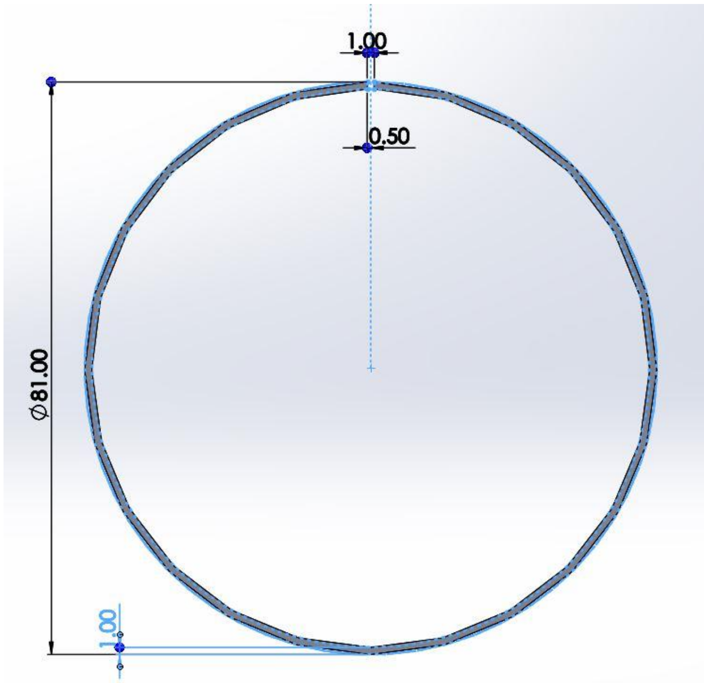

Figure 4: Longitudinal Beam Open Cross-Section

1.2 Load Configurations

The load configurations that have been analyzed are shown in ***Figure 5*** and ***Figure 6***:

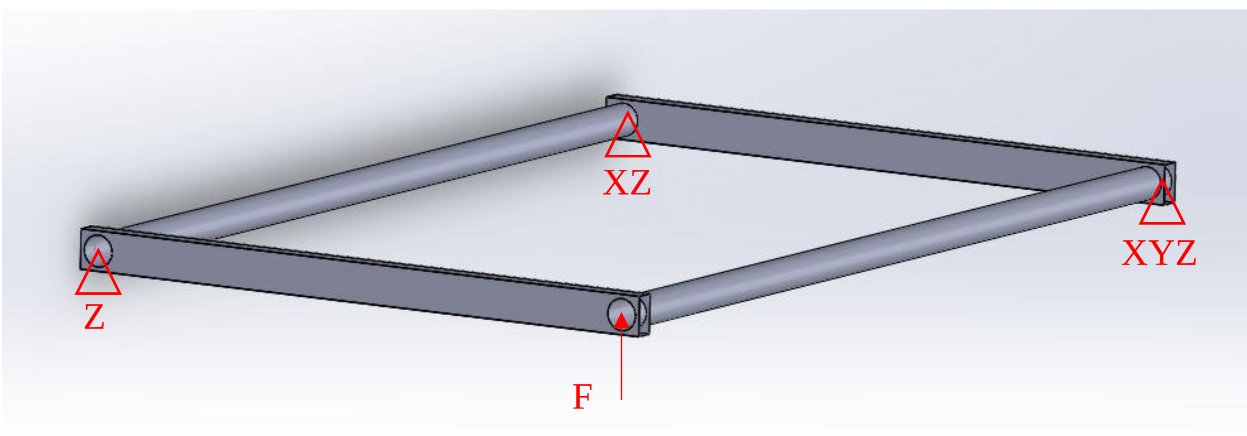

Figure 5: 1 Force Load Configuration

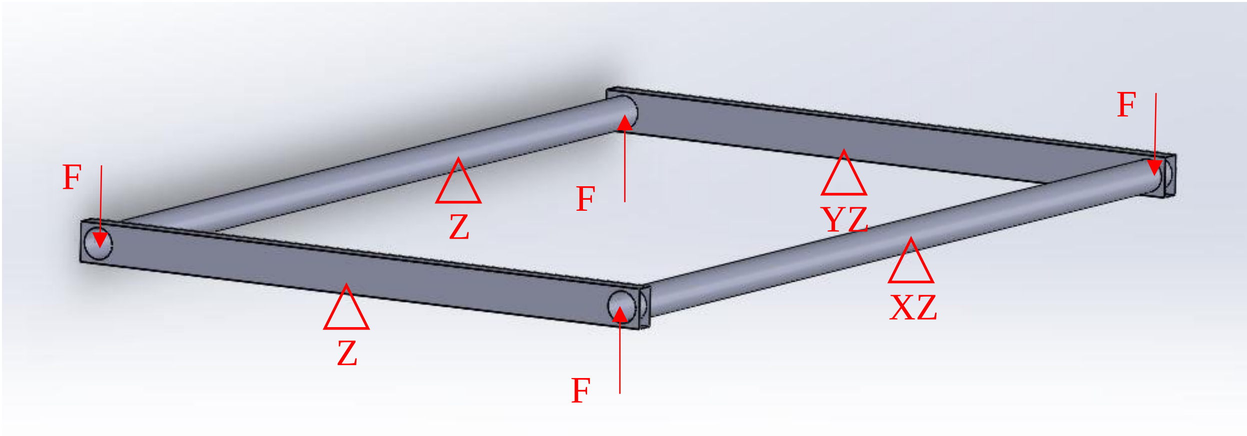

Figure 6: 4 Forces Load Configuration

The material used is steel with a modulus of elasticity equal to $2.1*10^{11}$ [Pa] and a modulus of rigidity equal to $81*10^{9}$ [Pa].

2. Analytical Method

2.1 Brief Description

The analytical method adopted in this work is the one proposed by the authors in reference[1]. A generic ladder frame is shown in *Figure 7*:

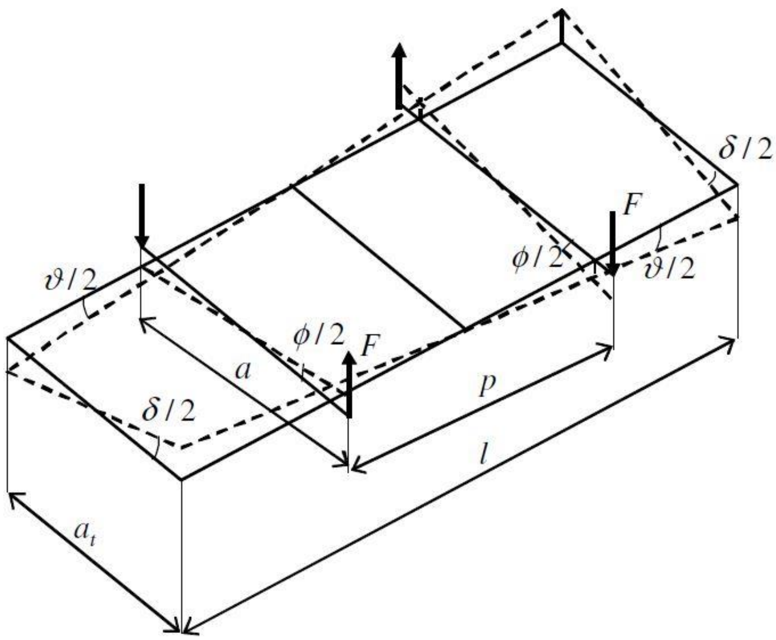

Figure 7: Ladder Frame

The model is based on a set of assumptions one of which is that the longitudinal and cross members have a straight axis and constant section. This is very important because it assumes that the energy stored due to deformation will be purely torsional and not due to bending. Hence, we can say that:

$$\frac{1}{2}K_{\phi}\phi^2 = n_t\frac{1}{2}K_t\vartheta^2 + 2\frac{1}{2}K_l\delta^2 \;\ldots (1)$$

This equation simply describes what has been stated earlier, that is, the elastic potential energy given by the sum of the contributions of the longitudinal and cross beams due to torsion is equal to the elastic energy of the whole frame. Moreover, we can say that:

$$\phi = \frac{p}{a_t}\theta \quad \text{and} \quad \delta = \frac{l}{a_t}\theta$$

Therefore, equation (1) becomes:

$$K_{\phi} = \left(\frac{a_t}{p}\right)^2\left[n_tK_t + 2K_l\left(\frac{l}{a_t}\right)^2\right] \;\ldots (2)$$

Another takeaway from the assumption is that the torsional stiffness of the longitudinal and cross beams can be expressed as:

$$K_t = \frac{GJ_{tt}}{a_t} \quad ; \quad K_l = \frac{GJ_{tl}}{l}$$

Where "J_{tt}" and "J_{tl}" are the torsional stiffness moduli of the sections of the cross and longitudinal beams, respectively. Finally, equation (2) becomes:

$$K_\phi = \frac{Ga_t}{p^2}\left(n_t J_{tt} + 2J_{tl}\frac{l}{a_t}\right) \ldots (3)$$

2.2 Application

The application of the analytical method requires some modification according to the specific configuration adopted and the definition of the parameters involved. In this work, there are only 2 cross beams and 2 longitudinal beams. Moreover, the dimensions are:

$$a = 1500\ [mm] \quad ; \quad a_t = 1600\ [mm] \quad ; \quad p = l = 2500\ [mm]$$

and the longitudinal beams are separated by the distance "a" instead of "a_t"; hence:

$$\delta = \phi \qquad \text{and} \qquad \vartheta = \phi\frac{a}{l}$$

As a result, the equations (3) reaches its final form as:

$$K_t = \frac{2G}{l}\left(J_{tt}\frac{a^2}{a_t l} + J_{tl}\right) \ldots (4)$$

Obviously, the calculation of the torsional stiffness modulus of each section is required. This calculation segregates between open cross-sections and closed cross-sections.

2.2.1 Closed Cross-Section

The torsional stiffness moduli corresponding to thin-walled closed cross-sections is derived from the Bredt equations. In short, the result is the following:

$$J_t = 4\Omega^2 \oint \frac{t}{dl} \ldots (5)$$

Where "Ω" is the are enclosed by the mid-thickness line, "t" is the wall thickness, and "l" is the perimeter of cross-section.

By applying equation (5) to the geometries in ***Figure 1*** and ***Figure 2***, we get:

$$J_{tt,c} = 4 * (50 * 100)^2 * \frac{1}{2 * (100 + 50)} = \frac{10^6}{3}\ [mm^4] = \frac{1}{3 * 10^6}\ [m^4]$$

$$J_{tl,c} = 2\pi * (40)^3 * 1 = 128{,}000\pi\ [mm^4] = 4 * 10^{-7}\ [m^4]$$

2.2.2 Open Cross-Section

For the open cross-sections, their corresponding torsional stiffness modulus is calculated by imagining the cross-section unwrapped into a straight beam loaded in bending. Therefore, using Beam Theory, we can say that:

$$\vartheta' = 3\frac{M_t}{Gab^3} = \frac{M_t}{GJ_t} \qquad => \qquad J_t = \frac{ab^3}{3} \ldots (6)$$

Where "*a*" is the total length of the mid-thickness line and "*b*" is the thickness of the wall. Hence, applying equation (6) on ***Figure 3*** and ***Figure 4***:

$$J_{tt,o} = \frac{(2 * 50 + 100)3^3}{3} = 1800\ [mm^4] = 1.8 * 10^{-9}\ [m^4]$$

$$J_{tl,o} = \frac{(2\pi * 40 - 1)1^3}{3} \cong \frac{80}{3}\pi\ [mm^4] = 8.34 * 10^{-11}\ [m^4]$$

Finally, we can use these values to calculate the torsional stiffness of the different combinations that can be obtained for the frame using equation (4). The results are summarized in ***Table 1***:

	Open Cross	***Closed Cross***
Open Longitudinal	**70.8**	**12120**
Closed Longitudinal	**26048**	**38098**

Table 1: Frame Torsional Stiffness for Different Combinations [Nm/rad]

3. Fine Element Analysis

3.1 Application

Over the past three decades, the availability of high computational power at reasonable cost on one side and of finite elements code and their integration with CAD software on the other has made in depth structural analysis on almost any component or assembly possible.

As for any kind of structure, the finite element model of a frame may be realized considering each of its components as a three dimensional solid; therefore, it's meshed with prismatic or tetrahedral solid elements. Nevertheless, the geometric complexity and the small thickness that characterizes the components in this case require the use of an extremely large number of elements and nodes. Thus, the complexity can be dramatically reduced by adopting plate elements instead of solid ones.

In this work, Altair Hyperworks® has been used to model the structure under study with R-Trias elements of size 5 [mm]. Moreover, a brief analysis to evaluate the effect of linear (1st order) and nonlinear (2nd order) elements on the computation of the frame torsional stiffness has been done. A representation of the meshed structure is shown in ***Figure 8***:

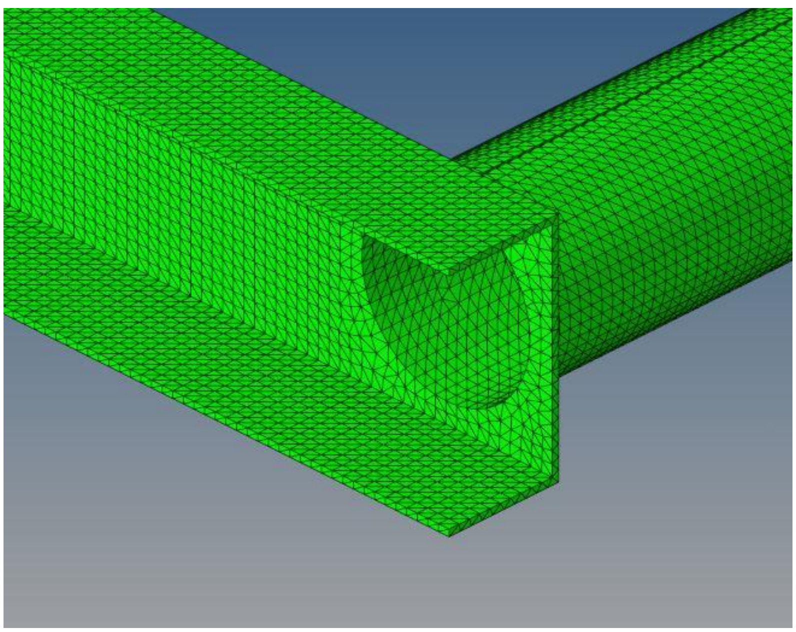

Figure 8: Frame Mesh

The connection between the longitudinal and cross beams has been analyzed by designing the whole frame as a single component on the CAD software. In this case, the FE code analyzes the whole frame as one component and generates the mesh in such a way that it is continuous between different subcomponents. This can be evidenced in ***Figure 9*****:**

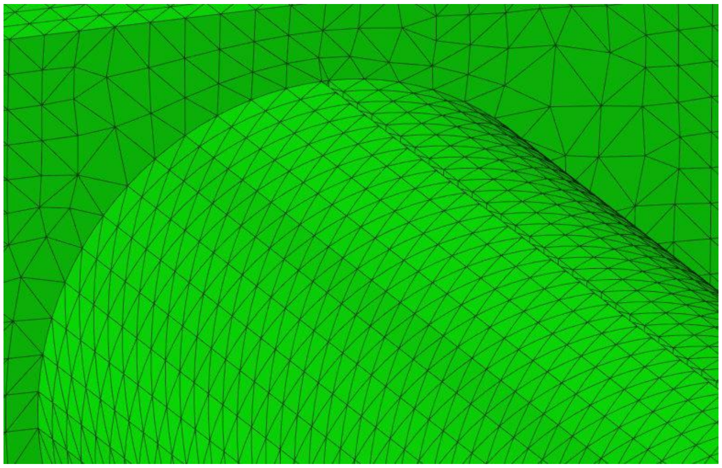

Figure 9: Mesh Continuity

3.2 Boundary Conditions

The boundary conditions are defined through the two different load configurations shown in ***Figure 5*** and ***Figure 6***. To ensure that these configurations are applied properly, "*RBE3*" one dimensional elements have been used to connect the nodes where the boundary conditions are applied to the structure. This connection is shown in ***Figure 10***:

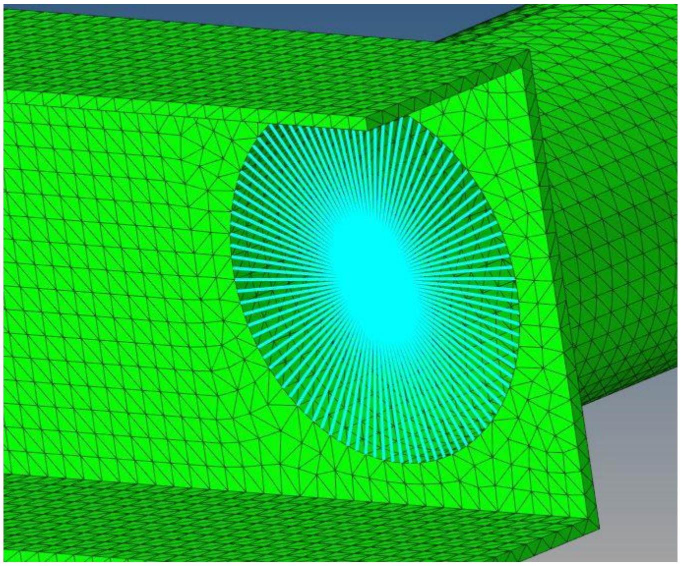

Figure 10: RBE3 Connection Elements

The boundary conditions could then be applied at the center of such connections as shown in [*Figure 11*](#) and [*Figure 12*](#):

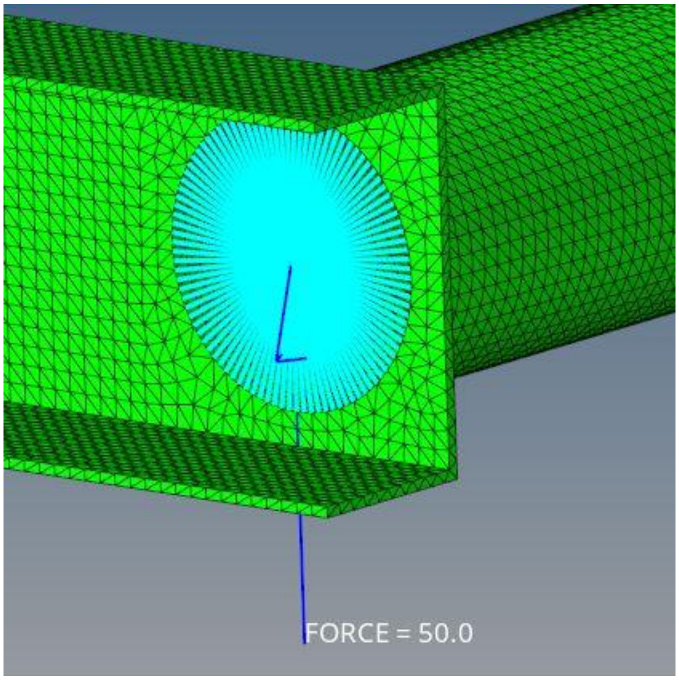

Figure 11: Boundary Condition – Force

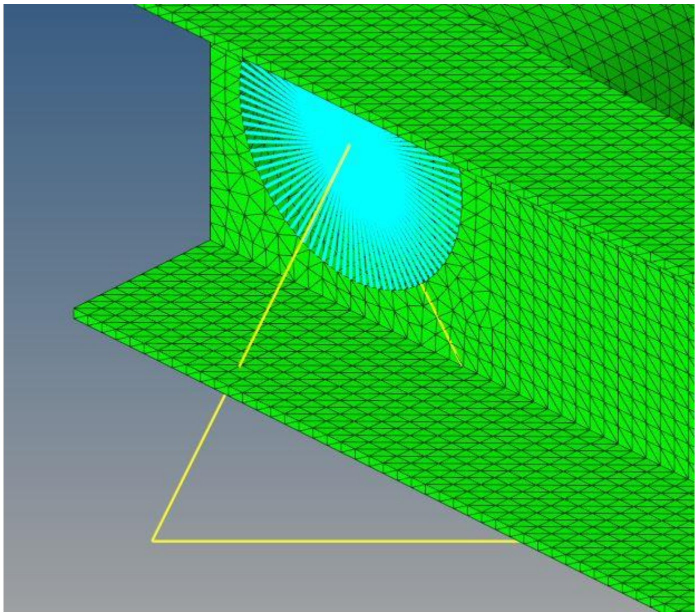

Figure 12: Boundary Condition - Constraint

3.3 Results

The evaluation of the torsional stiffness of the frame for all different configurations is based on the deformation obtained from FEA. The deformation is given in terms of displacement of the nodes where the force is applied. From that, we can calculate the angle of twist along the longitudinal beam and by knowing the load applied, we can calculate the torsional stiffness by:

$$K_t = \frac{F * a}{\phi} \ldots (7)$$

However, the computation of "*Φ*" differs between the two different load configurations. Denoting by "*f*" the displacement of the node(s) where the load(s) is(are) applied:

- 1 Force Configuration: $\phi = \frac{f}{a}$
- 4 Forces Configuration: $\phi = \frac{2*f}{a}$

A result for each configuration is shown in ***Figure 13*** and ***Figure 14***:

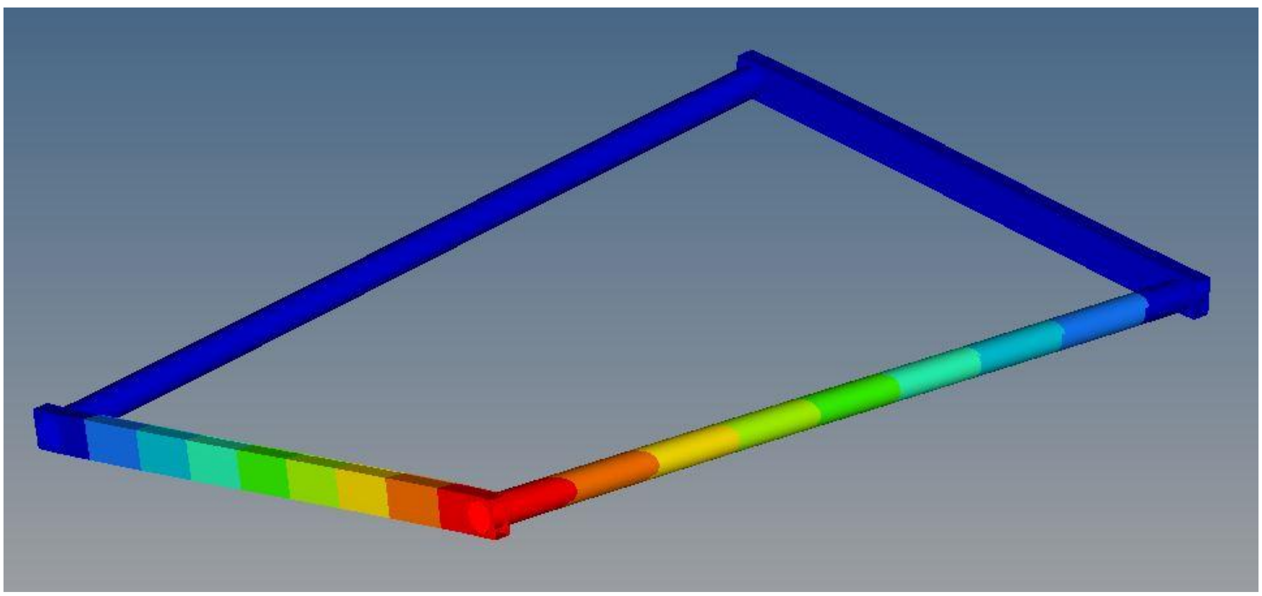

Figure 13: Results - 1 Force Configuration

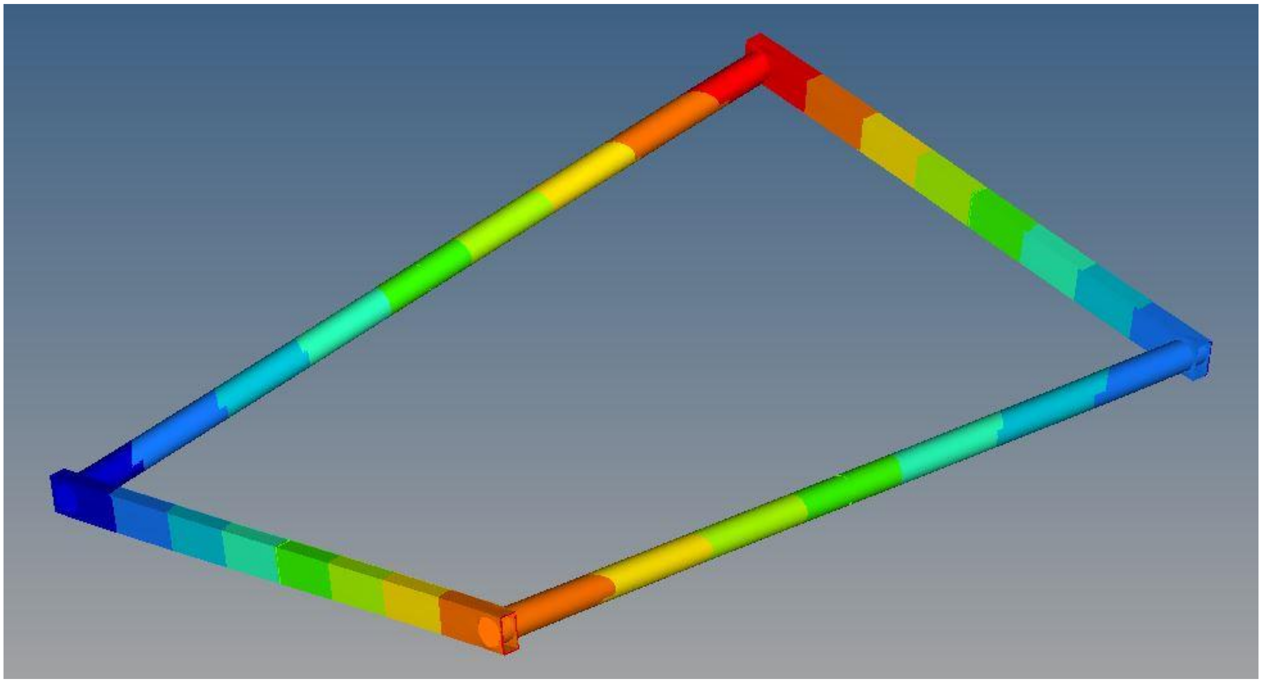

Figure 14: Results - 4 Forces Configuration

The total number of combinations simulated is 16 since we have 4 combinations between open and closed cross-sections, 2 combinations concerning load configurations, and 2 combinations concerning the element type. The results are shown in ***Table 2***, ***Table 3***, ***Table 4***, and ***Table 5***.

CoLo	*1F 1st Order*	*1F 2nd Order*	*4F 1st Order*	*4F 2nd Order*
Force [N]	50	50	100	100
Displacement [mm]	210.3	221.7	118	293
Angle [rad]	0.1402	0.1478	0.157	0.4
K_t [Nm/rad]	535	507	953	384

Table 2: Results - Open Cross-Beam Open Longitudinal Beam

CcLo	*1F 1st Order*	*1F 2nd Order*	*4F 1st Order*	*4F 2nd Order*
Force [N]	100	100	100	100
Displacement [mm]	27.94	27.25	7.221	7
Angle [rad]	0.018	0.018	0.0096	0.0093
K_t [Nm/rad]	8053	8256	15580	16071

Table 3: Results - Closed Cross-Beam Open Longitudinal Beam

CoLc	*1F* *1st Order*	*1F* *2nd Order*	*4F* *1st Order*	*4F* *2nd Order*
Force [N]	100	100	100	100
Displacement [mm]	4.69	4.74	1.2	1.226
Angle [rad]	0.0031	0.0031	0.0016	0.0016
K_t [Nm/rad]	47974	47468	93750	91761

Table 4: Results - Open Cross-Beam Closed Longitudinal Beam

CcLc	*1F* *1st Order*	*1F* *2nd Order*	*4F* *1st Order*	*4F* *2nd Order*
Force [N]	100	100	100	100
Displacement [mm]	1.906	3.821	0.975	1.344
Angle [rad]	0.00127	0.00254	0.0013	0.0018
K_t [Nm/rad]	118048	58885	115384	83705

Table 5: Results - Open Cross-Beam Open Longitudinal Beam

Results Analysis

Whether it is the analytical method or FEA, the results in both suggest that the configuration with all closed sections has the highest torsional stiffness, after which we have the configuration with open cross-beams and closed longitudinal beams, then the closed cross-beams and open longitudinal beams, and eventually all open cross-sections. Therefore, we can say that there's a huge impact on the torsional stiffness of the frame considering an open/closed cross-section. It's quite evident that the structure composed of longitudinal beams with closed cross-section has a higher torsional stiffness than the one composed of cross beams with closed cross-section. This is simply because the longitudinal beams have a circular cross-section which, by means of simple analytical derivations, has a higher modulus of torsional stiffness with respect to a rectangular cross-section with the same area.

The discrepancy between the results obtained from the analytical method and the FEM is due to the assumption imposed by the analytical method, that is, the beams' axes remain straight. This signifies that the deformation energy is all stored as torsional deformation; however, in reality and FEM, some of the energy is stored as bending deformation. Thus, the FEM must result in a stiffer frame which is true for 14 cases out of the 16. Since the bending deformation is much more significant in closed cross-sections than open cross-sections, we realize that the analytical results and the FEA results are closest in all open cross-sections and furthest in all closed cross-sections.

Another issue that is observed is the difference between the values obtained in the two different load configurations using FEM. The pattern shows that when 4 forces are applied, the frame is stiffer. This could be due to the constraining boundary condition chosen for the 4 forces configuration to ensure that the software can simulate the behavior when all 6 degrees of freedom are blocked. However, the pattern doesn't fit 2 conditions for unknown reasons.

Finally, we can comment on the difference between the results of the FEM based on the element type. We can predict that when bending is more significant, the behavior is more nonlinear; thus, the difference between the results will be higher. Therefore, we can clearly see that the highest difference is shown in the all closed cross-section configuration where bending is more significant. An outlier is the situation with all open cross-sections and 4 forces configuration for which the reasons remain unknown.

Conclusion

In this exercise, the computation of the torsional stiffness of a frame has been performed analytically and numerically to mainly analyze the effect of open cross-sections and closed cross-sections on the final value. Moreover, the effect of two different load configurations and two different element types has been analyzed. The results obtained show that closed cross-sections make the frame much stiffer and that the choice of the cross-section of the longer beams impacts the value of the torsional stiffness higher than the choice of the cross-section of the shorter beams.

References

1. Morello, Lorenzo, Lorenzo Rosti Rossini, Giuseppe Pia, and Andrea Tonoli. n.d. *The Automotive Body Volume II: System Design.*