

Mixture tree model for network inference

JOBIM

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- 1 Motivation
- 2 Network inference
- 3 Mixture tree

Context

Rising interest in **jointly analysed** species abundances:

- Metagenomics
- Microbiologie
- Ecology

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Ecological network

Tool to better understand species interactions (direct/indirect, nature),
eco-systems organizations (clusters ?)

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Ecological network

Tool to better understand species interactions (direct/indirect, nature), eco-systems organizations (clusters ?)

Allows for resilience analyses, pathogens control, ecosystem comparison, reaction prediction...

Example

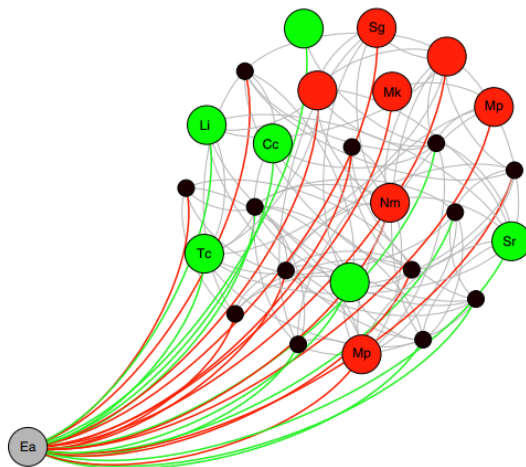


Figure: *Erysiphe althitoides* pathobiome on oak tree leaves.

Data

- **Species** : animal, bacteria, gene ...
- **Abundances** : ecological counts, Next-Generation Sequencing technologies...
- **Covariates** : coverage, temperature, water depth ...

Repeated signal : n samples of p abundances.

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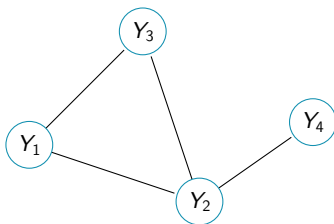
Notations

$$Y = [Y_{ij}]_{(i,j) \in \{1, \dots, n\} \times \{1, \dots, p\}}$$

- Y_{ij} : abundance of the j^{th} species in the i^{th} sample

Infer the species interaction network from Y

Graphical models



- All variables are dependant
- Some are **conditionnally independant** (i.e. indirectly dependant)

$$Y_4 \perp\!\!\!\perp (Y_1, Y_3) | Y_2$$

Graphical models

Factorization propriety

The joint distribution P is faithful to the graph G iff

$$P(Y_1, \dots, Y_p) \propto \prod_{C \in \mathcal{C}_G} \psi_C(Y_C)$$

where \mathcal{C}_G = set of cliques of G .

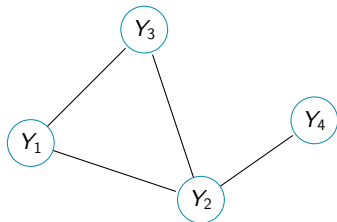
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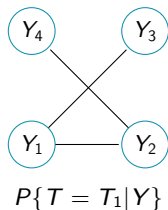
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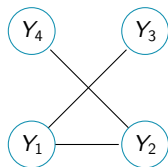


$$P(Y_1, Y_2, Y_3, Y_4) \propto \psi_1(Y_1, Y_2, Y_3) \times \psi_2(Y_2, Y_4)$$

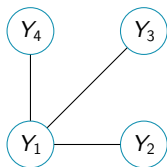
Tree averaging



Tree averaging

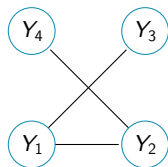


$$P\{T = T_1|Y\}$$

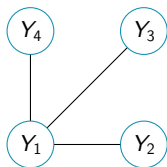


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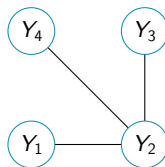
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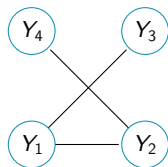


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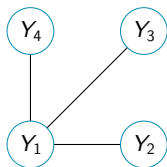


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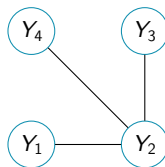
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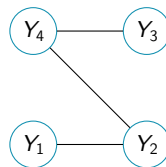
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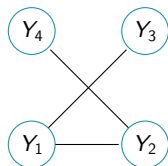
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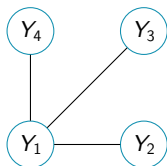
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...

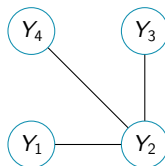
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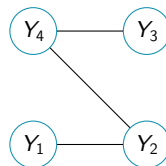
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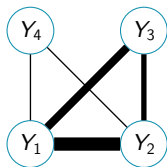
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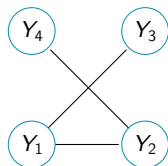
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Compute edge probabilities:

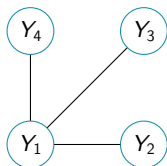


$$P\{(j, k) \in T|Y\}$$

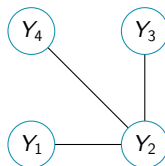
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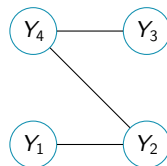
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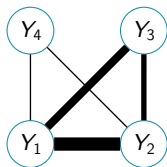
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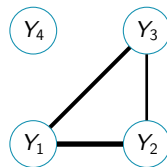
...

Compute edge
probabilities:



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Thresholding
probabilities:



$$P\{(j, k) \in T|Y\}$$