Theory Assignment 1: CS2233

August 12, 2024

Kindly adhere to the following instructions.

- Please neatly write your solutions on the answer sheet. Please clearly mention your name and roll number in the top right corner of the paper. The solutions should be self-explanatory with required mathematical arguments/proofs.
- Please submit the scanned copy of the solution to the google classroom portal and hardcopy to the classroom.
- Any form of plagiarism (web/chatGPT/with peers) will be severely penalised and will result in F grade.
- The submission (strict) timeline is 22nd August, Thursday, 11 AM (after the class).

1. For each of these questions, briefly explain your answer. 3 Marks
   - If I prove that an algorithm takes $O(n^2)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?
   - If I prove that an algorithm takes $O(n^2)$ worst-case time, is it possible that it takes $O(n)$ on all inputs?
   - If I prove that an algorithm takes $\Theta(n^2)$ worst-case time, is it possible that it takes $O(n)$ on some inputs?

2. Let $f(n)$ and $g(n)$ are non-negative functions over non-negative input. Prove or disprove the following and justify your claims via formal mathematical arguments.
   8 Marks
   - $f(n) + o\left(f(n)\right) = \Theta(f(n))$.
   - $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
   - $f(n) + g(n) = \Theta\left(\min\left\{f(n), g(n)\right\}\right)$
   - $n! = o(n^n)$ and $n! = \omega(2^n)$

3. Is the function $\lceil \ln n \rceil!$ polynomially bounded? Is the function $\lceil \ln \ln n \rceil!$ polynomially bounded? 3 Marks

4. Solve the following recursions. Mention any assumptions you might be making. 5 Marks

   - $T(n^2) = 7T(n^2/4) + cn^2$ and $T(1) = 1$
   - $T(n) = n * T(\sqrt{n})$ and $T(2) = 4$ (Assume n to be of the form 2^{2^i})
   - $T(n) = T(n/2) + 2T(n/4) + 3n/2$ (whenever $n > 3$) and $T(1) = 0, T(2) = 2$
   - $T(n) = 4T(n/2) + n^3$ and $T(1) = 1$
   - $T(n) = T(n/2) + c \log n$

5. Arrange the following function by order of growth. That is, find an ordering among the following functions $f_1, f_2, f_3, \ldots, f_6$ such that $f_1 = \Omega(f_2), f_2 = \Omega(f_3), \ldots f_5 = \Omega(f_6)$. Partition the list into equivalence class such that functions $f(n)$ and $g(n)$ are in the same class (that is, they are of the same order), if and only if $f(n) = \Theta(g(n))$. Justify your claims.

   3 Marks

   - $\frac{n^{1.2}}{\log n}$
   - n^2
   - $n \log n$
   - 1.1^n
   - 0.9^n
   - $\log^3 n$

6. What value is returned by the following function? Express your answer as a function of n. Give the worst-case running time using big-Oh notation. 3 Marks

```
function XYZ($n$)
$r := 0$
    for $i := 1$ to $n$ do
       for $j := 1$ to $i$ do
          for $k := j$ to $i + j$ do
             for $l := 1$ to $i + j - k$ do
                $r := r + 1$;
return($r$)
```