

Coding + Theory Assignment 4: CS2233

3th November, 2025

Max Marks 130

Instructions: Kindly adhere to the following instructions.

- Please write a C/C++ program corresponding to each problem. Your code should be well commented on, and variable names should be appropriately chosen.
 - Create a folder and put all the code files and name all the files as the question number (i.e. 1.cpp/1.c), add the scanned copy of the theory assignment solution to it, give a name to the folder as “yourRollNo”, zip the folder and submit it to the Google Classroom portal. Also, submit the hard copy of the theory assignment to the class.
 - Strictly follow the input and output format for each problem.
 - Any code that does not follow the input-output criteria won't be evaluated and will get **ZERO**.
 - Your code will also be checked against plagiarism (both from web and peer).
 - Any form of plagiarism (web/chatGPT/with peers) will be severely penalised and will result in an F grade.
 - For theory questions, the answer should consist of the following three parts: (i) algorithm/pseudocode of the problem, (ii) a theorem statement stating that the algorithm mentioned outputs the desired result, and a comment about the time/space complexity of the algorithm, (iii) a formal proof of the theorem.
 - The submission (strict) timeline is 17th November, Monday, 11:30 PM.
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1 Coding Problems:

In the following problems, suppose the graph is given as input in the adjacency list format.

1. Graph Traversal:

- (a) Suppose $G = (V, E)$ is an undirected graph, and a vertex $s \in V$ is given as input. Implement the **BFS traversal algorithm**. For every vertex $v \in V$, you need to output both the **length of the shortest path** from s , and the **list of vertices** in the shortest path.

[10 Marks]

Input Format:

- The first line contains an integer n denoting the number of vertices.
- The second line contains an integer m denoting the number of edges.
- The next m lines contain two integers u and v representing an undirected edge between u and v .
- The last line contains an integer s denoting the source vertex.

Output Format:

- For each vertex $v \in V$, print:
 - The length of the path from s to v .
 - The list of vertices in the path from s to v . (You can print any path among shortest paths.)

Sample Input:

```
6
7
1 2
1 3
2 4
2 5
3 6
4 5
5 6
1
```

Sample Output:

```
Vertex 1: Length = 0, Path = 1
Vertex 2: Length = 1, Path = 1 2
Vertex 3: Length = 1, Path = 1 3
Vertex 4: Length = 2, Path = 1 2 4
Vertex 5: Length = 2, Path = 1 2 5
Vertex 6: Length = 2, Path = 1 3 6
```

2. Minimum Spanning Tree (MST):

- (a) Suppose $G = (V, E)$ is an undirected weighted graph given as input. Compute the MST of this graph using **Prim's algorithm**. You need to output the list of edges that comprise the MST. [10 Marks]
- (b) Suppose $G = (V, E)$ an undirected weighted graph is given as input. Compute the MST of this graph using **Kruskal's algorithm**. You need to output the list of edges that comprises MST. [10 Marks]

Input Format:

- The first line contains an integer n denoting the number of vertices.
- The second line contains an integer m denoting the number of edges.
- The next m lines contain three integers u, v, w representing an undirected edge between u and v with weight w .

Output Format:

- Output the list of edges included in the MST in any order.
- Also print the total weight of the MST.

Sample Input:

```
5
7
1 2 2
1 3 3
2 3 1
2 4 4
3 5 5
4 5 7
3 4 6
```

Sample Output:

```
Edges in MST:
(2, 3) weight = 1
(1, 2) weight = 2
(2, 4) weight = 4
(3, 5) weight = 5
Total Weight = 12
```

3. Single Source Shortest Path:

- (a) Suppose $G = (V, E)$ is a directed weighted graph with non-negative edge weights, and a vertex $s \in V$ is given as input. Implement **Dijkstra's algorithm** for computing the shortest path from s to every vertex $v \in V \setminus \{s\}$. For each v , output both the **length of the shortest path** from s to v , and the **list of vertices** in the path.

[10 Marks]

Input Format:

- The first line contains an integer n denoting the number of vertices.
- The second line contains an integer m denoting the number of edges.
- The next m lines contain three integers u, v, w representing a directed edge from u to v with weight w .
- The last line contains an integer s denoting the source vertex.

Output Format:

- For each vertex $v \in V$, print:
 - The shortest distance from s to v .
 - The path from s to v .

Sample Input:

```
6
9
1 2 4
1 3 2
2 3 5
2 4 10
3 5 3
5 4 4
4 6 11
5 6 5
3 6 8
1
```

Sample Output:

```
Vertex 1: Length = 0, Path = 1
Vertex 2: Length = 4, Path = 1 2
Vertex 3: Length = 2, Path = 1 3
Vertex 4: Length = 9, Path = 1 3 5 4
Vertex 5: Length = 5, Path = 1 3 5
Vertex 6: Length = 10, Path = 1 3 5 6
```

2 Theory Questions:

1. Suppose that a weighted, directed graph $G = (V, E)$ has a negative-weight cycle. Give an efficient algorithm to list the vertices of one such cycle. Prove that your algorithm is correct. [10 Marks]
2. Given a graph G and a minimum spanning tree T , suppose that we decrease the weight of one of the edges not in T . Give an algorithm for finding the minimum spanning tree in the modified graph. [10 Marks]

3. Show that a graph has a unique minimum spanning tree if, for every cut of the graph, there is a unique **light edge** crossing the cut. Show that the converse is not true by giving a counterexample. [10 Marks]
4. Suppose that a graph G has a minimum spanning tree already computed. How quickly can we update the minimum spanning tree if we add a new vertex and incident edges to G ? [10 Marks]
5. Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers. Please elaborate on what points the proof of Dijkstra's algorithm breaks when negative-weight edges are allowed? [10 Marks]
6. Let $G = (V, E)$ be a weighted, directed graph with nonnegative weight function $w : E \mapsto \{0, 1, 2, \dots, W\}$ for some nonnegative integer W . Modify Dijkstra's algorithm to compute the shortest paths from a given source vertex s in $O(VW + E)$ time. [10 Marks]
7. Consider an array A over n distinct elements. Prove that the best case running time for running HEAPSORT on A is $\Omega(n \ln n)$. [10 Marks]
8. Consider a directed graph $G = (V, E)$ with no self loops. Consider the **incident Matrix** of the graph G , a matrix of size $|V| \times |E|$ denoted as B , whose each entry b_{ij} is defined as follows:

$$b_{ij} = \begin{cases} -1 & \text{1 if edge } j \text{ leaves vertex } i \\ 1 & \text{1 if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Describe what the entries of the matrix product BB^T represent, where B^T is the transpose of B . [10 Marks]

9. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$ such that **and** edge $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v . Describe efficient algorithms for computing G^2 from G for both the adjacency-list and adjacency-matrix representations of G . Analyze the running times of your algorithms.

[10 Marks]