Department of Physics

Computational Physics

Exercise Sheet 08

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Assignment N° #08

Vorwort

Die Aufgaben sind verhältnismäßig kurz.

Die Zeitersparnis durch u.A. das Fehlen von Fragen wird gerne angenommen. Dennoch sind die Aufgaben tatsächlich sehr demonstrativ und lehrreich, gerade Aufgabe 2.

Ein wenig Text und Erklärung befindet sich in den Bildunterschriften. Ich hoffe, dass diese minimale Abgabe euren Erwartungen entspricht.

Da sich die Aufgaben gegenseitig bedingen, ist die gesamte Programmausführung in einer `.cpp`-Datei geschrieben. Das Plotten geschieht entsprechend in einer einheitlichen `.ipynb`-Datei. Die Kommunikation zwischen diesen Programmen basiert auf `.csv`-Dateien.

E1: Linearly congruent random number generators

Task:

Generate pseudo-random number by implementing a linearly congruent generator

$$r_{n+1} = (ar_n + c) \bmod m$$

yourself.

(1) Write a program which determines the first N entries $(N < m)$ of the integer sequence r_n as a function of the four parameters r_0 (seed), a, c, and m (use 64-bit integers). Divide r_{n+1} by m to get a floating point generator for random numbers in $[0, 1[$.

(2) For the four sets of parameters:

   (a) $r_0 = 1234$, $a = 20$, $c = 120$, $m = 6075$

   (b) $r_0 = 1234$, $a = 137$, $c = 187$, $m = 256$

   (c) $r_0 = 123456789$, $a = 65539$, $c = 0$, $m = 2^{31} = 2147483648$ (RANDU Generator von IBM)

   (d) $r_0 = 1234$, $a = 7^5 = 16807$, $c = 0$, $m = 2^{31} - 1$ (ran1() aus Numerical Recipes, 2. Ausgabe, bzw. Matlab bis Version 4)

   investigate your floating point generator for equal partition by generating a histogram (consisting of $N = 10^5$ total values). Split the interval $[0, 1[$ into 10 bins of length 0.1.

(3) Now, test the four floating point generators for correlations by respectively plotting $N/2$ pairs $\{(r_n, r_{n-1}), (r_{n-2}, r_{n-3}), \ldots\}$ of two immediately consecutively generated values in the square $[0, 1]^2$. Use $N = 10^5$ values, and note that only $N < m$ makes sense.

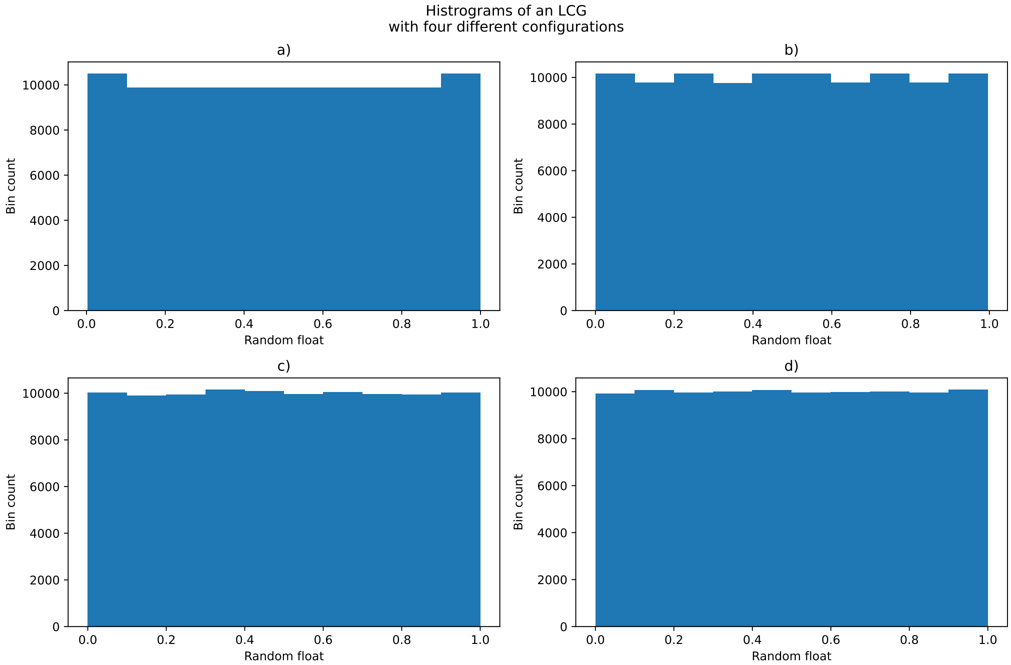

Figure 1.1: Histrograms of numbers generated by an LCG with four different configurations.

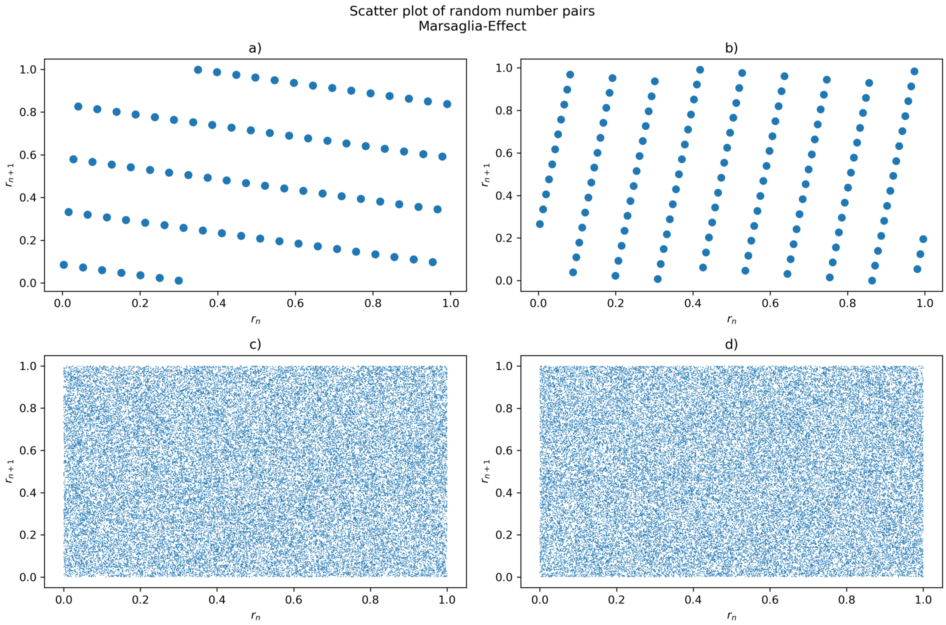

Figure 1.2: Scatter plot of random number pairs using the LCG. In the two plots above, a periodic and ordered pattern is emerging. The two distributions below show no apparent periodic or ordered behavior. The distributions looks similar to a random noise.

E2: Arbitrary distributions

Task:

A random number generator producing uniformly distributed numbers in the interval $[0,1[$ can also be used to generate numbers from an arbitrary distribution. Use of your linear congruential generator implemented in the previous exercise and parameter set 4 in the following tasks.

(a) Implement the Box-Muller algorithm to sample a Gaussian distribution.

(b) An alternative approach to generate a Gaussian distribution is based on the central limit theorem. Sum N uniformly distributed random numbers from the interval $[0,1[$. How can you get a distribution with mean 0 and standard deviation 1 from this sum? Derive a criterion on N from the condition on the standard deviation. Explain disadvantages of this method with respect to correctness and efficiency.

(c) Implement the von Neumann rejection sampling to generate the distribution

$$p_1 = \frac{\sin(x)}{2}$$

over the interval $[0,\pi[$.

(d) Apply the inversion method to draw random numbers from the distribution

$$p_2(x) = 3x^2$$

for $x \in [0,1[$.

Plot histograms of 10^5 random numbers for each subtask to verify the distributions and compare to the analytical distributions.

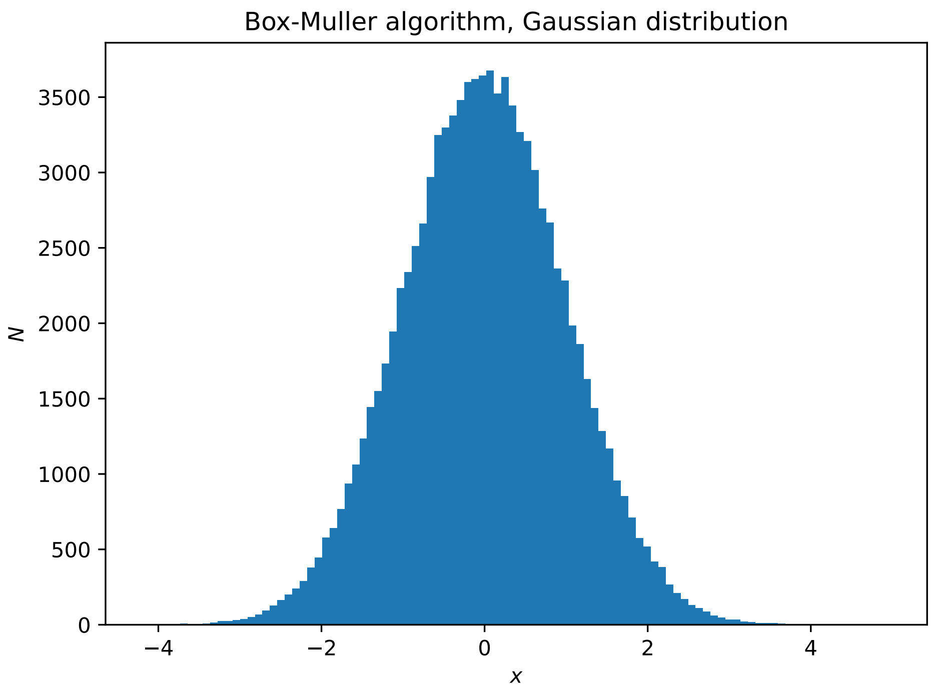

Figure 1.3: The histogram shows random numbers generated by the Box-Muller transformation. The generated numbers follow a Gaussian distribution, as shown by the good fit to the expected Gaussian shape.

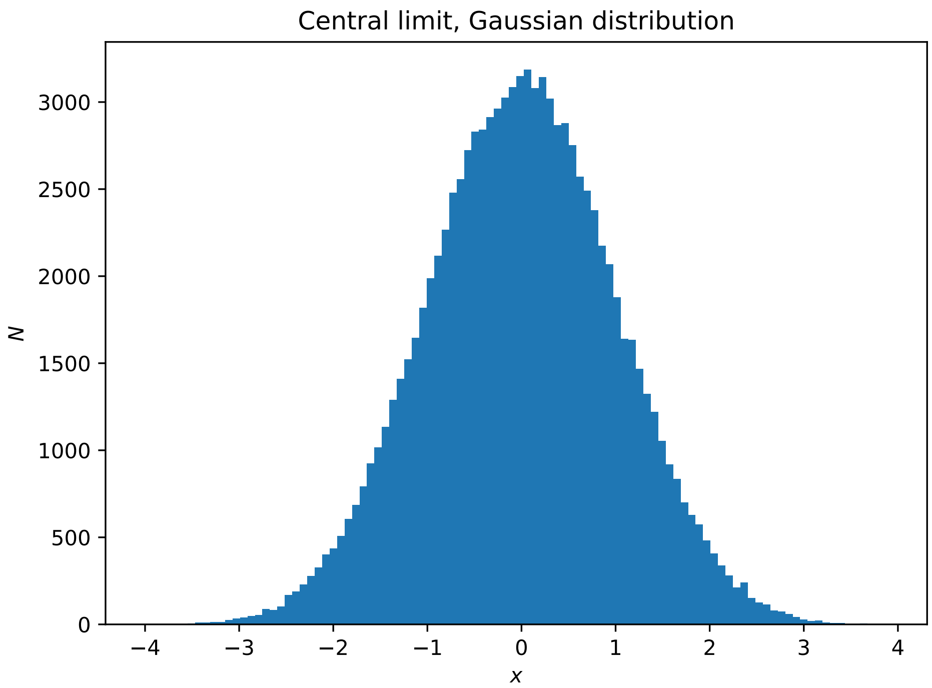

Figure 1.4: This histogram displays random numbers generated by summing multiple uniformly distributed random values. It demonstrates the central limit theorem, which states that the distribution of the sum of many independent and identically distributed variables tends towards a Gaussian distribution.

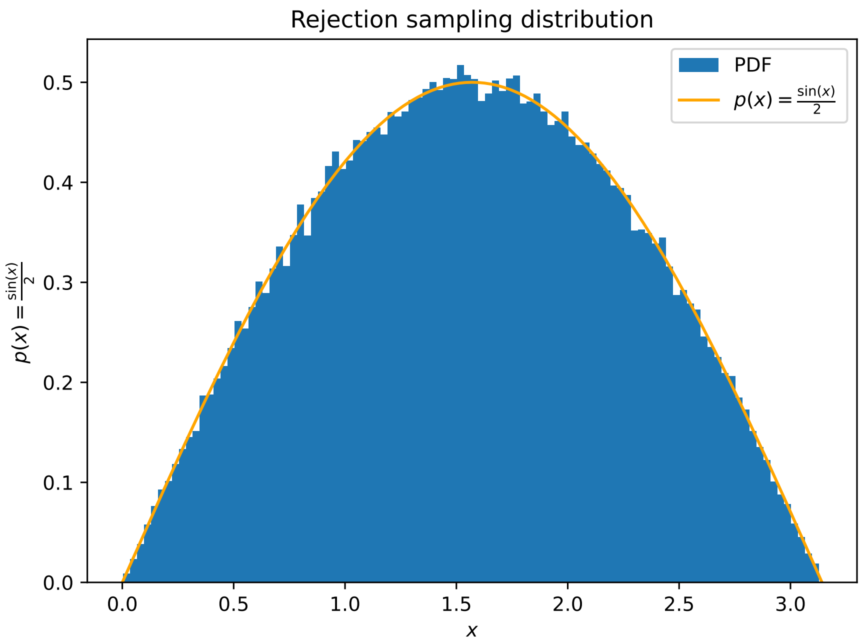

Figure 1.5: This histogram shows random numbers generated using the von Neumann rejection method for a function proportional to sin(x). The shape of the histogram closely matches the expected sinusoidal distribution over the interval [0, π).

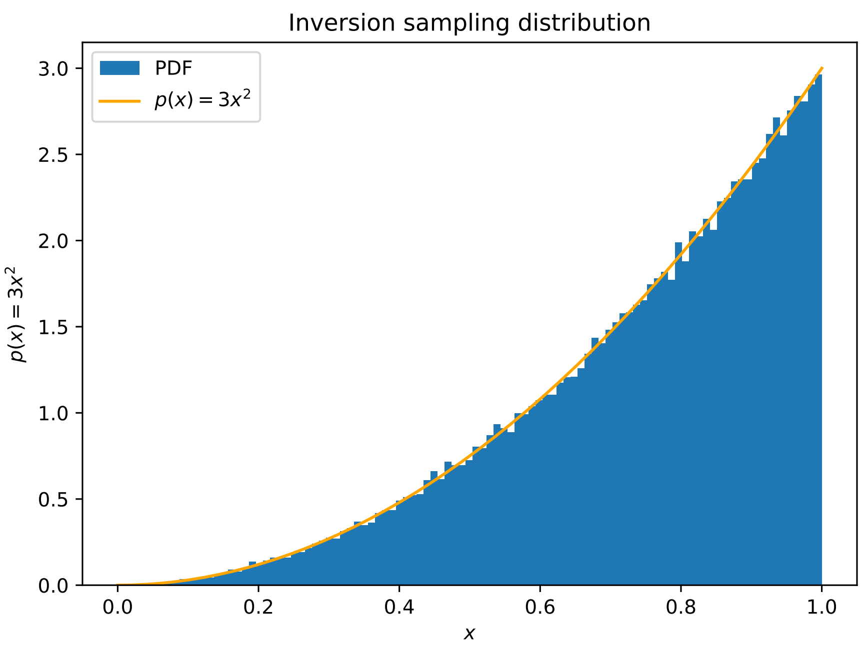

Figure 1.6: The histogram represents random numbers generated from the distribution $p(x) = 3x^2$ using the inversion method. The distribution of generated numbers closely matches the quadratic distribution, verifying the correctness of the method.