# Computational Physics 8. Exercise Sheet

SuSe 2023

**Handing out:** 9.6.2023
**Submission:** 16.6.2023 8 pm

Prof. H. Jelger Risselada

---

### Exercise 1: Linearly congruent random number generators — 10 Points

Generate pseudo-random number by implementing a linearly congruent generator

$$r_{n+1} = (ar_n + c) \bmod m \tag{1}$$

yourself.

1. Write a program which determines the first $N$ entries ($N < m$) of the integer sequence $r_n$ as a function of the four parameters $r_0$ (seed), $a$, $c$ und $m$ (use 64-bit integers).

   Divide $r_{n+1}$ by $m$ to get a floating point generator for random number in $[0, 1[$.

2. For the four sets of parameters

   (a) $r_0 = 1234$, $a = 20$, $c = 120$, $m = 6075$,

   (b) $r_0 = 1234$, $a = 137$, $c = 187$, $m = 256$,

   (c) $r_0 = 123456789$, $a = 65539$, $c = 0$, $m = 2^{31} = 2147483648$
   (RANDU Generator von IBM),

   (d) $r_0 = 1234$, $a = 7^5 = 16807$, $c = 0$, $m = 2^{31} - 1$
   (`ran1()` aus Numerical Recipes, 2. Ausgabe, bzw. Matlab bis Version 4)

   investigate your floating point generator for equal partition by generating a histogramm (consisting of $N = 10^5$ total values). Split the interval $[0, 1[$ into 10 bins of length 0.1.

3. Now, test the four floating point generators for correlations by respectively plotting $N/2$ pairs $\{(r_n, r_{n-1}), (r_{n-2}, r_{n-3}), \dots\}$ of two immediately consecutively generated values in the square $[0, 1]^2$. Use $N = 10^5$ values and note that only $N < m$ makes sense.

### Exercise 2: Arbitrary distributions — 10 Points

A random number generator producing uniformly distributed numbers in the interval $[0, 1[$ can also be used to generate numbers from an arbitrary distribution. Use of your linear congruential generator implemented in the previous exercise and parameter set 4 in the following tasks.

a) Implement the Box-Muller algorithm to sample a Gaussian distribution.

b) An alternative approach to generate a Gaussian distribution is based on the central limit theorem. Sum $N$ uniformly distributed random numbers from the interval $[0, 1[$. How can you get a distribution with 0 mean and standard deviation 1 from this sum? Derive a criterion on $N$ from the condition on the standard deviation. Explain disadvantages of this method with respect to correctness and efficiency.

c) Implement the von Neumann rejection sampling to generate the distribution

$$p_1 = \frac{\sin(x)}{2}$$

over the interval $[0, \pi[$.

d) Apply the inversion method to draw random numbers from the distribution

$$p_2(x) = 3x^2$$

for $x \in [0, 1[$.

Plot histograms of $10^5$ random numbers for each subtask to verify the distributions and compare to the analytical distributions.