

## Absolute band gaps of a two-dimensional triangular-lattice dielectric photonic crystal with different shapes

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Absolute band gaps of a two-dimensional triangular-lattice photonic crystal are calculated with the finite-difference time-domain method in this paper. Through calculating the photonic band structures of the triangular-lattice photonic crystal consisting of Ge rods immersed in air with different shapes, it is found that a large absolute band gap of 0.098 ( $2c/a$ ) can be obtained for the structures with hollow triangular Ge rods immersed in air, corresponding to 19.8% of the middle frequency. The influence of the different factors on the width of the absolute band gaps is also discussed.

**photonic crystal, absolute band gap, triangular lattice**

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Since the pioneering work of Yablonovitch [1] and John [2], photonic crystals (PCs), which are inhomogeneous periodic arrays of dielectric or metal with a lattice constant comparable to the wavelength, have attracted considerable attention and extensive research. One of the PC's remarkable features is that the propagation of electromagnetic waves within a frequency region, which is called the photonic band gap (PBG), is forbidden. It is well known that a large PBG is the cornerstone for various applications such as optical waveguides, defect-mode PC lasers, etc. Although three-dimensional PCs may provide most of the potential applications, two-dimensional (2D) PCs have also been intensively studied. The 2D PCs are easier to fabricate and may be employed in optical and electronic devices. The electromagnetic waves can be decomposed into the TM-polarized and the TE-polarized modes for the 2D PCs. If the PBGs in both polarization modes are present and overlap each other, an absolute PBG is achieved.

Many attempts have been made to design various kinds of PC structures to achieve a large absolute PBG [3–14]. It has been shown that an absolute PBG exists in both cases of

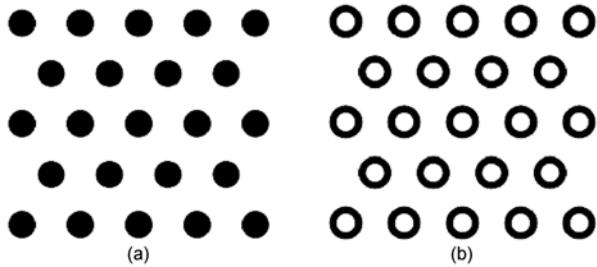
air rods immersed in a dielectric and dielectric rods in air for the square-lattice PC, while an absolute PBG exists in the case of air rods in a dielectric for the triangular-lattice PC, especially in the case of circular air holes in a dielectric. Cassagne et al. [3] have proved that a large absolute PBG can appear for triangular structures of air holes in a dielectric as well as for graphite structures of dielectric rods in air. Qiu et al. have found that the elliptic rods can get a larger PBG than circular rods in a rectangular lattice [4]. Wang et al. [5] have investigated the photonic band structures of 2D PCs consisting of varied lattices with five different shapes of scatterers (hexagon, circle, ellipse, square and rectangle). It was found that the hexagonal rods can generate the largest gap in a honeycomb lattice. The question of whether a large absolute PBG occurs in the triangular-lattice PC consisting of dielectric rods immersed in air has attracted many people's attention. In this paper, we show that an absolute PBG can be obtained for the triangular-lattice PC consisting of triangular Ge (Germanium) rods in air.

### 1 Numerical simulation and discussion

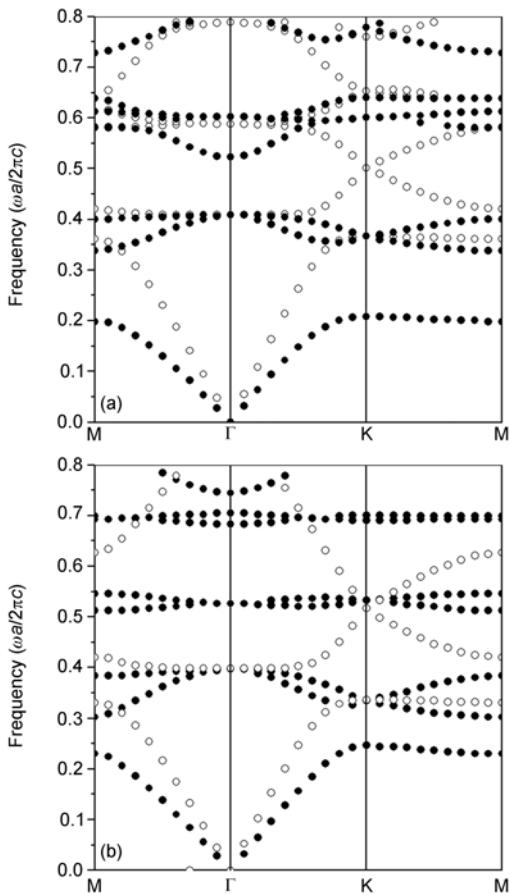
We first consider a 2D triangular-lattice PC consisting of

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circular Ge rods immersed in air, which is shown in Figure 1(a). The relative permittivity of Ge is set to be 18, the radius of the dielectric rods is  $r=0.233a$  (where  $a$  is the lattice constant of the PC), and the corresponding filling fraction of the rods is  $f=0.17$ . Figure 2(a) shows the band structure of the above PC for both the TM-polarized (solid dotted lines) and TE-polarized (hollow dotted lines) modes, which is calculated with the finite-difference time-domain method [15,16]. It can be seen that one large PBG lies between the first and the second TM-polarized bands, and two large PBGs appear among the higher TM bands. Although there



**Figure 1** Schematic diagrams of a 2D triangular lattice consisting of solid (a) and hollow (b) circular rods embedded in air.



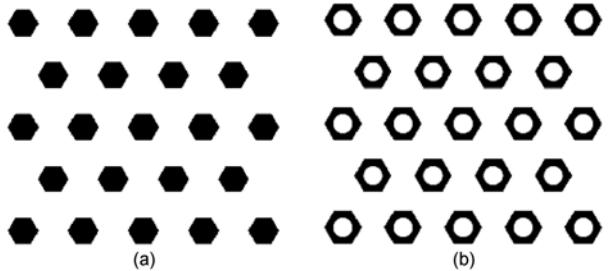
**Figure 2** The photonic band structures of the triangular-lattice PC consisting of solid (a) and hollow (b) circular Ge rods for TM (solid dots) and TE (hollow dots) modes.

is a small PBG between the first and the second TE-polarized bands, no absolute PBG is obtained since the PBGs for two polarization modes cannot overlap.

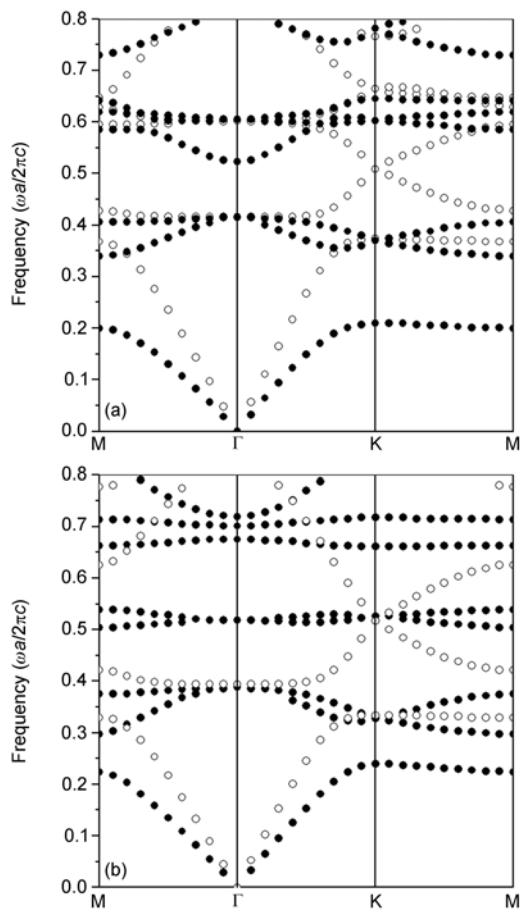
While keeping the filling fraction constant, we change the rod's shape from a solid circle to a hollow circle, which is shown in Figure 1(b). The inner radius of the circle is set to be  $0.16a$ , and the outer radius of the circle is  $0.282a$ . The calculated results of the band structures of this PC are shown in Figure 2(b), from which we can see that the width of the absolute PBG between the lowest and the second TM bands narrows, and a wider PBG appears among the highest several TM bands. The two lowest TE bands are almost unchanged with the alteration of the circle's shape. But the locations of the higher bands are more sensitively influenced by the rod's shape.

Figure 3 shows the PC structure when the rod's shape is changed from a circle to a hexagon. For the same filling fraction  $f=0.17$ , the corresponding side length of the solid hexagonal rod is  $0.256a$ , the outer side length of the hollow hexagonal rod is  $0.31a$ , and the inner radius of the hollow rod is  $0.16a$ . The corresponding calculated band structures are shown in Figure 4. Comparing Figure 2 with Figure 4, we see that the several lowest TM and TE bands are almost unchanged when the rod's shape changes from a circle to a hexagon. Also, no visible absolute PBG is obtained.

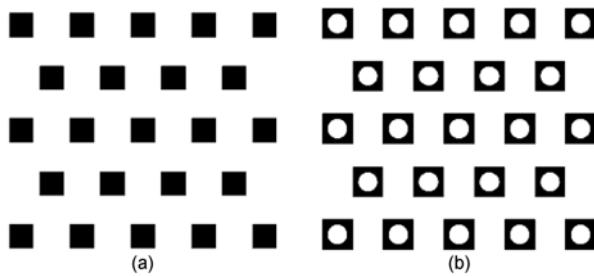
To reduce the symmetry of the rod to a greater degree, we utilize the square rods to constitute the triangular-lattice PC. The side length of the square rod is placed along the  $\Gamma$ X direction of the PC, which is shown in Figure 5. The side length of the solid square is  $l=0.412a$ , the outer side length of the hollow square is  $0.5a$ , and the inner radius of the square is  $0.16a$ , the filling factors of the dielectric rods in the above two structures are both 0.17. Figure 6 shows the calculated band structures in the above PC. It can be found from Figure 6(a) that a small absolute PBG appears between the third TM-polarized band and the second TE-polarized band, and it lies between the frequencies 0.41 and 0.418 ( $2c/a$ ) with a normalized band width (gap-midgap ratio) of about 1.9%. Another small absolute PBG also appears between the third and fourth TE bands, between the frequencies 0.5 and 0.508 ( $2c/a$ ) with a normalized band width of about 1.6%. Figure 6(b) shows that more TM PBGs appear when the square rods are changed from solid to hollow,



**Figure 3** Schematic diagrams of a 2D triangular lattice consisting of solid (a) and hollow (b) hexagonal Ge rods embedded in air.

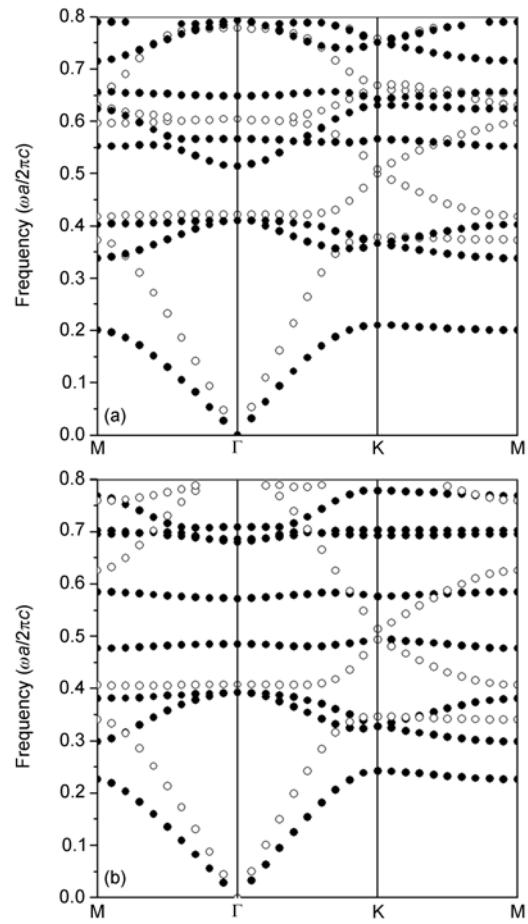


**Figure 4** The photonic band structures of the triangular-lattice PC consisting of solid (a) and hollow (b) hexagonal Ge rods for TM (solid dots) and TE (hollow dots) modes.



**Figure 5** Schematic diagrams of a 2D triangular lattice consisting of solid (a) and hollow (b) square rods embedded in air.

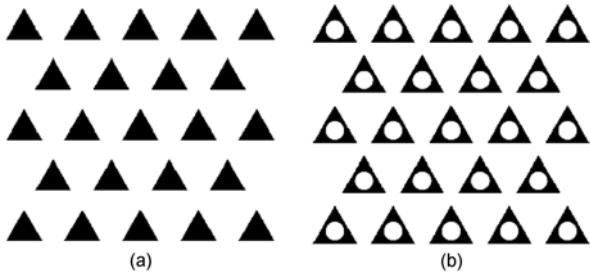
while the width of the PBG between the first and second bands narrows. The two lowest TE bands are almost not influenced by the alteration of the rod's shape. An absolute PBG appears between the third TM-polarized band and the second TE-polarized band, and lies between the frequencies 0.392 and 0.408 ( $2c/a$ ) with a normalized band width (gap-midgap ratio) of about 4%. Another absolute PBG also appears between the second and the third TE-polarized bands, and lies between the frequencies 0.493 and 0.503



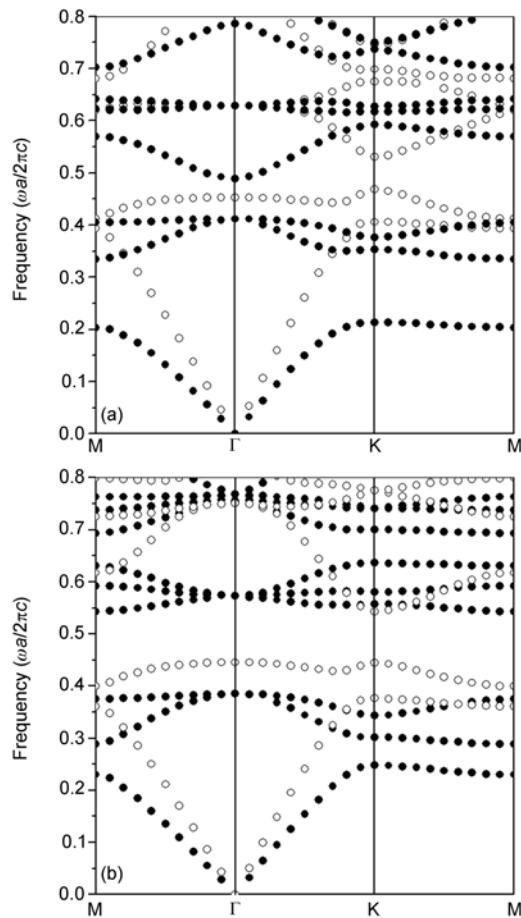
**Figure 6** The photonic band structures of the triangular-lattice PC consisting of solid (a) and hollow (b) square Ge rods for TM (solid dots) and TE (hollow dots) modes.

( $2c/a$ ) with a normalized band width of about 4%. The width of the absolute PBG increases when the rod's shape is changed from solid to hollow, although the width is still small.

When the triangular-lattice PCs are composed of triangular rods, which are shown in Figure 7, the rods have the same shapes as the coordination polygon of a lattice point and the basis of a PC has the same symmetry as that of the lattice. The side length of the solid triangle is  $0.626a$ , the outer side length of the hollow triangle is  $0.76a$ , and the inner radius of the air hole is  $0.16a$ . The filling factors of the dielectric rods for the above two structures are both 0.17. We can see from Figure 8(a) that an obvious absolute PBG appears between the second TE band and the fourth TM band; the lower and upper band edges are 0.468 and 0.489 ( $2c/a$ ), respectively. The normalized band width is 4.4%. When the rod's shape is changed to a hollow triangle, a large absolute PBG is also obtained between the second TE band and the fourth TM band, which lies between the frequencies of 0.444 and 0.542 ( $2c/a$ ). A large absolute PBG with a width of 0.098 ( $2c/a$ ) can be obtained, and the corre-



**Figure 7** Schematic diagrams of a 2D triangular lattice consisting of solid (a) and hollow (b) triangular rods embedded in air.



**Figure 8** The photonic band structures of the triangular-lattice PC consisting of solid (a) and hollow (b) triangular Ge rods for TM (solid dots) and TE (hollow dots) modes.

sponding normalized band width is 19.8%. Beyond the wide absolute PBG, another narrow PBG exists between the second TE band and the third TM band, which lies between the frequencies of 0.385 and 0.4 ( $2c/a$ ), with a normalized band width of 3.8%.

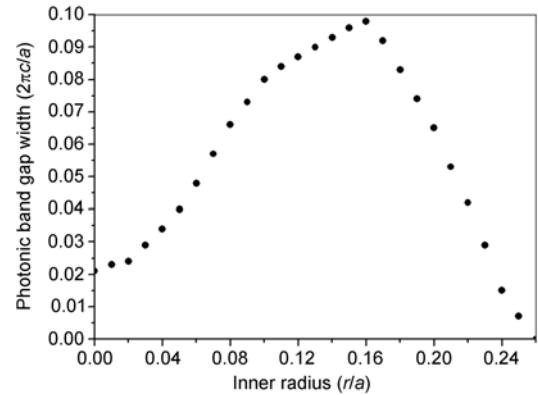
Keeping the filling factor of the dielectric rods constant at 0.17, the width of the largest PBG is influenced by the inner air hole's radius, which is shown in Figure 9. It can be seen that a large PBG, whose width is more than 0.04 ( $2c/a$ ),

exists when the inner radius varies from  $0.06a$  to  $0.21a$ , and the corresponding sidelength of the outer triangle varies from  $0.65a$  to  $0.844a$ .

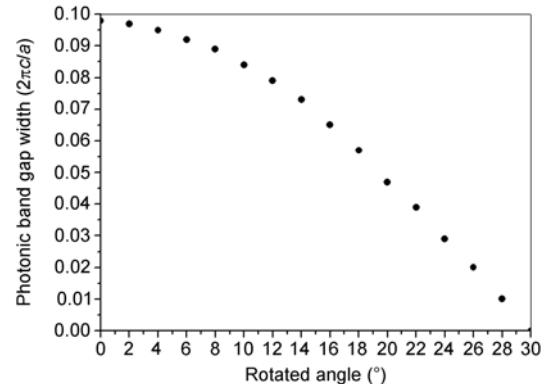
If the hollow-triangular rod is rotated in the unit cell, it can be shown in Figure 10 that the width of the absolute PBG is also influenced by the angle between the direction of the triangular side and the PC's  $\Gamma K$  direction. The absolute PBG's width narrows with the direction of the triangular side deviating from the  $\Gamma K$  direction of the PC. When the angle is less than 20°, a large absolute PBG occurs with a width more than 0.04 ( $2c/a$ ). Finally, the silicon rod is used to replace the germanium rods, and similar results are achieved.

## 2 Conclusion

In conclusion, the photonic band structures of the triangular-lattice PCs consisting of dielectric rods immersed in air are studied. The calculated results show that a large absolute PBG can be produced in a triangular lattice by using hollow-triangular rods which have the same shape as the coor-



**Figure 9** The width of the absolute band gap is influenced by the size of the inner radius under the same filling factor of dielectric rods  $f=0.17$ .



**Figure 10** The width of the absolute band gap is influenced by the angle between the direction of the triangular side and the PC's  $\Gamma K$  direction, for the structure, where the outer triangular sidelength is  $0.76a$  and the inner radius is  $0.16a$ .

dination polygon of a lattice point. These simulated results should be useful in the design of 2D PCs when a large absolute PBG is desired.

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