

Time	Group	Submission in Moodle; Mails with subject: [SMD2023]
Th. 12:00–13:00	A	lukas.beiske@udo.edu and tristan.gradetzke@udo.edu
Fr. 08:45–09:45	B	jonas.hackfeld@ruhr-uni-bochum.de and ludwig.neste@udo.edu
Fr. 10:00–11:00	C	stefan.froese@udo.edu and vincent.latko@udo.edu

Exercise 1 *Numerical stability*

4 p.

Consider the functions

(a) $f(x) = (x^3 + 1/3) - (x^3 - 1/3)$ and

(b) $g(x) = ((3 + x^3/3) - (3 - x^3/3))/x^3$.

Determine empirically for which ranges of x (roughly) the numerical result

- deviates from the algebraic by more than 1 %,
- is equal to zero.

(c) Graphically represent the result in a suitable form (for exaple, logarithmic x -scale)!

`x = np.logspace(start, stop, num)`

(d) How does the representation change when you create the data points with the float32 or float64 data type?

`x_32 = np.logspace(start, stop, num, dtype='float32')`
`x_64 = np.logspace(start, stop, num, dtype='float64')`

Exercise 2 *Numerical stability and condition*

6 p.

The expression $f(E, \Theta)$ represents a summand of the differential cross section for the reaction $e^-e^+ \rightarrow \gamma\gamma$

$$f(E, \Theta) = \frac{2 + \sin^2\Theta}{1 - \beta^2 \cos^2\Theta},$$

with

$$\beta = \sqrt{1 - \gamma^{-2}},$$

$$\gamma = \frac{E}{m},$$

$$m = 511 \text{ keV}.$$

- (a) In which range for Θ is the equation for $E = 50 \text{ GeV}$ numerically unstable?
- (b) Resolve the numerical instability by a suitable analytic transformation. (Hint: Make use of $1 - \beta^2 = 1/\gamma^2$ and $1 = \sin^2\Theta + \cos^2\Theta$)
- (c) Show that you have fixed the stability problems by plotting both equations in the critical interval.
- (d) Calculate the condition number. How does this depend on Θ ?
- (e) Plot the value of the condition number as a function of Θ ($0 \leq \Theta \leq \pi$). In which area is the problem well or poorly conditioned?
- (f) What is the difference between stability and condition?