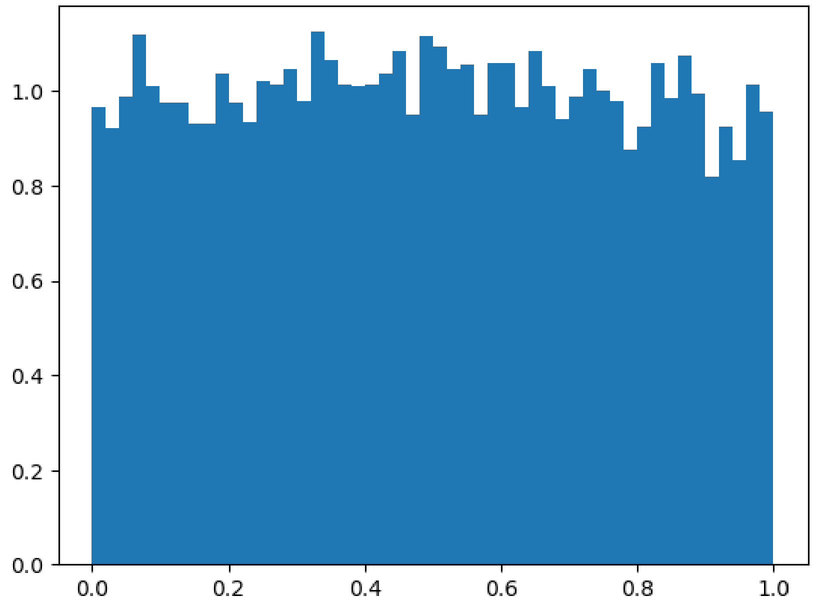
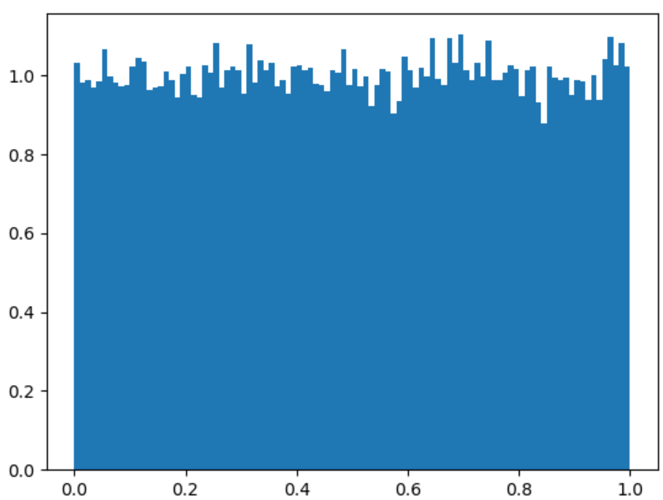
1. a) Uniform (0,1, size=10000)



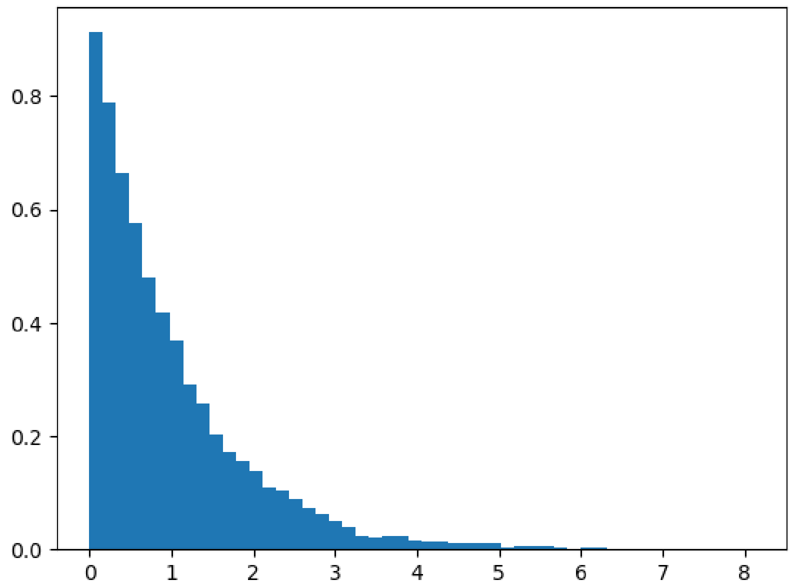
Mean: 0.499556439000049745
Variance: 0.08348752220470869
Skewness: 0.005933011588735974
Kurtosis: 1.7993824440387962

Uniform (0,1, size=50000)



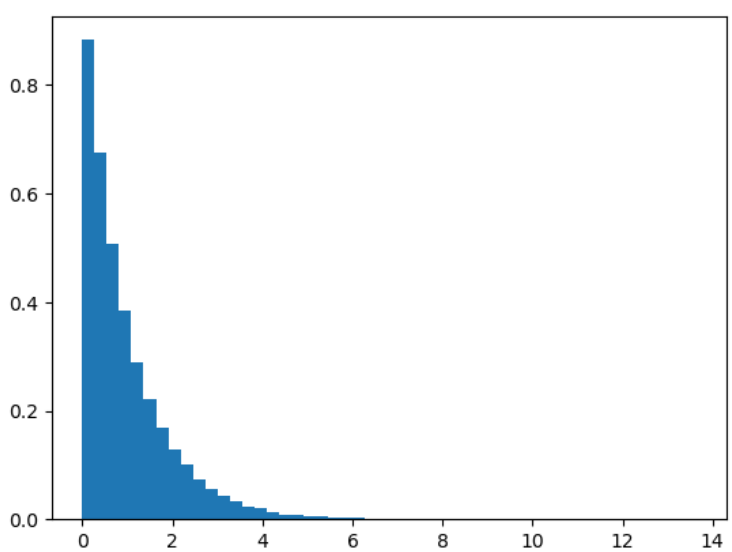
Mean: 0.4997946755328393
Variance: 0.083271514949579 54
Skewness: 0.0018130995357547558
Kurtosis: 1.7987984251204487

b) exponential(mean=1, size=10000)



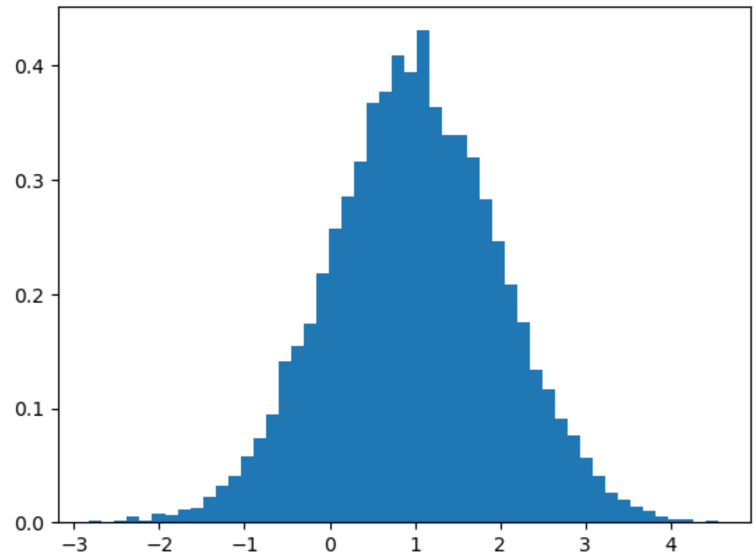
Mean: 1.0057271053824353
Variance: 1.0200823197311706
Skewness: 1.9430571206509764
Kurtosis: 8.327422347403072

exponential(mean=1, size=50000)



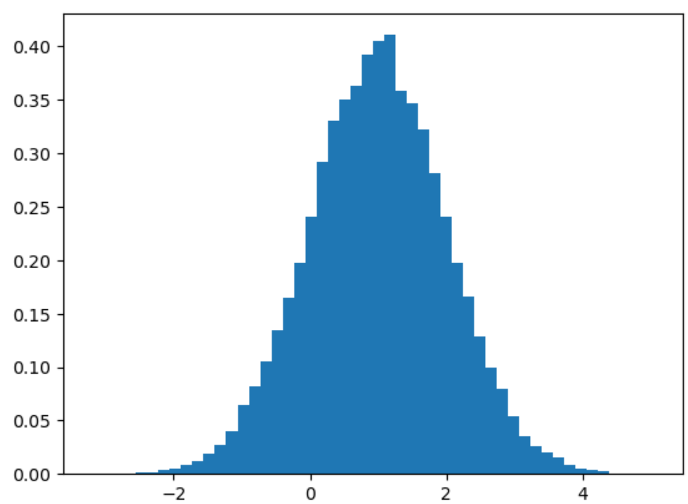
Mean: 0.9918047897782858
Variance: 0.9851591601277931
Skewness: 2.00262437860857
Kurtosis: 9.163033035946428

c) Normal (mean=1, standard deviation=1, size=10000)



Mean: 0.9925738047977164
Variance: 1.010785525513204
Skewness: -0.02237481445114067
Kurtosis: 3.038966938039418

Normal (mean=1, standard deviation=1, size=50000)



Mean: 0.9938811659733331
Variance: 1.0136638628284775
Skewness: 0.004489259800865832
Kurtosis: 3.0191221810955855

A Jupyter Notebook is attached to the submission, and can be run on JupyterHub Uvic.

2. a) What is the average service time at the Security-Check?

Theoretical average inter-arrival time for terminal 1 passengers: 60 / 10 = 6 min
Theoretical average inter-arrival time for terminal 2 passengers: 60 / 20 = 3 min

$\rho = \lambda S / \mu S$
$0.8 = (1/6 + 1/3) / \mu S$
$0.8 = (1/2) / \mu S$
$\mu S = 0.625$
average service time at the Security-Check = 1/0.625 = 1.6 minutes per customer

b) What are the performance metrics (ρ, L, Lq, w, wq) for Terminal 1 Check-in?

Terminal 1 check-in is a M/M/1 model
$\rho = \lambda 1 / \mu 1 = (1/6) / (1/2) = 1/3 \approx 0.3333$
$L = \rho / (1 - \rho) = (1/3) / (1-(1/3)) = 1/2 \approx 0.5$ customers
$Lq = \rho^2 / (1 - \rho) = (1/3)^2 / (1-(1/3)) = 1/6 \approx 0.1667$ customers
$w = 1/ \mu 1(1-p) = 1/(1/2)*(1-(1/3)) = 3$ minutes
$wq = \rho / \mu 1(1-\rho) = (1/3) / ((1/2)*(1-1/3)) = 1$ minutes

c) What are the performance metrics (ρ, L, LQ, w, wQ) for Security-Check?

Security-check is a M/M/1 model
$\rho = 0.8$
$L = \rho / (1 - \rho) = (4/5) / (1-(4/5)) = 4$ customers
$Lq = \rho^2 / (1 - \rho) = (4/5)^2 / (1-(4/5)) = 3\frac{1}{5}$ customers
$w = 1/ \mu s(1-p) = 1/(5/8)*(1-(4/5)) = 8$ minutes
$wq = \rho / \mu s(1-\rho) = (4/5) / ((5/8)*(1-4/5)) = 32/5$ minutes = 6.4 minutes

d) What is the total average time (system time) spent by Terminal 1 passengers in the system?

System time of Terminal 1 passengers in the system: w1 + ws = 3 + 8 = 11 minutes

e) What is the total average number of Terminal 1 passengers in the system?

Total average number of Terminal 1 passengers in the system:
$L1 + (\lambda 1 / (\lambda 1 + \lambda 2)) * Ls = \frac{1}{2} + ((1/6) / (1/6+1/3)) * 4 = 11/6$ customers

f) What is the average time spent (System time) by Terminal 2 passengers in the system?

Average system of terminal 2 passengers: w2 = ws = 8 minutes

3.

Seed	ρ	W / min	Wq / min	L	Lq
1234	0.7970451911	9.255773079	7.655157306	4.626210051	3.826192083
12345	0.7931346868	9.276920035	7.681329949	4.641624052	3.843284808
3456	0.8021360148	9.397733923	7.798805499	4.674277567	3.878996989
34567	0.8077294009	10.1316047	8.525084375	5.03091918	4.233190276
5789	0.8033484491	9.816060142	8.206624241	4.914453334	4.108682228
Mean	0.8006787485	9.575618376	7.973400274	4.777496837	3.978069277

Original Excel sheet can be found inside the .zip file

4. ρ comperison: M/M/2 $\qquad$ M/M/1 × 2

$$\rho = \frac{2\lambda}{2\mu} = \frac{\lambda}{\mu} \qquad \rho = \frac{\left(\frac{\lambda}{\mu}+\frac{\lambda}{\mu}\right)}{2} = \frac{\lambda}{\mu}.$$

Both servers have the same ρ

L comparison:

M/M/2

$$P_0 = \left\{\left[\sum_{n=0}^{1} \frac{(2\cdot\frac{\lambda}{\mu})^n}{n!}\right] + \left[(2\frac{\lambda}{\mu})^2 (\frac{1}{2!})(\frac{1}{1-\frac{\lambda}{\mu}})\right]\right\}^{-1}$$

$$= \left\{\frac{2\lambda+\mu}{\mu} + \frac{2\lambda^2}{\mu(\mu-\lambda)}\right\}^{-1}$$

$$= \left\{\frac{\mu+\lambda}{\mu-\lambda}\right\}^{-1}$$

$$= \frac{\mu-\lambda}{\mu+\lambda}$$

$$L = c\rho + \frac{c(c\rho)^{c+1} P_0}{c(c!)(1-\rho)^2}$$

$$= 2\frac{\lambda}{\mu} + \frac{(2\cdot\frac{\lambda}{\mu})^3 \frac{\mu-\lambda}{\mu+\lambda}}{2\times(2!)(1-\frac{\lambda}{\mu})^2}$$

$$= \frac{2\lambda}{\mu} + \frac{2\lambda^3}{\mu(\mu-\lambda)(\mu+\lambda)}.$$

$$= \frac{2\lambda\mu}{(\mu+\lambda)(\mu-\lambda)}$$

M/M/1 × 2

$$L = \frac{\lambda}{\mu-\lambda} + \frac{\lambda}{\mu-\lambda}$$

$$= \frac{2\lambda}{\mu-\lambda}$$

$$L_{M/M/2} = L_{M/M/1\times2} \cdot \frac{\mu}{\mu+\lambda}$$

$\frac{\mu}{\mu+\lambda} < 1$, average number of customers in the system for two M/M/1 queues are greater than M/M/2 queue.

W comparsion:

M/M/2

$$W = \frac{L}{\lambda} = \frac{\frac{2\lambda\mu}{(\mu+\lambda)(\mu-\lambda)}}{2\lambda} = \frac{\mu}{(\mu+\lambda)(\mu-\lambda)}$$

M/M/1 × 2

$$W = \frac{\left(\frac{1}{\mu-\lambda}+\frac{1}{\mu-\lambda}\right)}{2} = \frac{1}{\mu-\lambda}$$

$W_{M/M/2} = W_{M/M/1\times2} \cdot \frac{\mu}{\mu+\lambda}$, $\frac{\mu}{\mu+\lambda} < 1$

Customers spend less time in M/M/2 system than two M/M/1 System

W_Q comparsion

M/M/2

$$W_Q = W - \frac{1}{\mu} = \frac{\mu}{(\mu+\lambda)(\mu-\lambda)} - \frac{1}{\mu} = \frac{\lambda^2}{\mu(\mu+\lambda)(\mu-\lambda)}.$$

M/M/1×2

$$W_Q = \frac{\left(\frac{\lambda}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu(\mu-\lambda)}\right)}{2} = \frac{\lambda}{\mu(\mu-\lambda)}$$

Customers in queue for M/M/2 spend less time on average than 2 M/M/1 models.

$$\frac{\lambda^2}{\mu(\mu+\lambda)(\mu-\lambda)} = \frac{\lambda}{\mu(\mu-\lambda)} \times \frac{\lambda}{(\mu+\lambda)}, \quad \frac{\lambda}{\mu+\lambda} < 1$$

L_Q comparsion

M/M/2

$$L_Q = \lambda W_Q = 2\lambda \cdot \frac{\lambda^2}{\mu(\mu+\lambda)(\mu-\lambda)} = \frac{2\lambda^3}{\mu(\mu+\lambda)(\mu-\lambda)}$$

M/M/1×2.

$$L_Q = \frac{\left(\frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda^2}{\mu(\mu-\lambda)}\right)}{2} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

It's hard to determine which system would have more customers in the queue.

Overall, we can see that both servers have the same server utilization rate. Although customers in pooling servers spend less time in the system, there are also fewer customers in the pooling servers on average. Pooling server also has a faster wait time in queues.

5. average arrival rate λ : $\frac{34}{60} = \frac{17}{30}$ (in min)

average service time $\frac{1}{\mu}$ (in min) : $3\times0.2+7\times0.7+12\times0.1 = 6.7$ min.

average service rate μ : $\frac{10}{67}$.

Original queue : M/M/4/7/∞

$$\rho = \frac{\lambda}{c\mu} = \frac{\frac{17}{30}}{4\times\frac{10}{67}} = \frac{1139}{1200} \approx 0.949$$

$$a = \frac{\lambda}{\mu} = \frac{\frac{17}{30}}{\frac{10}{67}} = \frac{1139}{300} \approx 3.796$$

$$P_0 = \left[1+\sum_{n=1}^{c}\frac{a^n}{n!}+\frac{a^c}{c!}\sum_{n=c+1}^{N}\rho^{n-4}\right]^{-1}$$

$$= \left[1+\sum_{n=1}^{4}\frac{(3.796)^n}{n!}+\frac{(3.796)^4}{4!}\sum_{n=5}^{7}(0.949)^{n-4}\right]^{-1}$$

$$= \left[1+28.76883478+23.39612\right]^{-1}$$

$$\approx 0.019$$

$$P_N = \frac{a^N P_0}{c!\,c^{N-c}} = \frac{(3.796)^7\times 0.019}{4!\,4^{7-4}} \approx 0.140$$

Customer loss rate: $\lambda P_N = \frac{17}{30}\times 0.140 \approx 0.079$ (opposite of λ_e)

(4.74 in hour).

Proposed System : M/M/5/7/∞

λ and μ remain the same.

$$\rho = \frac{\lambda}{c\mu} = \frac{\frac{17}{30}}{5\times\frac{10}{67}} = \frac{1139}{1500} \approx 0.759$$

$$a = 3.796$$

$$P_0 = \left[1+\sum_{n=1}^{c}\frac{a^n}{n!}+\frac{a^c}{c!}\sum_{n=c+1}^{N}\rho^{n-5}\right]^{-1}$$

$$= \left[1+\sum_{n=1}^{5}\frac{(3.796)^n}{n!}+\frac{(3.796)^5}{5!}\sum_{n=6}^{7}0.759^{n-5}\right]^{-1}$$

$$= \left[1+35.33708626+8.769147762\right]^{-1}$$

$$\approx 0.022$$

$$P_N = \frac{a^N P_0}{c!\, c^{N-c}} = \frac{(3.796)^7 \times 0.022}{5!\, 5^{7-5}} \approx 0.083$$

customer loss rate: $\frac{17}{30} \times 0.083 \approx 0.047$

(2.82 in hour)

The proposed system would have a lower customer loss rate.

6. a) mean of total examation time: $E(X) = \frac{1}{4} \times 3 = \frac{3}{4}$ hr.

variance of total examation: $V(X) = \frac{1}{4^2} \times 3 = \frac{3}{16}$ hr.

b) It is not a exponential distribution since $\mu \neq \sigma$ $(\sigma = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4})$.

c) $L_Q = \dfrac{\rho^2 (1+\sigma^2\mu^2)}{2(1-\rho)}$ $\quad \rho = \dfrac{\lambda}{\mu} = \dfrac{1}{\frac{4}{3}} = \dfrac{3}{4}$

$$= \frac{(\frac{3}{4})^2 (1 + \frac{3}{16} \times (\frac{4}{3})^2)}{2(1-\frac{3}{4})}$$

$$= \frac{3}{2}.$$

d) $W = \dfrac{1}{\mu} + \dfrac{\lambda(1/\mu^2 + \sigma^2)}{2(1-\rho)}$

$$= \frac{1}{\frac{4}{3}} + \frac{1(1/(\frac{4}{3})^2 + \frac{3}{16})}{2(1-\frac{3}{4})}$$

$$= \frac{9}{4}$$

7. Since $Y = \sum_{i=1}^{n} X_i$, $X_i = \eta$, and $E(X+Y) = E(X) + E(Y)$

$E(Y) = \sum_{i=1}^{n} E(X_i) = n \cdot \eta$

$V(Y) = \sum_{i=1}^{n} V(X_i) = n \cdot \sigma^2.$