CS 303-01: Introduction to Machine Learning

HW 2 (Given Sept. 11, 2018; Due Sept. 18, 2018)

Email your answers to the TA before midnight on the day it is due

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Numbers in the parentheses indicate points allocated to the question.

1. Write the Taylor series expansion of,

   (a) Write the Taylor series expansion of e^x. **(10 points)**

   (b) Write a small program to plot what happens as you progressively take higher order terms i.e. the approximation when you only take the linear term, the approximation when you also include the second order term, the approximation when you include the third order term, and the approxination when you include the fourth order term. Plot these in different colors overlaid on e^x. **(20 points)**

2. Assume an error function to be minimized is given by,

$$J(w) = \frac{1}{2}\left[(w_2 - w_1)^2 + (1 - w_1)^2\right] \tag{1}$$

   (a) Sketch pictorially (or use or some other package) to draw the contour map of this error function. **(10 points)**

   (b) Find analytically the gradient vector,

$$\nabla(w) = \begin{bmatrix} \frac{\partial J}{\partial w_1} \\ \\ \frac{\partial J}{\partial w_2} \end{bmatrix} \tag{2}$$

   (10 points)

   (c) Identify on the picture you obtained in part (a) using the expression you obtained in part (b), the gradient vector at three arbitrary locations. **(10 points)**

   (d) Find analytically the weight vector that minimizes the error function such that $\nabla(w) = 0$. **(10 points)**

3. Suppose instead of $x = [x_1, x_2]^T$ being the input to a neuron, a pre-processing step is added such that the input to the neuron is $x = [x_1, x_1^2, x_1x_2, x_2^2, x_2]^T$. You may assume that a bias input to the neuron is present and that the activation function being used is linear.

   What does the decision boundary look like (note: decision boundary is drawn in the originial input space, i.e. in the x_1, x_2 space)? What use can you think for this? Be very specific. **(20 points)**