

# Analyzing and composing music with algorithms and machine learning

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### **Abstract**

The goal of the project is to compare, experiment and prove some mathematical algorithms for analyzing music and composing a similar pieces of music with random numbers. The algorithms are based on analyzing some relations from given note notations of given melodies, in form of standart MIDI files.

# 1 Introduction

## 1.1 Frequency relations (tuning)

The sound is type of mechanical vibration (waves) so it has the parameters of all the normal mechanical waves. However in the field of music we are most interested in the frequency of the wave, because it makes the variety of tones we hear. But not every sequence of frequencies sounds pleasant to the human. So the people developed some algorithms to “synchronize” the sequence of tunes. Here are the most used of them:

### 1.1.1 Pythagorean tuning

The Pythagorean tuning is the base of almost all tuning systems. It is based on the Pythagorean fractions.

$$f = \frac{i+1}{i}; i = 1, 2, \dots, n$$

Figure 1: Pythagorean fractions

It is assumed that if the frequencies correlate like these numbers, we hear them in harmony and the music will sound pleasant (if we select appropriate sequence of these tones of course).

If we pick a string with given length we know that the frequency of the produced sound depends on the length of the string. So to pick the “perfect” fraction we have to divide it like the Pythagorean fractions.

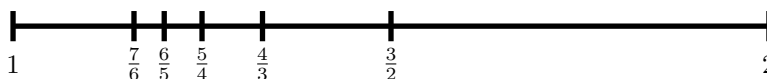


Figure 2: The “perfect” string separation

### 1.1.2 Equal temperament

Equal temperament is a tuning scale developed by Vincenzo Galilei. His idea is that the frequency ratio between each pair of adjacent notes is the same. In equal temperament tunings, the generating interval is often found by dividing some larger desired interval into a number of smaller equal steps (equal frequency ratios between neighbouring notes). The tones are divided in groups called

octaves. The ratio between the frequencies of last and the first tone of the octave is two.

$$\frac{\text{last tune}}{\text{first tune}} = \frac{2}{1}$$

In the Western music, the most commonly used tuning system since the 18th century has been the twelve-tone equal temperament (or 12-TET). The 12-TET divides the octave in 12 equal parts, all of them equal on logarithmic scale, with ratio  $\sqrt[12]{2} = 2^{\frac{1}{12}} \approx 1.05946$ . In modern times 12-TET is usually tuned relatively to the standard frequency of 440 Hz, called A4, meaning that the note A is tuned to 440 Hz and all other notes are derived by the logarithmic scale described above.

Equal temperament is an approximation of the Pythagorean scale, which is more convenient for use.

Name	12-TET	Pythagorean scale
Unison ( <i>C</i> )	$2^{\frac{0}{12}} = 1$	$\frac{1}{1} = 1$
Minor second ( <i>C<math>\sharp</math>/D<math>\flat</math></i> )	$2^{\frac{1}{12}} \approx 1.05946$	$\frac{16}{15} \approx 1.06666$
Major second ( <i>D</i> )	$2^{\frac{2}{12}} \approx 1.12246$	$\frac{9}{8} = 1.125$
Minor third ( <i>D<math>\sharp</math>/E<math>\flat</math></i> )	$2^{\frac{3}{12}} \approx 1.18920$	$\frac{6}{5} = 1.2$
Major third ( <i>E</i> )	$2^{\frac{4}{12}} \approx 1.25992$	$\frac{5}{4} = 1.25$
Perfect fourth ( <i>F</i> )	$2^{\frac{5}{12}} \approx 1.33484$	$\frac{4}{3} \approx 1.33333$
Tritone ( <i>F<math>\sharp</math>/G<math>\flat</math></i> )	$2^{\frac{6}{12}} \approx 1.41421$	$\frac{7}{5} = 1.4^*$
Perfect fifth ( <i>G</i> )	$2^{\frac{7}{12}} \approx 1.49830$	$\frac{3}{2} = 1.5$
Minor sixth ( <i>G<math>\sharp</math>/A<math>\flat</math></i> )	$2^{\frac{8}{12}} \approx 1.58740$	$\frac{8}{5} = 1.6^*$
Major sixth ( <i>A</i> )	$2^{\frac{9}{12}} \approx 1.68179$	$\frac{5}{3} \approx 1.66666^*$
Minor seventh ( <i>A<math>\sharp</math>/B<math>\flat</math></i> )	$2^{\frac{10}{12}} \approx 1.78179$	$\frac{16}{9} \approx 1.77777^*$
Major seventh ( <i>B</i> )	$2^{\frac{11}{12}} \approx 1.88774$	$\frac{15}{8} = 1.875^*$
Octave ( <i>C</i> )	$2^{\frac{12}{12}} = 2$	$\frac{2}{1} = 2$

Note: the values with \* can't be represented like Pythagorean fractions with decent accuracy but the human ear can't differentiate this (in the most cases).

Figure 3: Values of the tones in 12-TET and Pythagorean scale

## 1.2 MIDI standart

The purpose of MIDI is to define a way compact representation of musical sheets, which makes it appropriate for disk storage.

The standart MIDI(SMID) file is devided into chunks. Each chunk has a type and length represented as ASCII and 32 bit number respectively. There are two types of chunks: “MThd”(e.g. Header chunk) and “MTrk”(e.g. Track chunk). There is always one header while there can be more than one track.

### 1.2.1 Header chunk

The header chunk contains data for the whole file. It has fixed length of 6 bytes. The first 2 bytes represent the format of file:

- 0 → One track
- 1 → Multiple tracks but one song
- 2 → Multiple song

The second 2 bytes represent the number of tracks. For format 0 it is always 1. And the last two bytes represent how many ticks are in a quarter note.

### 1.2.2 Track chunks

Each track is composed of events. Each event has delta time - the time (in ticks) which passes after the previous event before executing this event. There are two types of events: MIDI events - the ones which represent the actual music and META - these which supply additional info like titles, lyrics, etc.

Each MIDI event that has to play note uses this standart for encoding the frequency of the played tune:

$$f = 440 * 2^{\frac{d-69}{12}} \text{ where } d \text{ is the note number and } f \text{ is the frequency}$$

Therefore the note number is:

$$d = 69 + 12 \log_2 \frac{f}{440}$$

## 1.3 Markov chains

Markov chain is a stochastic model which describes a sequence of events in which the probability of each event depends entierly on the previous event (in some cases the previous  $n$  but the table which decribes the chain size becomes big quickly because it equals to  $k^n$  where  $k$  is the number of events).

Markov chains are represented as table in the most cases. Each cell of the table hold the probability of triggering an event from the row/column after the event in the collumn/row respectively (see Figure 4).

				Next event		
				A	B	C
A	A	16%	16%	68%		
A	B	100%	0%	0%		
A	C	12%	75%	13%		
B	A	37%	25%	38%		
B	B	0%	0%	100%		
B	C	100%	0%	0%		
C	A	33%	33%	34%		
C	B	83%	17%	0%		
C	C	100%	0%	0%		

Figure 4: Markov chains for the string  
“AABAACBABACBABACCACAACBAAACBACBBCABAACBA”

## 1.4 Used algorithms

Each algorithm is based on analitics with Markov chains.

### 1.4.1 Algorithm with one tune pitch

This algorithm constructs a Markov table based only on the frequencies of the tones. The size of the table depends on the variety of tunes used. Then it selects starting note and each next note is picked using random number generator and the chances in the Markov chain.

### 1.4.2 Algorithm with one tune pitch and note duration

This algorithm constructs a Markov table based on the frequencies of the tones and their length. Because of this the size of the table increases and the size depends ont only on the variety of notes but on the variety of note lengths. Then it selects starting note and each next note is picked using random number generator and the chances in the Markov chain.

### 1.4.3 Algorithm with two tune pitch

This algorithm constructs a Markov table based only on the frequencies of the tones but it calculates the chance of next event, based ont he previous two, so the size of the table increases and is the size of the table in 1.4.1 squared. Then it selects starting note and each next note is picked using random number generator and the chances in the Markov chain.

### 1.4.4 Rust

All the implementations of the algorithms for the project are written in Rust programming language. It is based on C but it gaurantees memory safety,

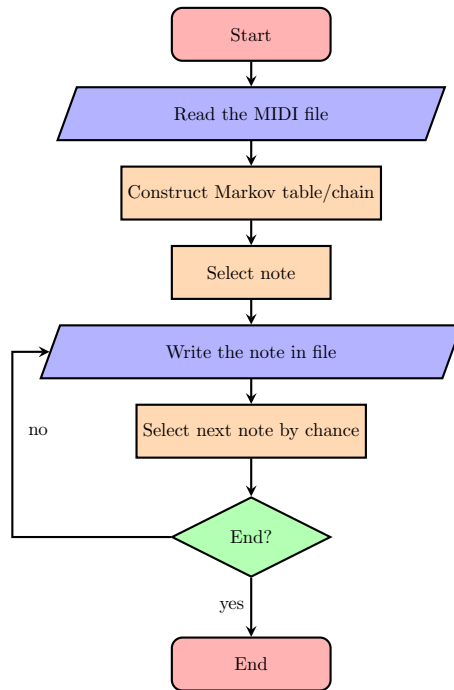


Figure 5: A common block diagram of the algorithms

threads without data races, generics(templates) and the standart library is bigger and has some bindings for functional programming.



## 2 Results

Here are listed some results. The tests are runned with original music “Noc-tune op.9 no.2” written by Chopin. Here is the music sheet of the piece(all the sheets are given in the github repo of the project like midi files):

♩ = 100

Piano, 

Piano 

8 

Pno. 

16 

Pno. 

23 

Pno. 

30 

Pno. 

♩ = 60

4

7

11

♩ = 111

5

10

14

The first system of the musical score for 'The Rose Tree' is written in 4/4 time with a key signature of one flat (B-flat). It consists of two staves: a bass staff on the left and a treble staff on the right. The melody is primarily in the treble staff, with some notes in the bass staff. The notes are: G2 (bass), A2 (bass), Bb2 (bass), C3 (bass), D3 (bass), E3 (bass), F3 (bass), G3 (bass), A3 (bass), Bb3 (bass), C4 (bass), D4 (bass), E4 (bass), F4 (bass), G4 (bass), A4 (bass), Bb4 (bass), C5 (bass), D5 (bass), E5 (bass), F5 (bass), G5 (bass), A5 (bass), Bb5 (bass), C6 (bass), D6 (bass), E6 (bass), F6 (bass), G6 (bass), A6 (bass), Bb6 (bass), C7 (bass), D7 (bass), E7 (bass), F7 (bass), G7 (bass), A7 (bass), Bb7 (bass), C8 (bass), D8 (bass), E8 (bass), F8 (bass), G8 (bass), A8 (bass), Bb8 (bass), C9 (bass), D9 (bass), E9 (bass), F9 (bass), G9 (bass), A9 (bass), Bb9 (bass), C10 (bass), D10 (bass), E10 (bass), F10 (bass), G10 (bass), A10 (bass), Bb10 (bass), C11 (bass), D11 (bass), E11 (bass), F11 (bass), G11 (bass), A11 (bass), Bb11 (bass), C12 (bass), D12 (bass), E12 (bass), F12 (bass), G12 (bass), A12 (bass), Bb12 (bass), C13 (bass), D13 (bass), E13 (bass), F13 (bass), G13 (bass), A13 (bass), Bb13 (bass), C14 (bass), D14 (bass), E14 (bass), F14 (bass), G14 (bass), A14 (bass), Bb14 (bass), C15 (bass), D15 (bass), E15 (bass), F15 (bass), G15 (bass), A15 (bass), Bb15 (bass), C16 (bass), D16 (bass), E16 (bass), F16 (bass), G16 (bass), A16 (bass), Bb16 (bass), C17 (bass), D17 (bass), E17 (bass), F17 (bass), G17 (bass), A17 (bass), Bb17 (bass), C18 (bass), D18 (bass), E18 (bass), F18 (bass), G18 (bass), A18 (bass), Bb18 (bass), C19 (bass), D19 (bass), E19 (bass), F19 (bass), G19 (bass), A19 (bass), Bb19 (bass), C20 (bass), D20 (bass), E20 (bass), F20 (bass), G20 (bass), A20 (bass), Bb20 (bass), C21 (bass), D21 (bass), E21 (bass), F21 (bass), G21 (bass), A21 (bass), Bb21 (bass), C22 (bass), D22 (bass), E22 (bass), F22 (bass), G22 (bass), A22 (bass), Bb22 (bass), C23 (bass), D23 (bass), E23 (bass), F23 (bass), G23 (bass), A23 (bass), Bb23 (bass), C24 (bass), D24 (bass), E24 (bass), F24 (bass), G24 (bass), A24 (bass), Bb24 (bass), C25 (bass), D25 (bass), E25 (bass), F25 (bass), G25 (bass), A25 (bass), Bb25 (bass), C26 (bass), D26 (bass), E26 (bass), F26 (bass), G26 (bass), A26 (bass), Bb26 (bass), C27 (bass), D27 (bass), E27 (bass), F27 (bass), G27 (bass), A27 (bass), Bb27 (bass), C28 (bass), D28 (bass), E28 (bass), F28 (bass), G28 (bass), A28 (bass), Bb28 (bass), C29 (bass), D29 (bass), E29 (bass), F29 (bass), G29 (bass), A29 (bass), Bb29 (bass), C30 (bass), D30 (bass), E30 (bass), F30 (bass), G30 (bass), A30 (bass), Bb30 (bass), C31 (bass), D31 (bass), E31 (bass), F31 (bass), G31 (bass), A31 (bass), Bb31 (bass), C32 (bass), D32 (bass), E32 (bass), F32 (bass), G32 (bass), A32 (bass), Bb32 (bass), C33 (bass), D33 (bass), E33 (bass), F33 (bass), G33 (bass), A33 (bass), Bb33 (bass), C34 (bass), D34 (bass), E34 (bass), F34 (bass), G34 (bass), A34 (bass), Bb34 (bass), C35 (bass), D35 (bass), E35 (bass), F35 (bass), G35 (bass), A35 (bass), Bb35 (bass), C36 (bass), D36 (bass), E36 (bass), F36 (bass), G36 (bass), A36 (bass), Bb36 (bass), C37 (bass), D37 (bass), E37 (bass), F37 (bass), G37 (bass), A37 (bass), Bb37 (bass), C38 (bass), D38 (bass), E38 (bass), F38 (bass), G38 (bass), A38 (bass), Bb38 (bass), C39 (bass), D39 (bass), E39 (bass), F39 (bass), G39 (bass), A39 (bass), Bb39 (bass), C40 (bass), D40 (bass), E40 (bass), F40 (bass), G40 (bass), A40 (bass), Bb40 (bass), C41 (bass), D41 (bass), E41 (bass), F41 (bass), G41 (bass), A41 (bass), Bb41 (bass), C42 (bass), D42 (bass), E42 (bass), F42 (bass), G42 (bass), A42 (bass), Bb42 (bass), C43 (bass), D43 (bass), E43 (bass), F43 (bass), G43 (bass), A43 (bass), Bb43 (bass), C44 (bass), D44 (bass), E44 (bass), F44 (bass), G44 (bass), A44 (bass), Bb44 (bass), C45 (bass), D45 (bass), E45 (bass), F45 (bass), G45 (bass), A45 (bass), Bb45 (bass), C46 (bass), D46 (bass), E46 (bass), F46 (bass), G46 (bass), A46 (bass), Bb46 (bass), C47 (bass), D47 (bass), E47 (bass), F47 (bass), G47 (bass), A47 (bass), Bb47 (bass), C48 (bass), D48 (bass), E48 (bass), F48 (bass), G48 (bass), A48 (bass), Bb48 (bass), C49 (bass), D49 (bass), E49 (bass), F49 (bass), G49 (bass), A49 (bass), Bb49 (bass), C50 (bass), D50 (bass), E50 (bass), F50 (bass), G50 (bass), A50 (bass), Bb50 (bass), C51 (bass), D51 (bass), E51 (bass), F51 (bass), G51 (bass), A51 (bass), Bb51 (bass), C52 (bass), D52 (bass), E52 (bass), F52 (bass), G52 (bass), A52 (bass), Bb52 (bass), C53 (bass), D53 (bass), E53 (bass), F53 (bass), G53 (bass), A53 (bass), Bb53 (bass), C54 (bass), D54 (bass), E54 (bass), F54 (bass), G54 (bass), A54 (bass), Bb54 (bass), C55 (bass), D55 (bass), E55 (bass), F55 (bass), G55 (bass), A55 (bass), Bb55 (bass), C56 (bass), D56 (bass), E56 (bass), F56 (bass), G56 (bass), A56 (bass), Bb56 (bass), C57 (bass), D57 (bass), E57 (bass), F57 (bass), G57 (bass), A57 (bass), Bb57 (bass), C58 (bass), D58 (bass), E58 (bass), F58 (bass), G58 (bass), A58 (bass), Bb58 (bass), C59 (bass), D59 (bass), E59 (bass), F59 (bass), G59 (bass), A59 (bass), Bb59 (bass), C60 (bass), D60 (bass), E60 (bass), F60 (bass), G60 (bass), A60 (bass), Bb60 (bass), C61 (bass), D61 (bass), E61 (bass), F61 (bass), G61 (bass), A61 (bass), Bb61 (bass), C62 (bass), D62 (bass), E62 (bass), F62 (bass), G62 (bass), A62 (bass), Bb62 (bass), C63 (bass), D63 (bass), E63 (bass), F63 (bass), G63 (bass), A63 (bass), Bb63 (bass), C64 (bass), D64 (bass), E64 (bass), F64 (bass), G64 (bass), A64 (bass), Bb64 (bass), C65 (bass), D65 (bass), E65 (bass), F65 (bass), G65 (bass), A65 (bass), Bb65 (bass), C66 (bass), D66 (bass), E66 (bass), F66 (bass), G66 (bass), A66 (bass), Bb66 (bass), C67 (bass), D67 (bass), E67 (bass), F67 (bass), G67 (bass), A67 (bass), Bb67 (bass), C68 (bass), D68 (bass), E68 (bass), F68 (bass), G68 (bass), A68 (bass), Bb68 (bass), C69 (bass), D69 (bass), E69 (bass), F69 (bass), G69 (bass), A69 (bass), Bb69 (bass), C70 (bass), D70 (bass), E70 (bass), F70 (bass), G70 (bass), A70 (bass), Bb70 (bass), C71 (bass), D71 (bass), E71 (bass), F71 (bass), G71 (bass), A71 (bass), Bb71 (bass), C72 (bass), D72 (bass), E72 (bass), F72 (bass), G72 (bass), A72 (bass), Bb72 (bass), C73 (bass), D73 (bass), E73 (bass), F73 (bass), G73 (bass), A73 (bass), Bb73 (bass), C74 (bass), D74 (bass), E74 (bass), F74 (bass), G74 (bass), A74 (bass), Bb74 (bass), C75 (bass), D75 (bass), E75 (bass), F75 (bass), G75 (bass), A75 (bass), Bb75 (bass), C76 (bass), D76 (bass), E76 (bass), F76 (bass), G76 (bass), A76 (bass), Bb76 (bass), C77 (bass), D77 (bass), E77 (bass), F77 (bass), G77 (bass), A77 (bass), Bb77 (bass), C78 (bass), D78 (bass), E78 (bass), F78 (bass), G78 (bass), A78 (bass), Bb78 (bass), C79 (bass), D79 (bass), E79 (bass), F79 (bass), G79 (bass), A79 (bass), Bb79 (bass), C80 (bass), D80 (bass), E80 (bass), F80 (bass), G80 (bass), A80 (bass), Bb80 (bass), C81 (bass), D81 (bass), E81 (bass), F81 (bass), G81 (bass), A81 (bass), Bb81 (bass), C82 (bass), D82 (bass), E82 (bass), F82 (bass), G82 (bass), A82 (bass), Bb82 (bass), C83 (bass), D83 (bass), E83 (bass), F83 (bass), G8

The first system of the musical score for 'The Rose Tree' consists of two staves. The top staff is in treble clef and the bottom staff is in bass clef. The key signature has one flat (B-flat). The melody is written in the treble staff, starting with a quarter note G4, followed by a quarter note A4, a quarter note B-flat4, and a quarter note G4. The bass staff provides a simple accompaniment with quarter notes. The system is numbered 7 in the top left corner.

10

13

