Something with music

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Abstract

The goal of the project is to compare, experiment and proove some mathematical algorithms for creating music with random numbers. The algorithms are based on analyzing some relations from given note notations of exsisting melodies.

1 Introduction

1.1 Frequency relations (tuning)

The sound is type of mechanical vibration (waves) so it has the parameters of all the normal mechanical waves. However in the field of music we are most interested in the frequency of the wave, because it makes the variety of tones we hear. But not every sequence of frequencies sound pleasent to the human. So the pepole developed some algorithms to "syncronize" the sequence of tunes. Here are the most used of them:

1.1.1 Pythagorean tuning

The pythagorean tuning is the base of almost all tuning sysytems. It is based on the pythagorean fractions.

$$f = \frac{i+1}{i};\ i = 1, 2, ..., n$$

Figure 1: Pythagorean fractions

It is assumed that if the frequencies corelate like these numbers, we hear them in harmony and the music will sound pleasent (if we select appropriate sequence of these tones of courses).
If we pick a string with given length we know that the frequency of the produced sound depends on the length of the string. So to pick the "perfect" fraction we have to devide it like the pythagorean fractions.

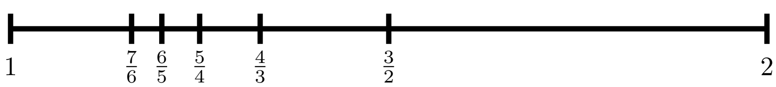

Figure 2: The "pefect" string separation

1.1.2 Equal temperament

Equal temperament is a tuning scale developed by Vincenzo Galilei. His idea is that the frequency interval between each pair of ajacent notes is the same. In equal temperament tunings, the generating interval is often found by dividing some larger desired interval into a number of smaller equal steps (equal frequency ratios between neighbouring notes). The tones are devided in groups called octaves. The ratio between the frequencies of last and the first tone of the octave is two.

$$\frac{last\ tune}{first\ tune} = \frac{2}{1}$$

In the Western music, the most common used tunning system sice the 18th century has been the twelve-tone equal temperament(or 12-TET). The 12-TET devides the octave in 12 eqal parts, all of them eqal on logaritmic scale, with ratio $\sqrt[12]{2} = 2^{\frac{1}{12}} \approx 1.05946$. In modern times 12-TET is usually tunned relatively to the standart frequency of 440 Hz, called A440, meaning that the note A is tuned to 440 Hz and all others are derived by the logaritmic scale described above.
Equal temperament is an approximation of the pythagorean scale, which is more convinient for use.

Name	**12-TET**	**Pythagorean scale**
Unision (C)	$2^{\frac{0}{12}} = 1$	$\frac{1}{1} = 1$
Minor second ($C\sharp/D\flat$)	$2^{\frac{1}{12}} \approx 1.05946$	$\frac{16}{15} \approx 1.06666$
Major second (D)	$2^{\frac{2}{12}} \approx 1.12246$	$\frac{9}{8} = 1.125$
Minor third ($D\sharp/E\flat$)	$2^{\frac{3}{12}} \approx 1.18920$	$\frac{6}{5} = 1.2$
Major third (E)	$2^{\frac{4}{12}} \approx 1.25992$	$\frac{5}{4} = 1.25$
Perfect fourth (F)	$2^{\frac{5}{12}} \approx 1.33484$	$\frac{4}{3} \approx 1.33333$
Tritone ($F\sharp/G\flat$)	$2^{\frac{6}{12}} \approx 1.41421$	$\frac{7}{5} = 1.4$*
Perfect fifth (G)	$2^{\frac{7}{12}} \approx 1.49830$	$\frac{3}{2} = 1.5$
Minor sixth ($G\sharp/A\flat$)	$2^{\frac{8}{12}} \approx 1.58740$	$\frac{8}{5} = 1.6$*
Major sixth (A)	$2^{\frac{9}{12}} \approx 1.68179$	$\frac{5}{3} \approx 1.66666$*
Minor seventh ($A\sharp/B\flat$)	$2^{\frac{10}{12}} \approx 1.78179$	$\frac{16}{9} \approx 1.77777$*
Major seventh (B)	$2^{\frac{11}{12}} \approx 1.88774$	$\frac{15}{8} = 1.875$*
Octave (C)	$2^{\frac{12}{12}} = 2$	$\frac{2}{1} = 2$

Note: the values with * can't be represented with decent accuracy but the human ear can't differentiate this (in the most cases).

Figure 3: Values of the tones in 12-TET and pythagorean scale

1.2 MIDI standart

The purpose of MIDI is to define a way compact representation of musical sheets, which makes it appropriate for disk storage.
The standart MIDI(SMID) file is devided into chunks. Each chunk has a type and length represented as ASCII and 32 bit number respectively. There are two types of chunks: "MThd"(e.g. Header chunk) and "MTrk"(e.g. Track chunk). There is always one header while there can be more than one track.

1.2.1 Header chunk

The header chunk contains data for the whole file. It has fixed length of 6 bytes. The first 2 bytes represent the format of file:

- 0 $\rightarrow$ One track
- 1 $\rightarrow$ Multiple tracks but one song
- 2 $\rightarrow$ Multiple song

The second 2 bytes represent the number of tracks. For format 0 it is always 1. And the last two bytes represent how many ticks are in a quarter note.

1.2.2 Track chunks

Each track is composed of events. Each event has delta time - the time (in ticks) which passes after the previous event before executing this event. There are two types of events: MIDI events - the ones which represent the actual music and META - these which supply additional info like titles, lyrics, etc.
Each MIDI event that has to play note uses this standart for encoding the frequency of the played tune:

$$f = 440 * 2^{\frac{d-69}{12}} \ where\ d\ is\ the\ note\ number\ and\ f\ is\ the\ frequency$$

Therefore the note number is:

$$d = 69 + 12 \log_2 \frac{f}{440}$$

1.3 Markov chains

Markov chain is a stochastic model which describes a sequence of events in which the probability of each event depends entierly on the previous event (in some cases the previous n but the table which decribes the chain size becomes big quickly because it equals to n^2).
Markov chains are represented as table in the most cases. Each cell of the table hold the probability of triggering an event from the row/collumn after the event in the collumn/row respectively.

1.4 Used algorithms

	Next event		
	A	B	C
A	20%	60%	20%
B	40%	20%	40%
C	10%	20%	70%

		Next event		
		A	B	C
A	A	20%	60%	20%
A	B	40%	20%	40%
A	C	10%	20%	70%
B	A	30%	50%	20%
B	B	35%	25%	40%
B	C	70%	14%	16%
C	A	90%	6%	4%
C	B	10%	10%	80%
C	C	7%	43%	50%

Figure 4: Example Markov chains

2 Future

Sonething interesting

3 Acknowledgments

Special thanks to:

- I still don't know

Thanks also to:

- I still don't know