



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix



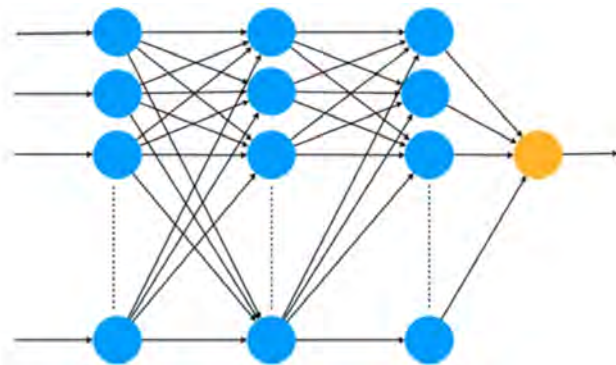
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# Solving System of Linear Equations

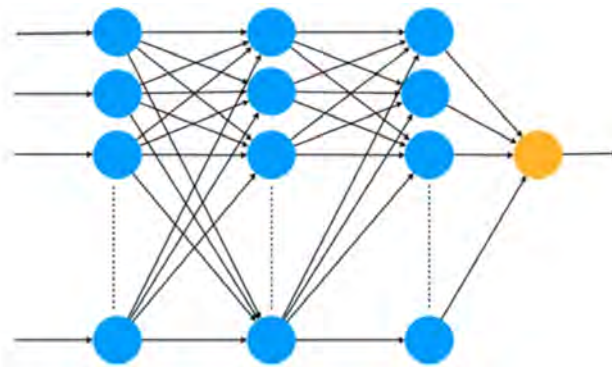
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## **Machine learning motivation**

# Neural networks - Matrix operations

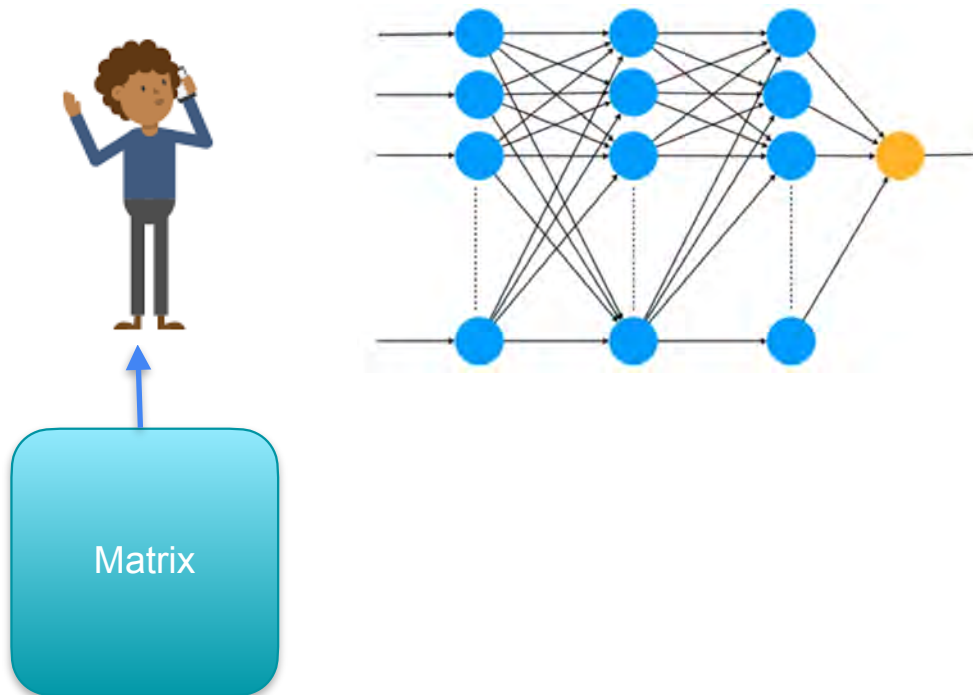


# Neural networks - Matrix operations



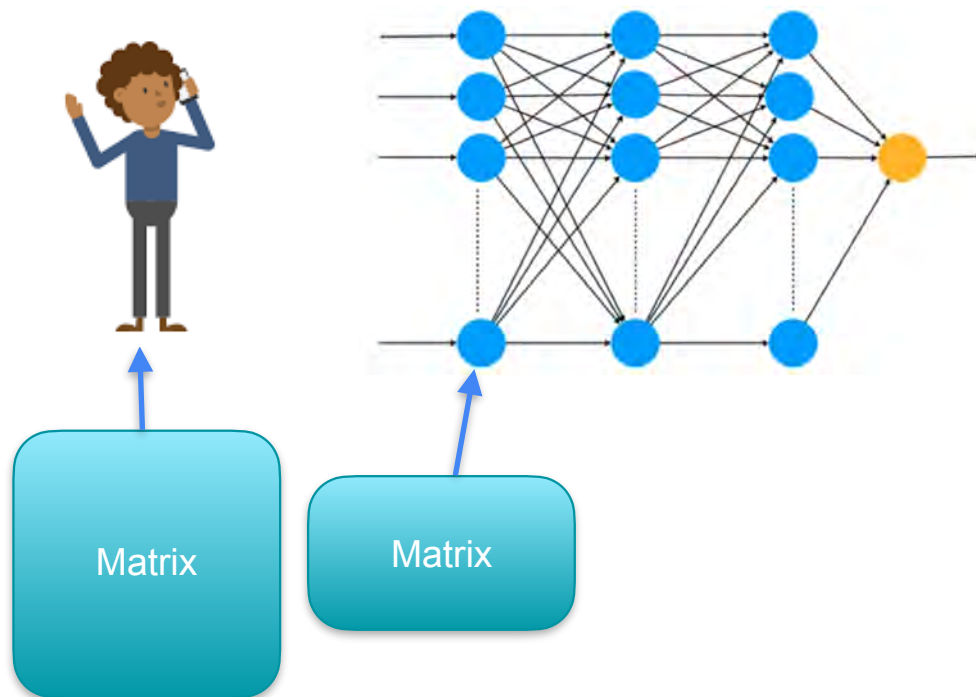
"Hello! Welcome to  
Math for Machine  
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# Neural networks - Matrix operations



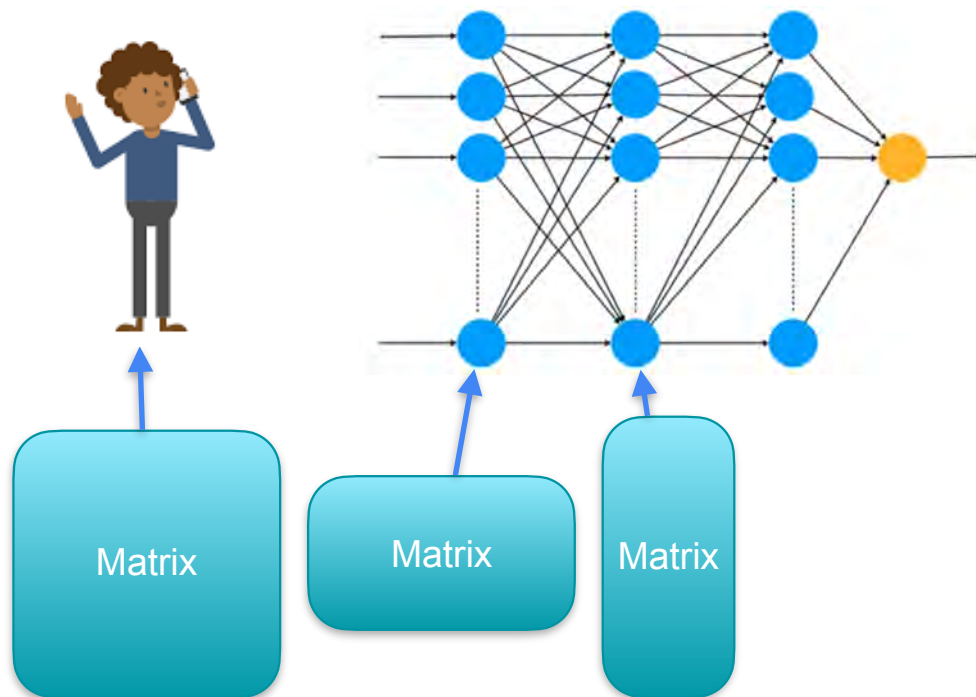
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# Neural networks - Matrix operations



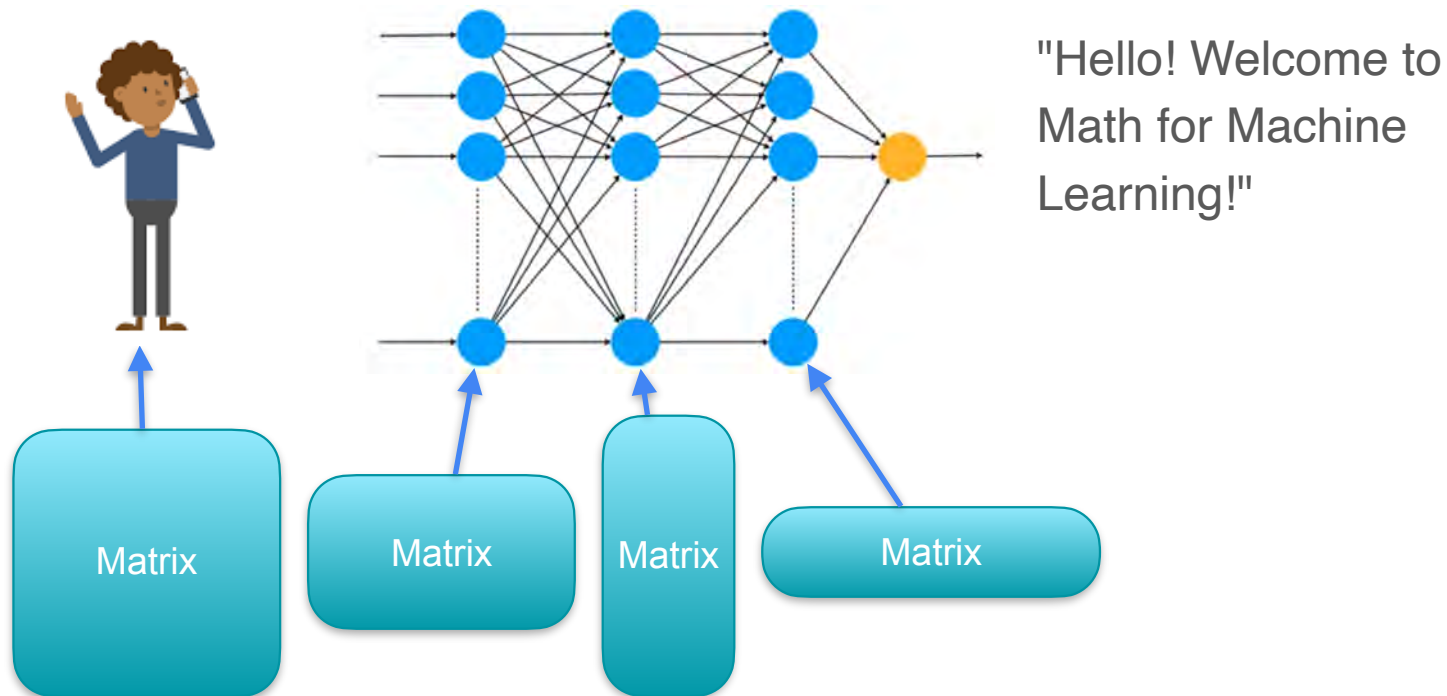
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# Neural networks - Matrix operations



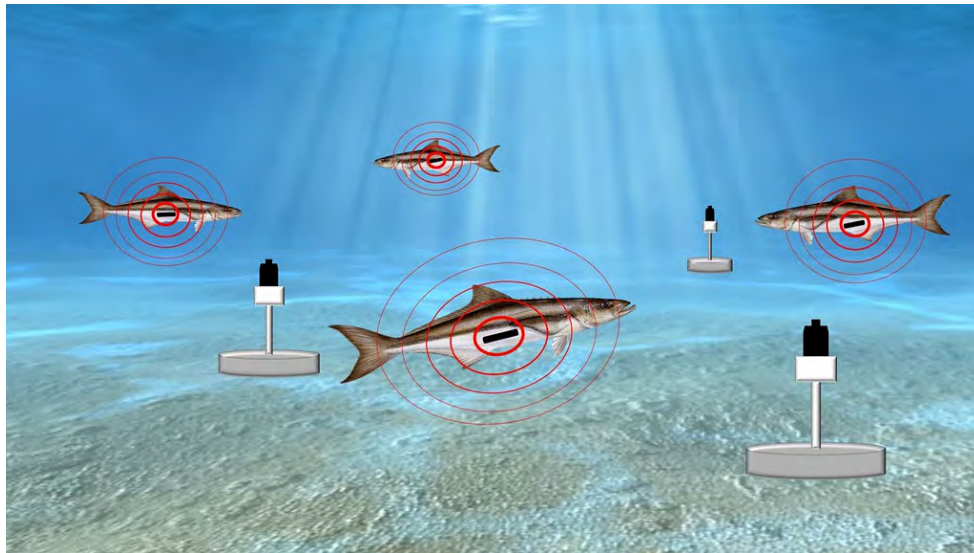
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# Neural networks - Matrix operations





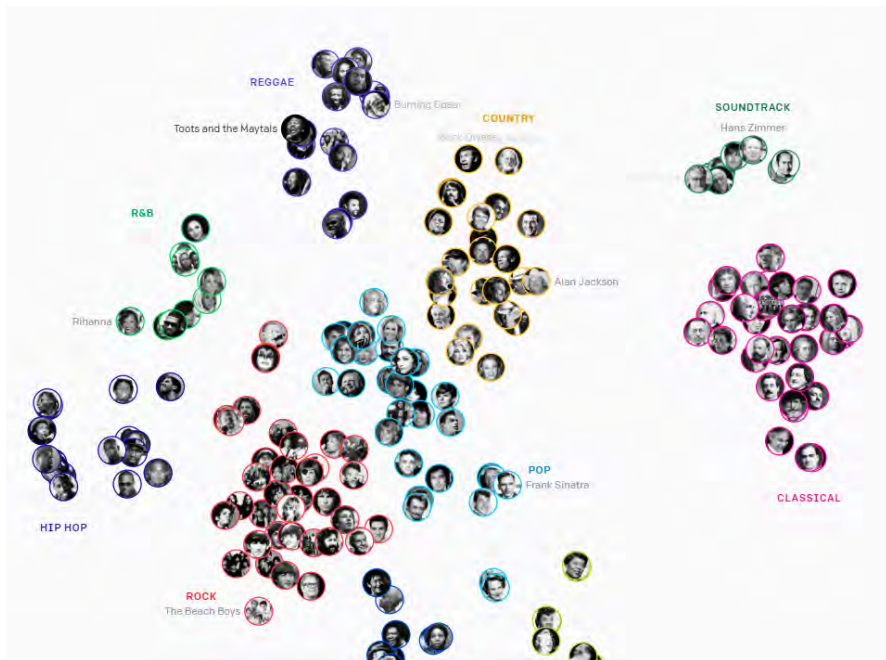
# Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

# Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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# Solving System of Linear Equations

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**Solving non-singular system  
of linear equations**

# Solving systems of equations



## System

- $a + b = 10$   
 

- $a + 2b = 12$   
 

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{🍏} + \text{🍌} = \$10$$

$$\text{🍏} + \text{🍌} + \text{🍌} = \$12$$

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$12$$

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$

# Solving systems of equations

## System


- $a + b = 10$   
 + 

- $a + 2b = 12$   
 + 

$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$

\$2





# Solving systems of equations

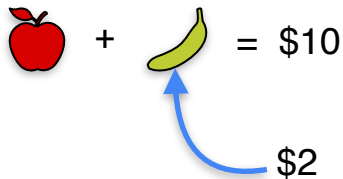
## System

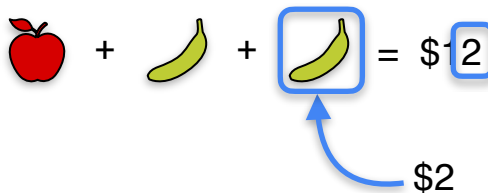
- $a + b = 10$



- $a + 2b = 12$





A diagram illustrating the first equation. It shows a red apple icon, a plus sign, a yellow banana icon, an equals sign, and the text '\$10'. A blue curved arrow points from the banana icon to the text '\$2' below it.





A diagram illustrating the second equation after substitution. It shows a red apple icon, a plus sign, a yellow banana icon, a plus sign, a yellow banana icon enclosed in a blue square box, an equals sign, and the text '\$10' followed by a blue square box containing the number '2'. A blue curved arrow points from the boxed banana icon to the text '\$2' below it.






# Solving systems of equations

## System

- $a + b = 10$   
 

- $a + 2b = 12$   
  

 +  = \$10  
 \$8  \$2

 +  +  = \$   
 \$2

# Solving systems of equations

# Solving systems of equations

## System

- $a + b = 10$







- $a + 2b = 12$





# Solving systems of equations

**System**





- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

- $a = 8$   

- $b = 2$   




# Solving systems of equations

**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 





Some process

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations



**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Some process





Manipulating equations

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations

**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Some process**



**Manipulating equations**

**Swapping equations**

**Adding equations**

**Multiplying equations by a constant**





**Solved system**

- $a = 8$   

- $b = 2$   






# Solving systems of equations

**System**


- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations


**System**


•  $a + b = 10$   
 

•  $a + 2b = 12$   
 

Eliminate 'a' from this equation

**Solved system**

•  $a = 8$   


•  $b = 2$   


# Manipulating equations

# Manipulating equations

**Multiplying by a constant**

# Manipulating equations

**Multiplying by a constant**

$$a + b = 10$$

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\underline{x \qquad 7}$$

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

# Manipulating equations

**Multiplying by a constant**

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

**Adding two equations**



# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \quad 2a + 3b = 26 \\ \hline \end{array}$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \quad 2a + 3b = 26 \\ \hline 3a + 4b = 36 \end{array}$$

Let's do a harder example

# Systems of equations

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

## Solved system

- $a = ?$
- $b = ?$



# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$



Eliminate 'a'  
from this equation

## Solved system

- $a = ?$
- $b = ?$

# Systems of equations

## System

- $5a + b = 17$

- $4a - 3b = 6$

## Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

## Solved system

- $a = ?$

- $b = ?$

↑  
Eliminate 'a'  
from this equation

# Systems of equations

## System

- $5a + b = 17$

- $4a - 3b = 6$

## Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

## Solved system

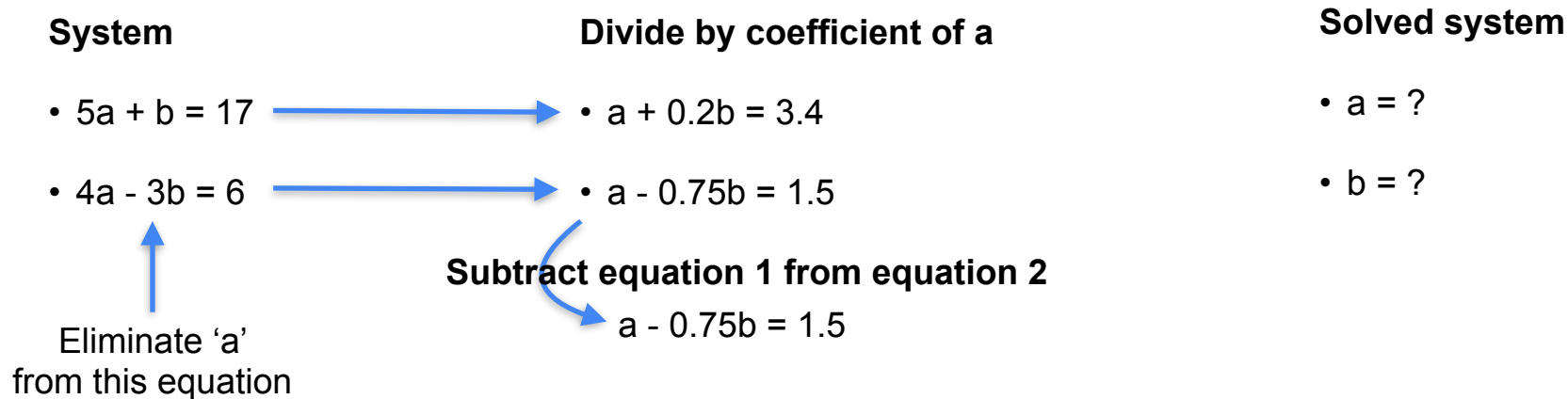
- $a = ?$

- $b = ?$

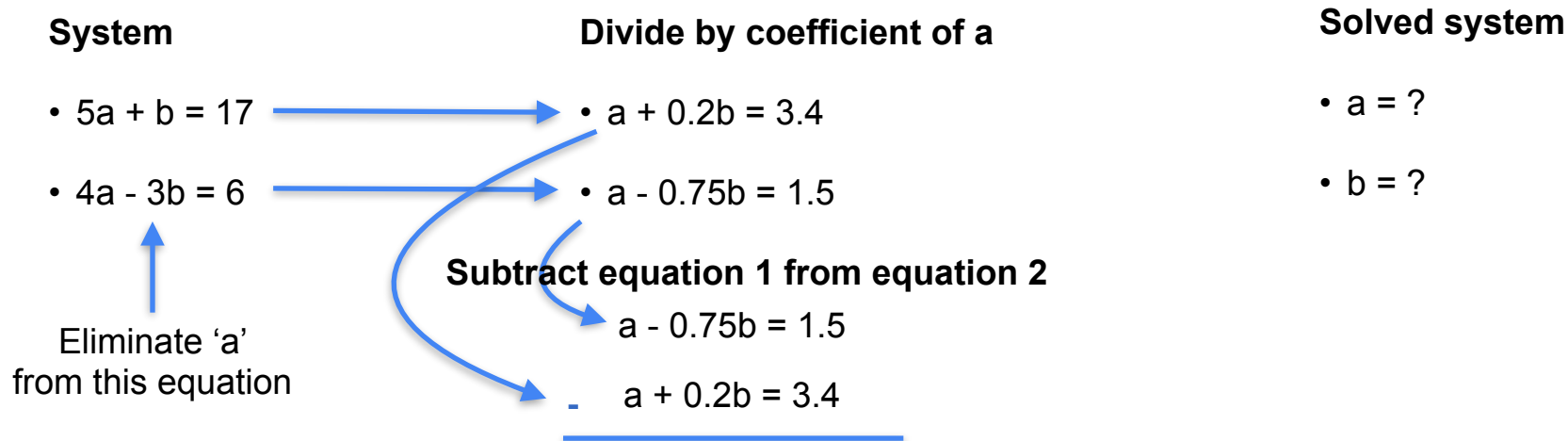
↑  
Eliminate 'a'  
from this equation

Subtract equation 1 from equation 2

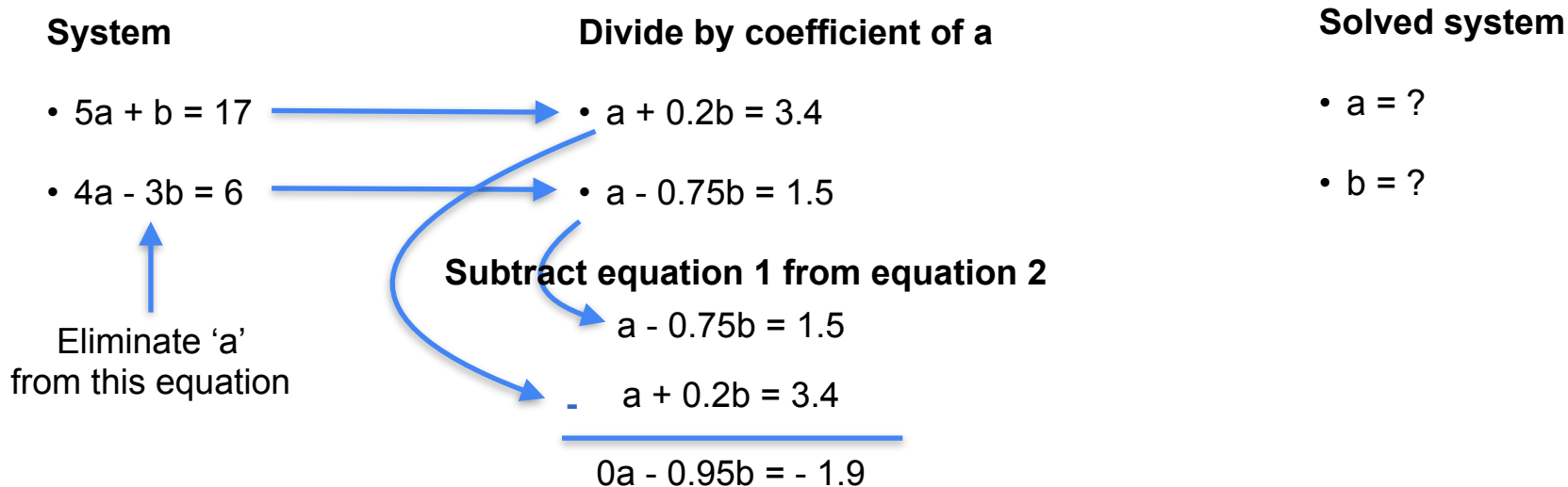
# Systems of equations



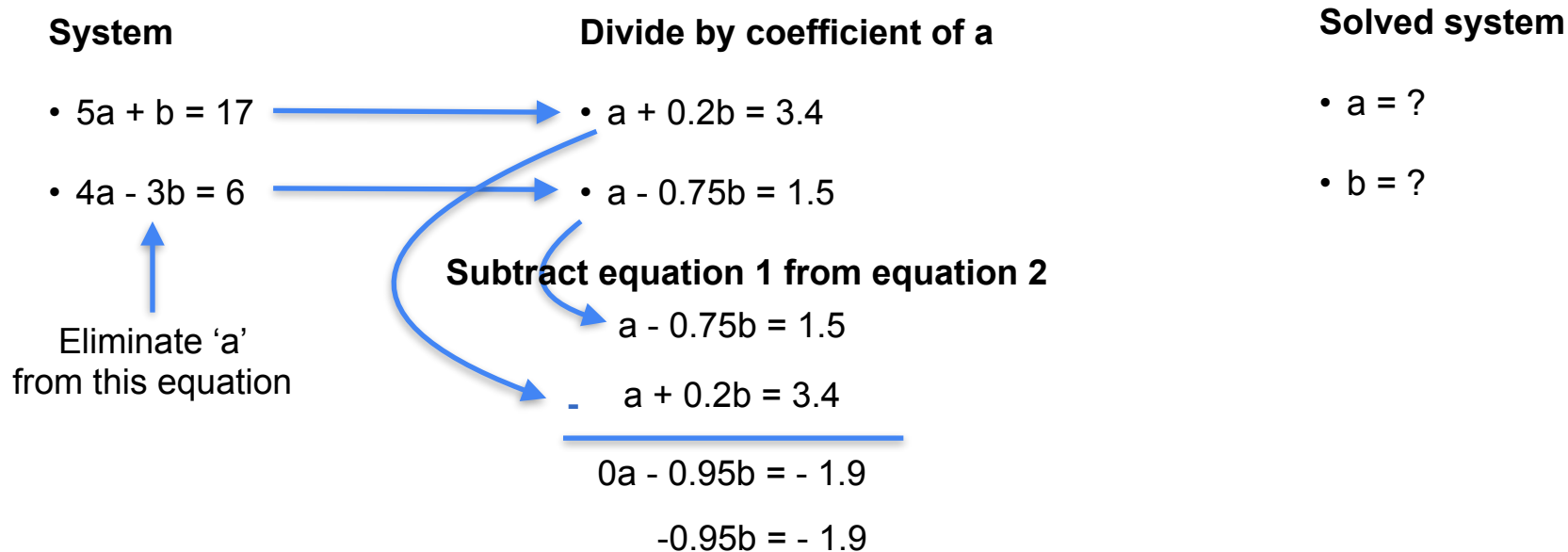
# Systems of equations



# Systems of equations



# Systems of equations

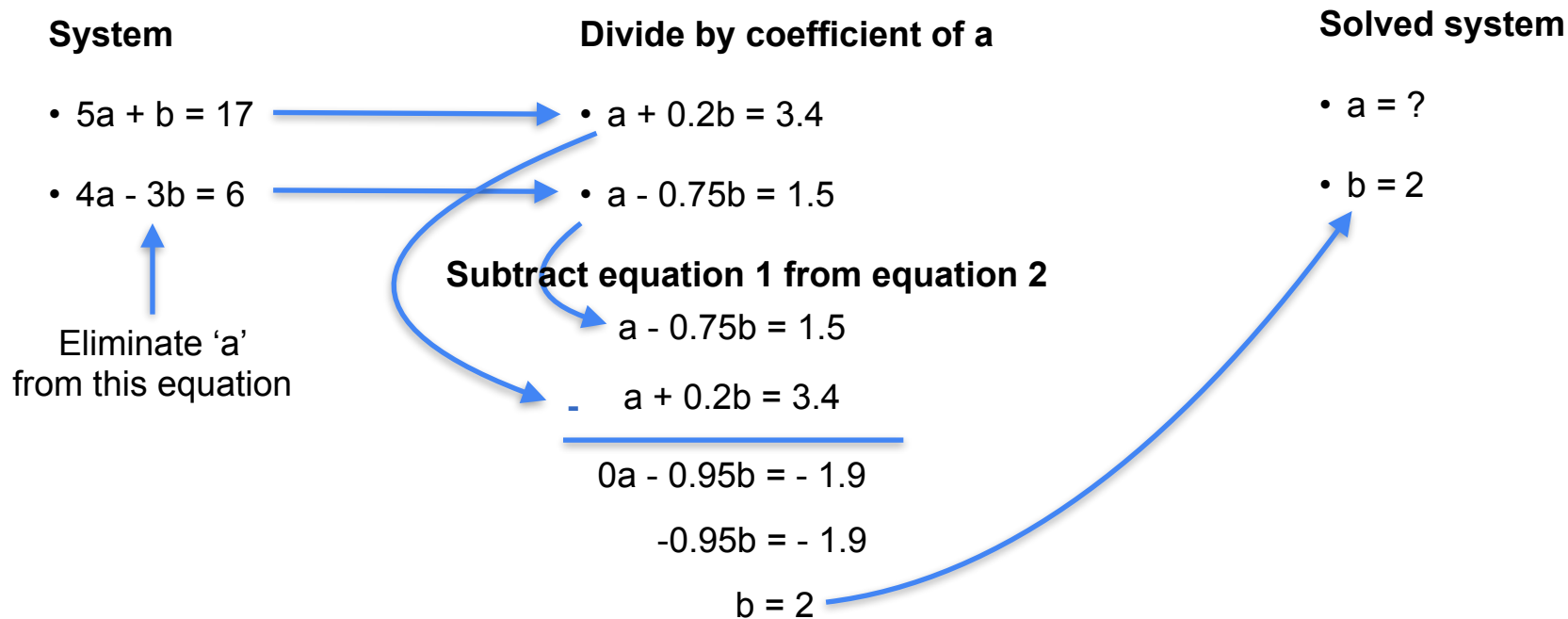


# Systems of equations

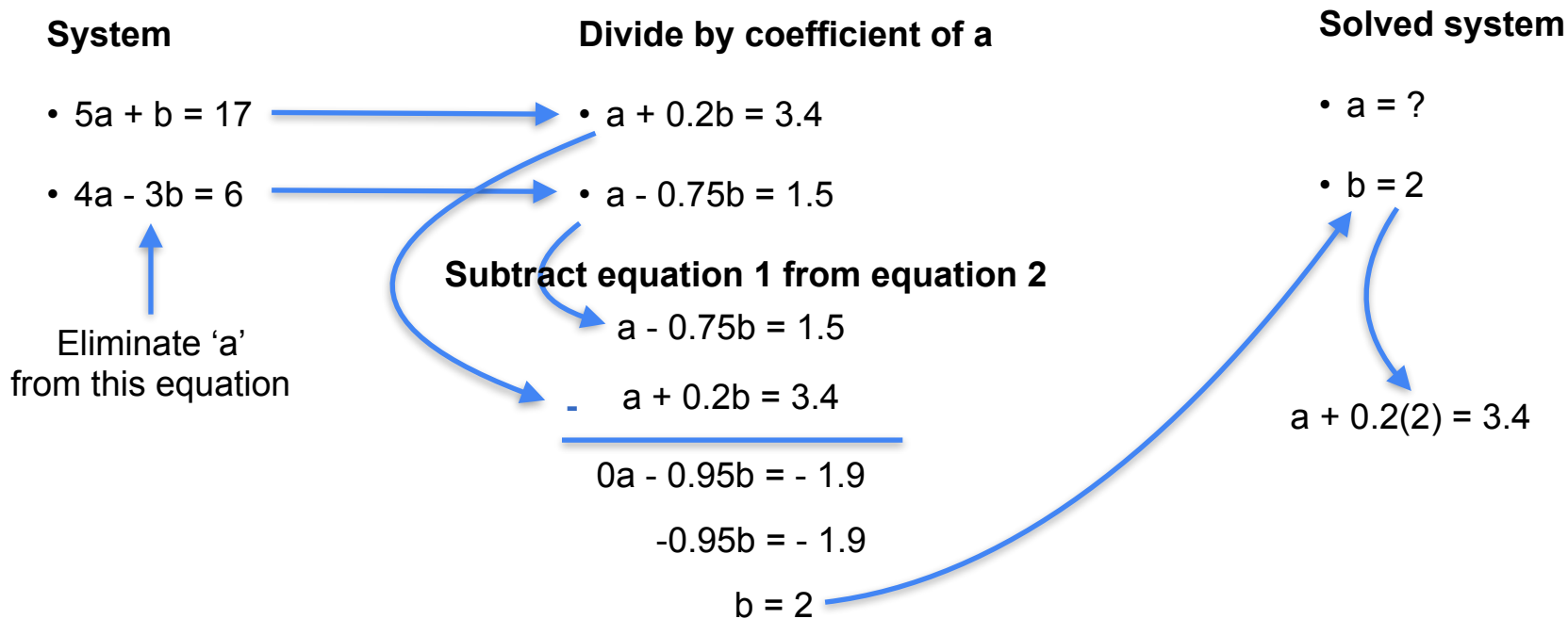
System	Divide by coefficient of a	Solved system
<ul style="list-style-type: none"><li>• <math>5a + b = 17</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a + 0.2b = 3.4</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a = ?</math></li></ul>
<ul style="list-style-type: none"><li>• <math>4a - 3b = 6</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a - 0.75b = 1.5</math></li></ul>	<ul style="list-style-type: none"><li>• <math>b = ?</math></li></ul>
<p>↑ Eliminate 'a' from this equation</p>	<p><b>Subtract equation 1 from equation 2</b></p> $\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline 0a - 0.95b = -1.9 \\ -0.95b = -1.9 \\ b = 2 \end{array}$	



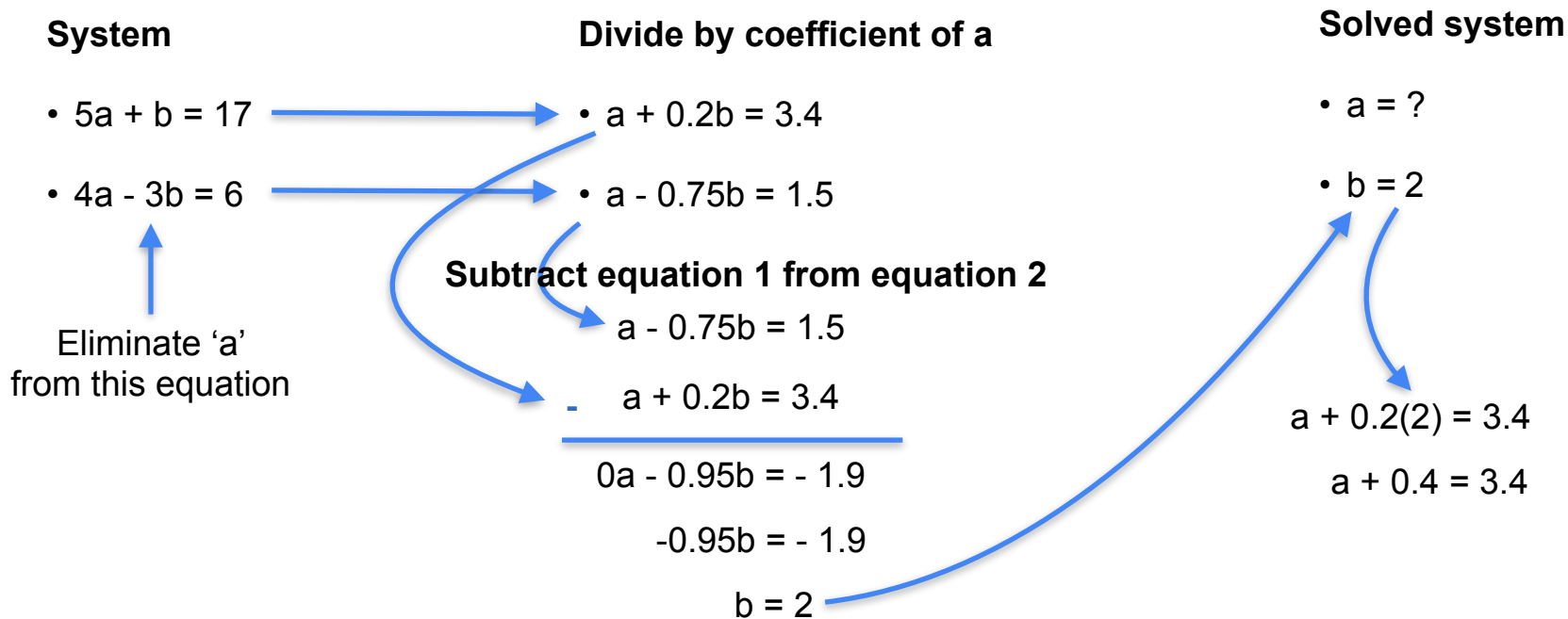
# Systems of equations



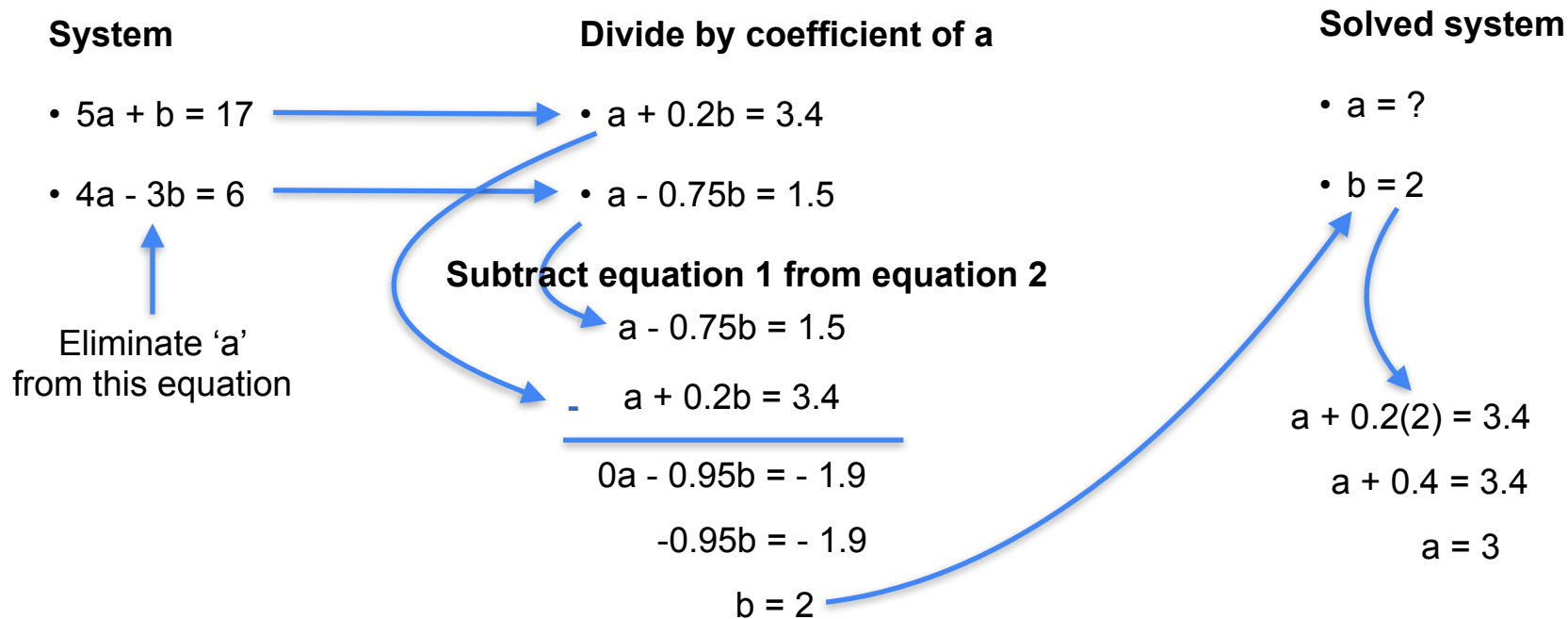
# Systems of equations



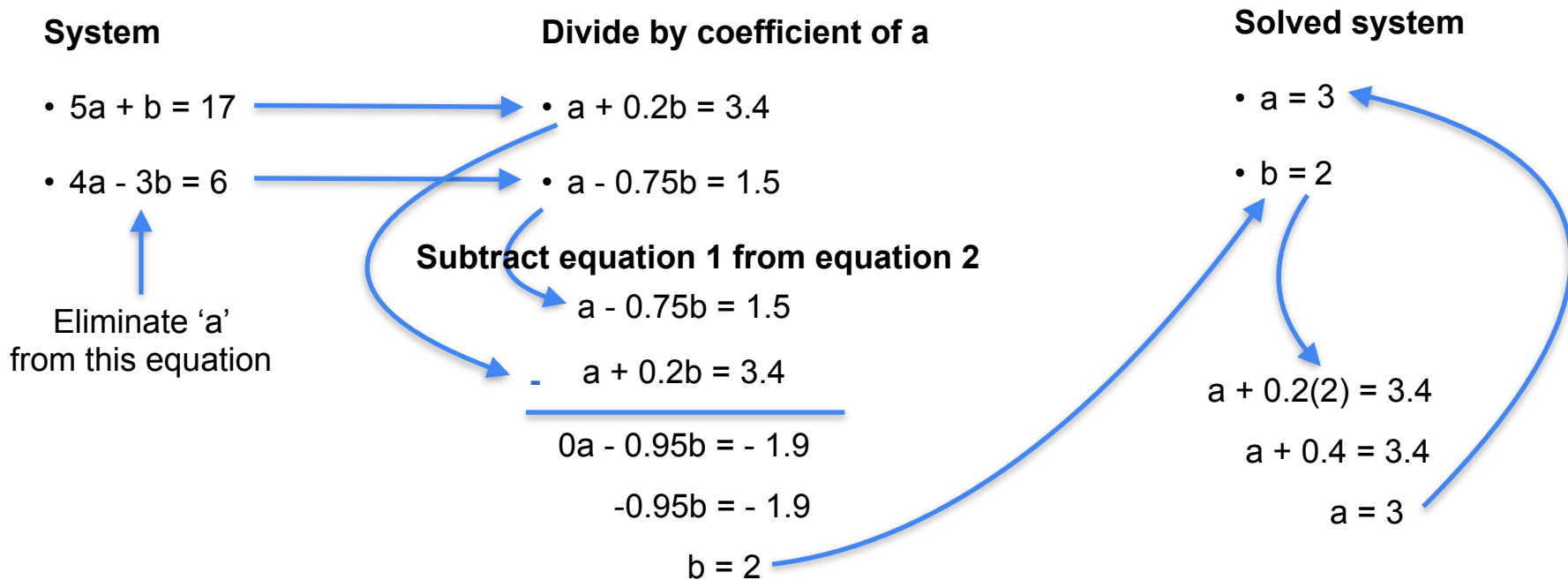
# Systems of equations



# Systems of equations



# Systems of equations



# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$
- $3b = 6$

## Solved system

- $a = ?$
- $b = ?$

# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$

- $3b = 6$





Eliminate 'a'  
from this equation


## Solved system

- $a = ?$

- $b = ?$




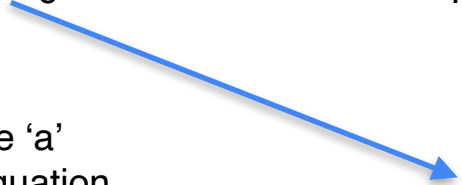
# What if one of the coefficients of a is zero?

System		Divide by coefficient of a	Solved system
• $5a + b = 17$		• $a + 0.2b = 3.4$	• $a = ?$
• $3b = 6$		• ???	• $b = ?$

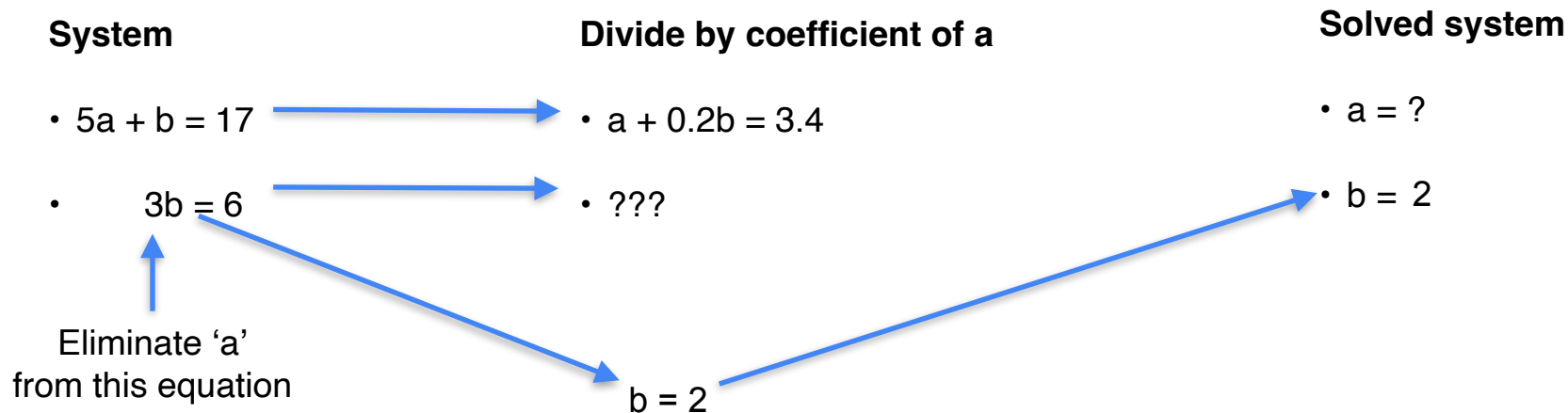
 Eliminate 'a' from this equation



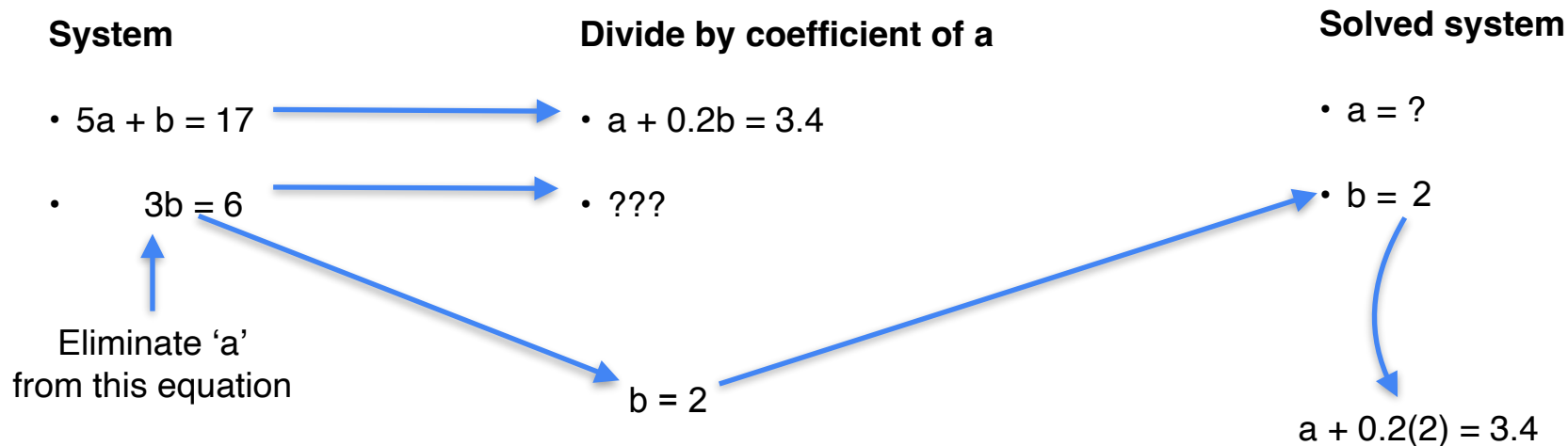
# What if one of the coefficients of a is zero?

System	Divide by coefficient of a	Solved system
<ul style="list-style-type: none"><li>• <math>5a + b = 17</math></li></ul>	 <ul style="list-style-type: none"><li>• <math>a + 0.2b = 3.4</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a = ?</math></li></ul>
<ul style="list-style-type: none"><li>• <math>3b = 6</math></li></ul>	 <ul style="list-style-type: none"><li>• ???</li></ul>	<ul style="list-style-type: none"><li>• <math>b = ?</math></li></ul>
<p>Eliminate 'a' from this equation</p> 	 $b = 2$	

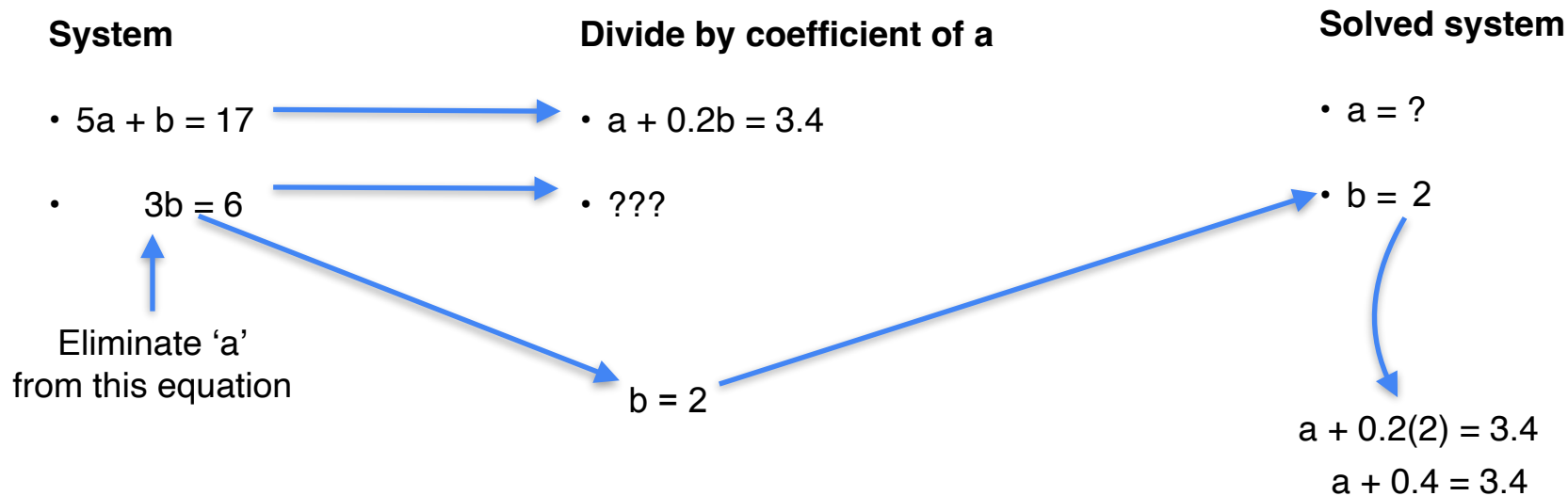
# What if one of the coefficients of a is zero?



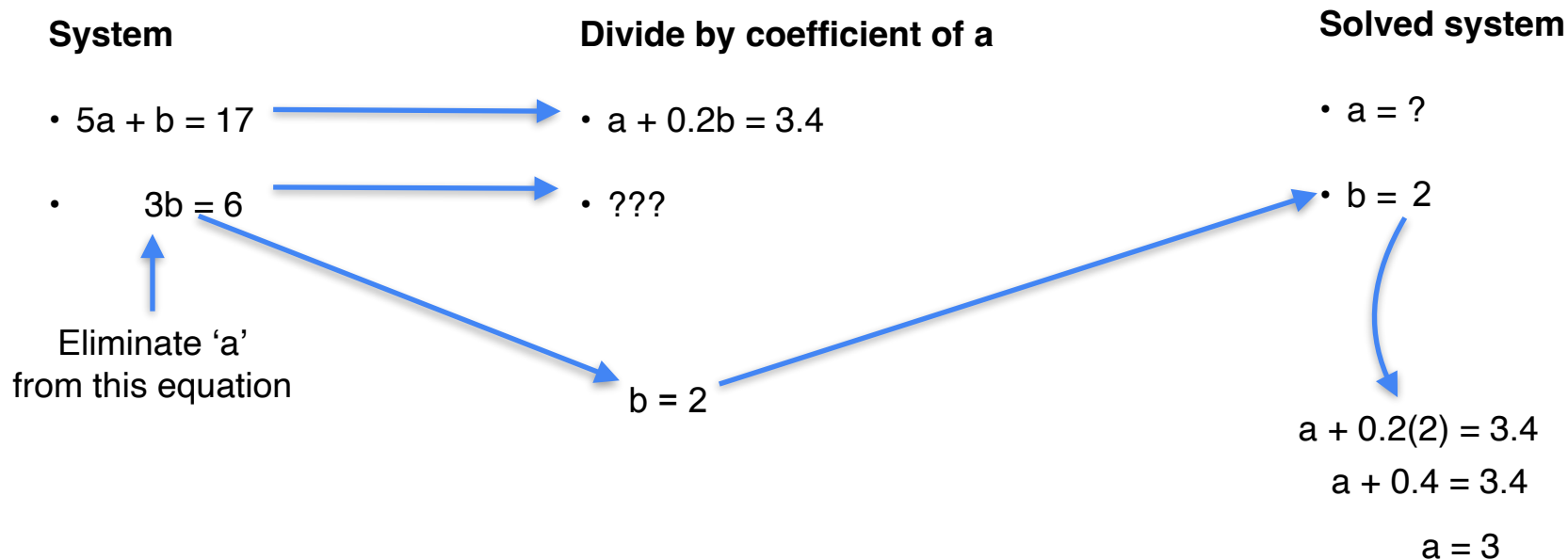
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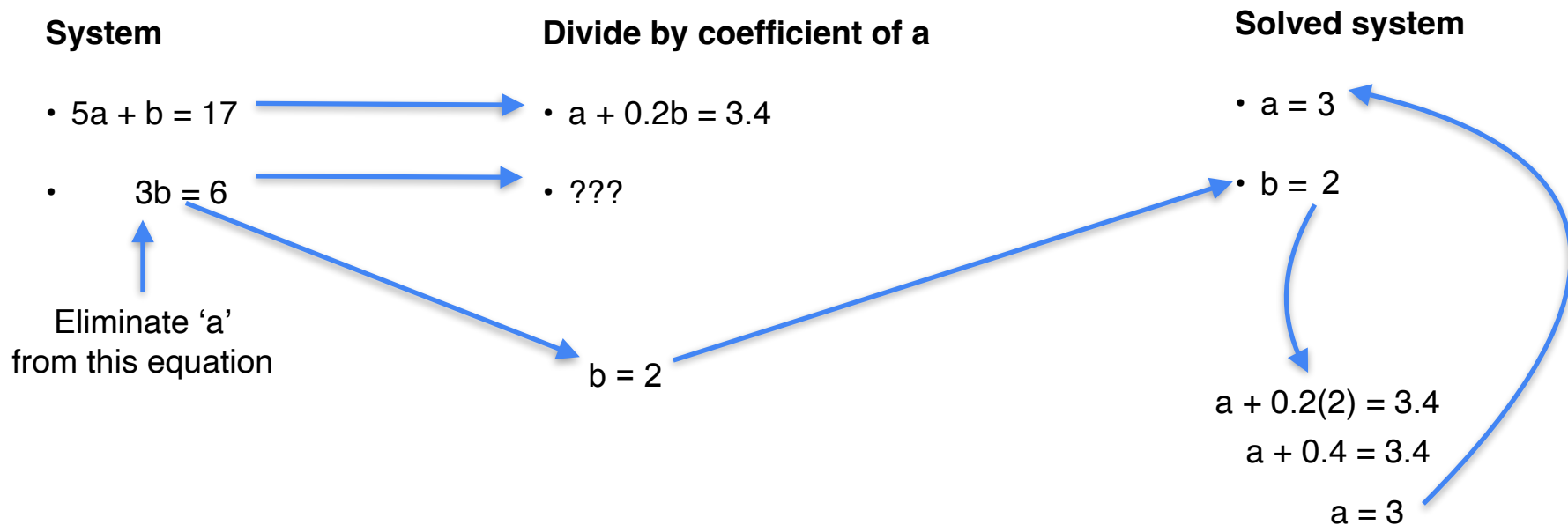
# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# Quiz

- Solve the following system of equations

## **System**

- $2a + 5b = 46$
- $8a + b = 32$

# Solution

- Solve the following system of equations

## **System**

- $2a + 5b = 46$
- $8a + b = 32$

## **Solution**

- $a = 3$
- $b = 8$





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# Solving System of Linear Equations

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**Solving singular system of  
linear equations**

# What if the system is singular (redundant)?



## System


- $a + b = 10$
- $2a + 2b = 20$

## Solved system

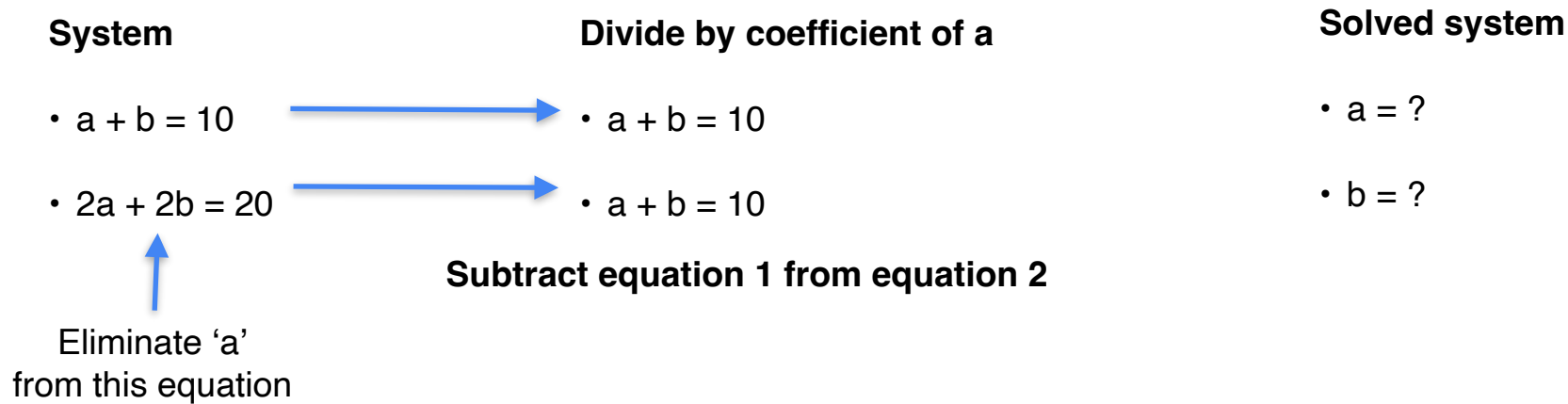
- $a = ?$
- $b = ?$

# What if the system is singular (redundant)?

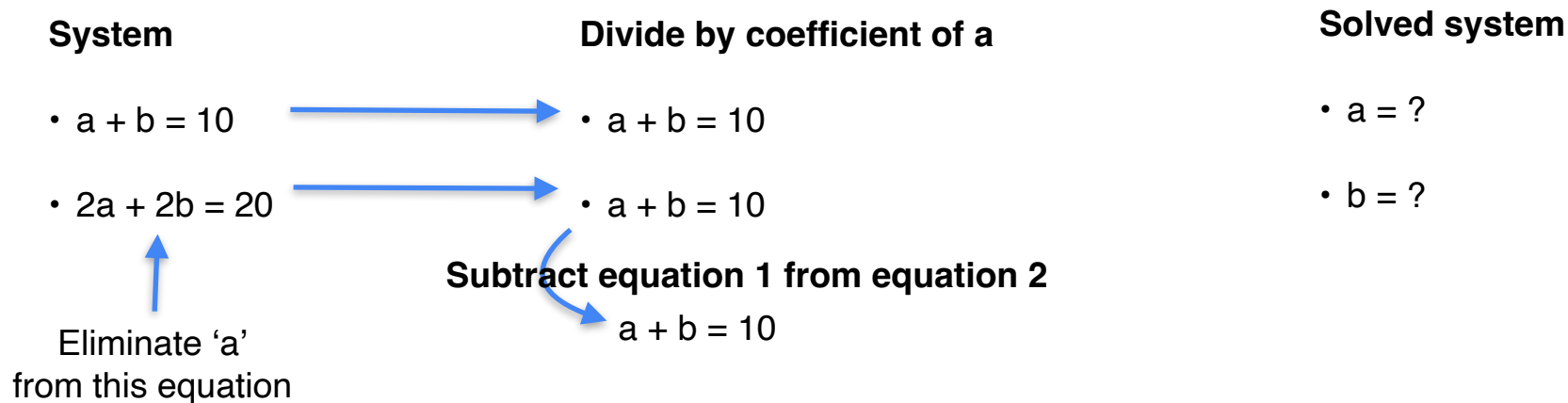
System		Divide by coefficient of a		Solved system
• $a + b = 10$		• $a + b = 10$		• $a = ?$
• $2a + 2b = 20$		• $a + b = 10$		• $b = ?$

 Eliminate 'a' from this equation

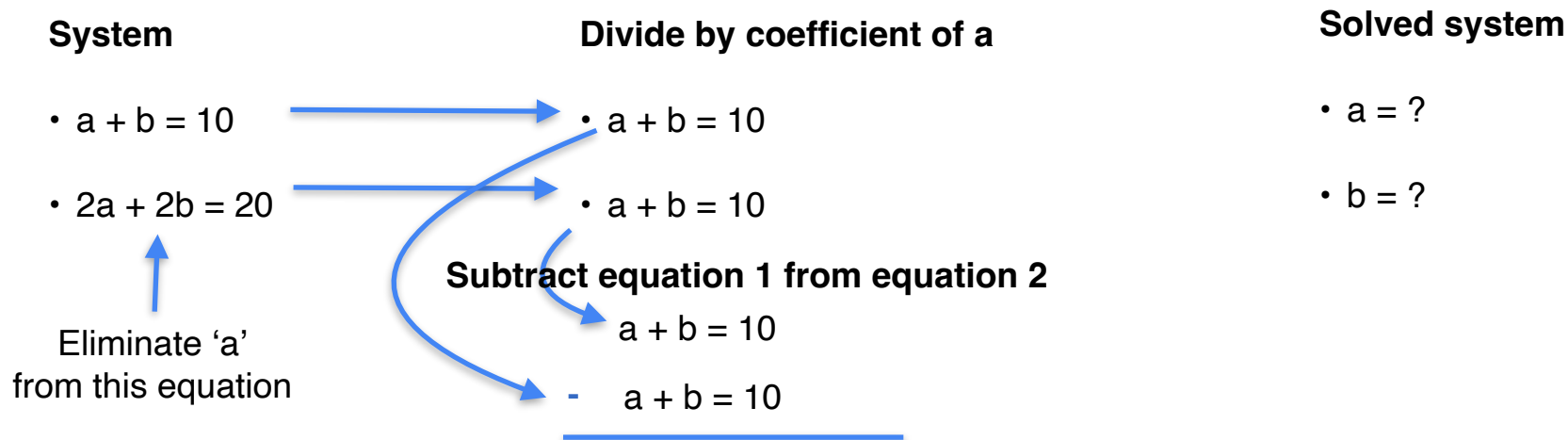
# What if the system is singular (redundant)?



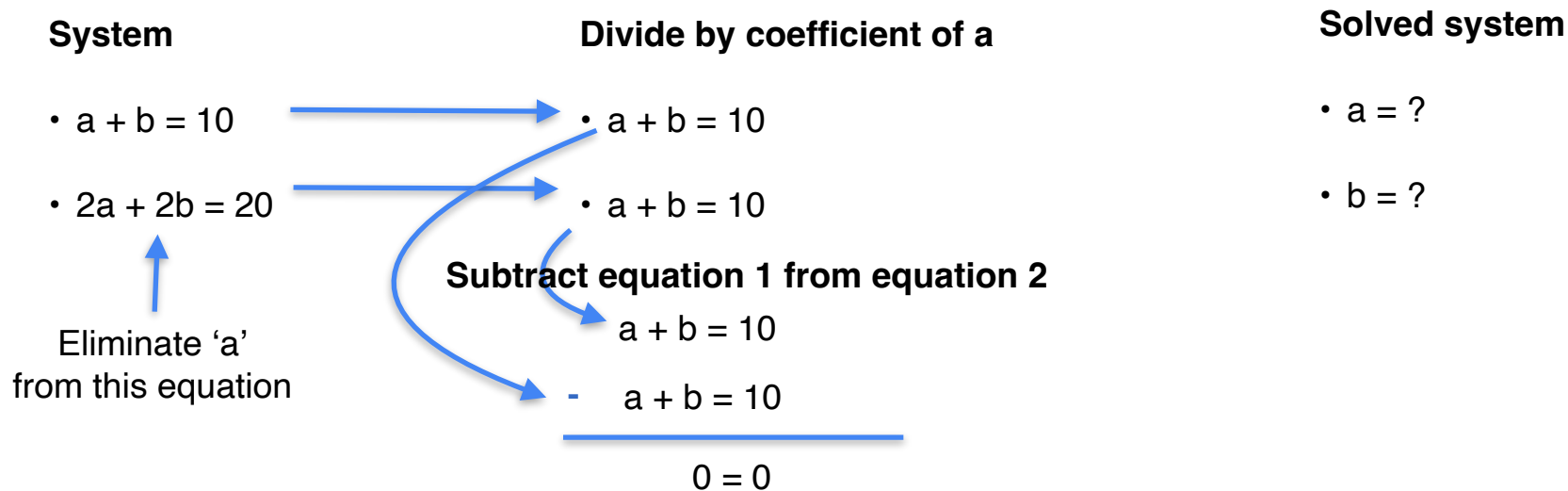
# What if the system is singular (redundant)?



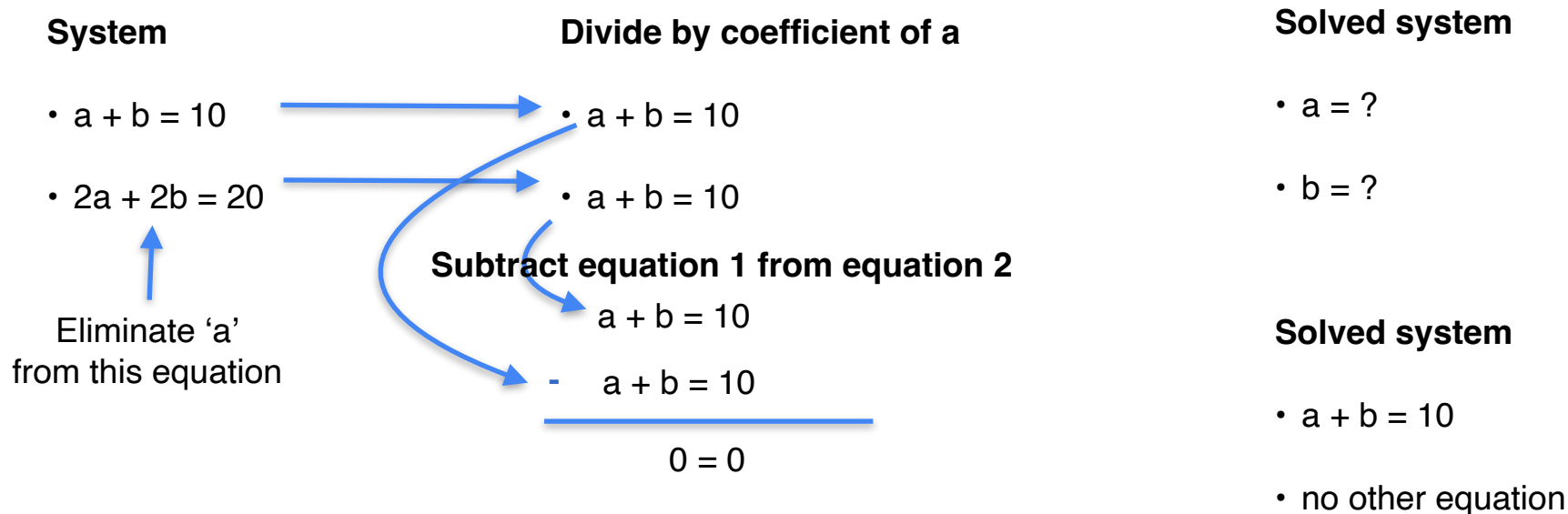
# What if the system is singular (redundant)?



# What if the system is singular (redundant)?

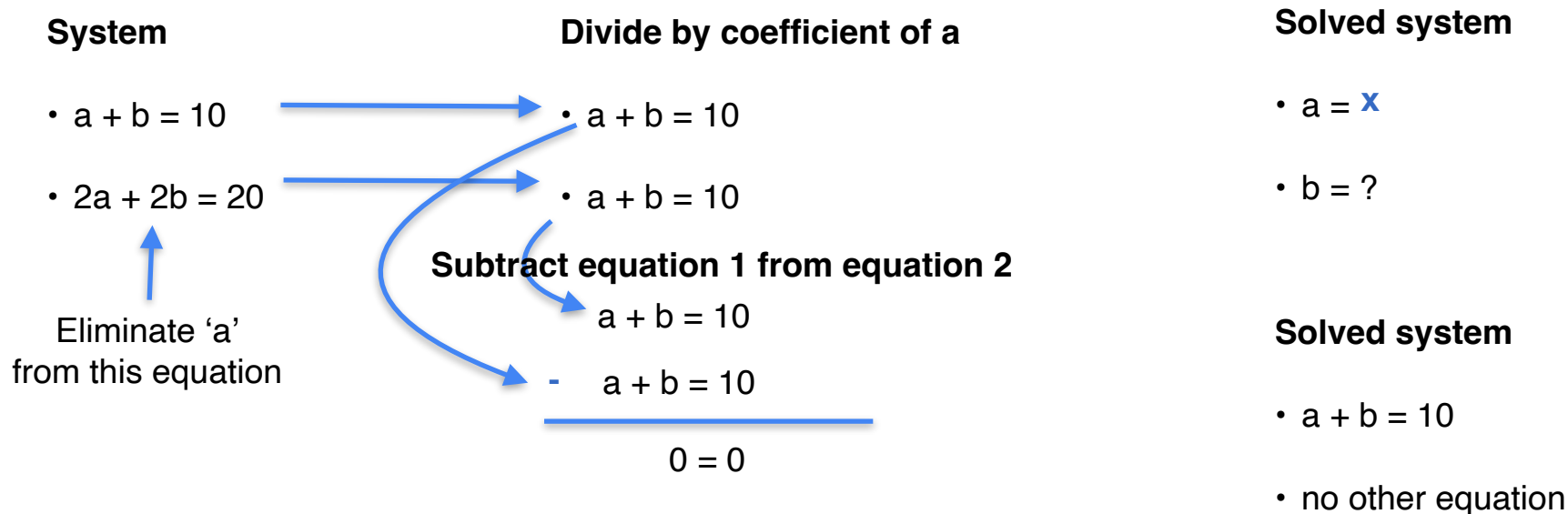


# What if the system is singular (redundant)?

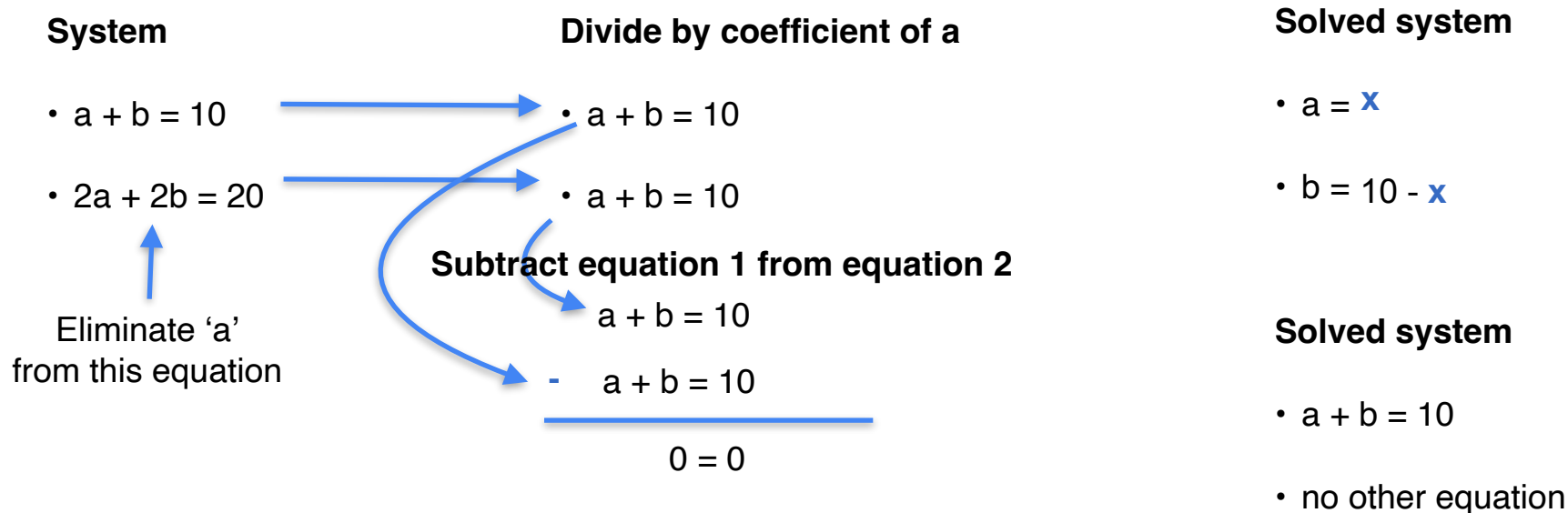




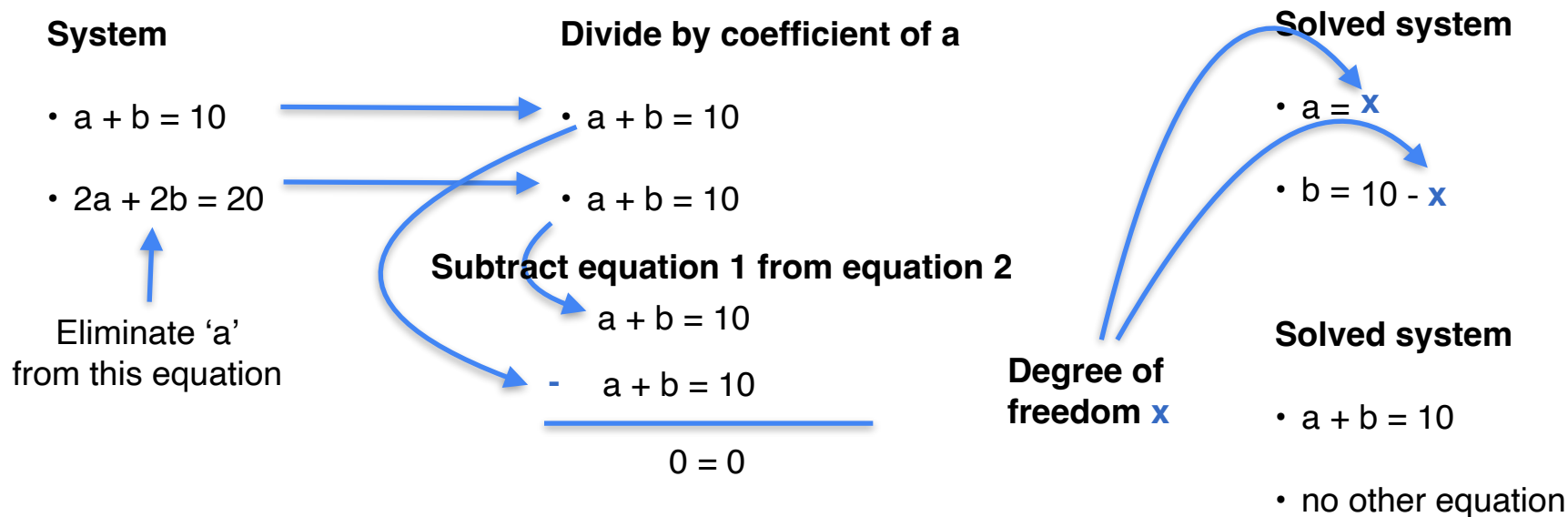
# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (contradictory)?

## System

- $a + b = 10$
- $2a + 2b = 24$

## Solved system

- $a = ?$
- $b = ?$

# What if the system is singular (contradictory)?

## System

- $a + b = 10$
- $2a + 2b = 24$






Eliminate 'a'  
from this equation

## Solved system

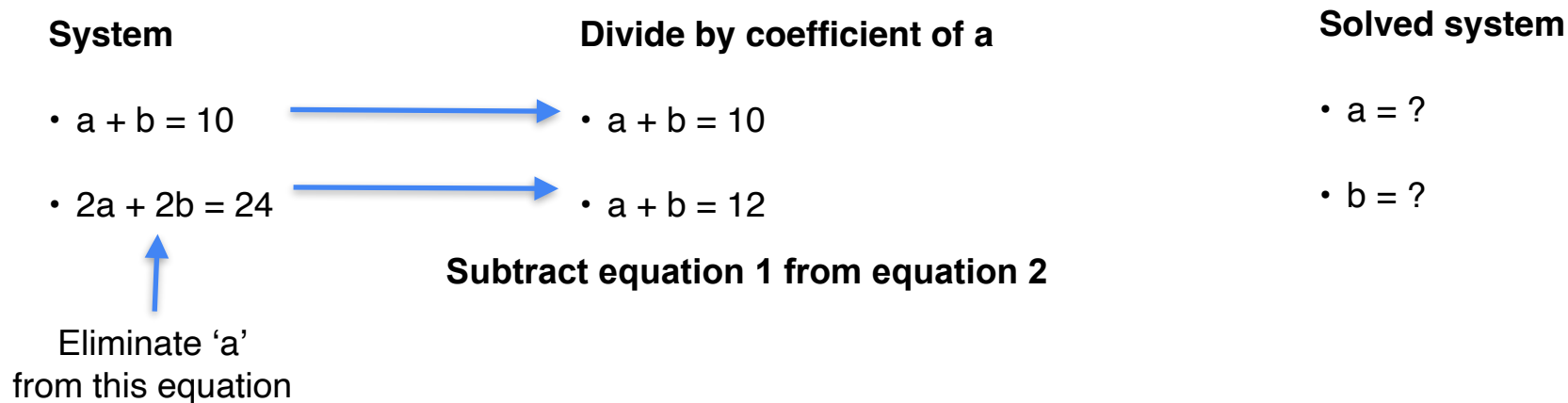
- $a = ?$
- $b = ?$

# What if the system is singular (contradictory)?

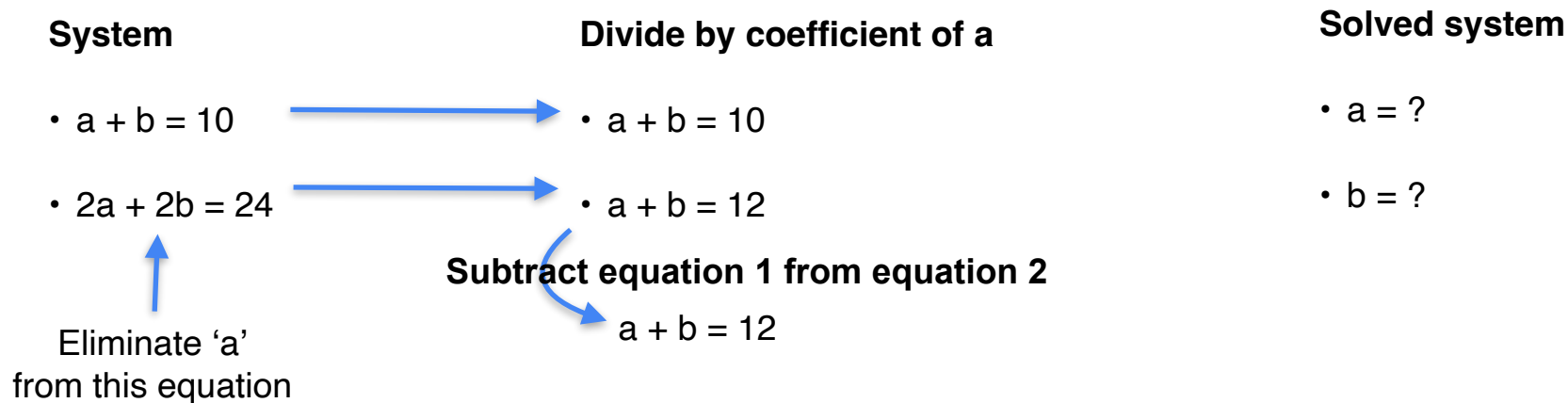
System		Divide by coefficient of a	Solved system
• $a + b = 10$		• $a + b = 10$	• $a = ?$
• $2a + 2b = 24$		• $a + b = 12$	• $b = ?$

 Eliminate 'a' from this equation

# What if the system is singular (contradictory)?

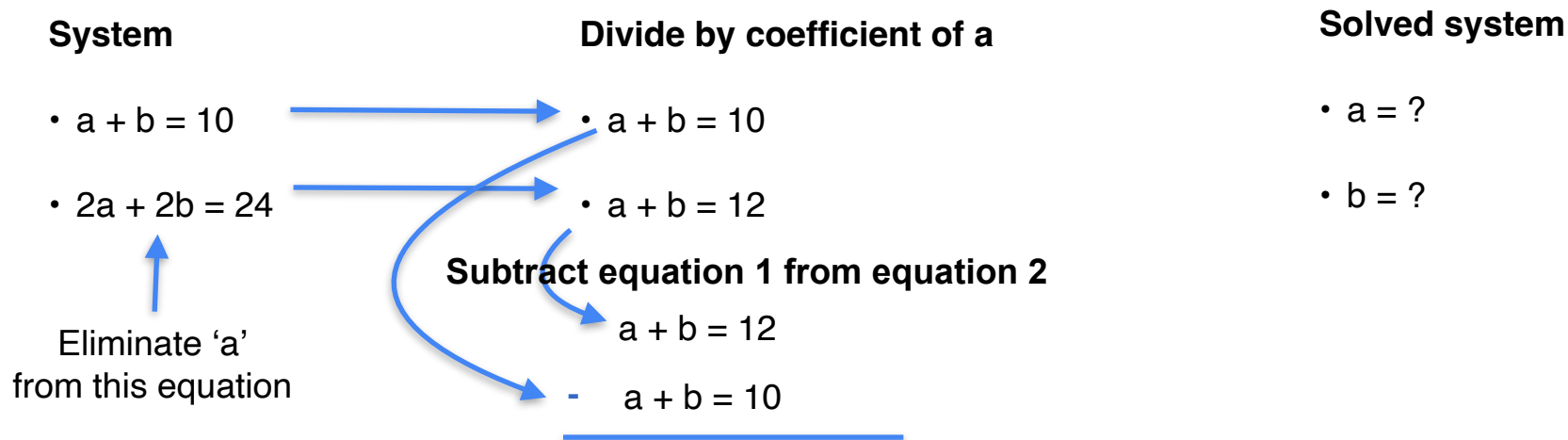


# What if the system is singular (contradictory)?

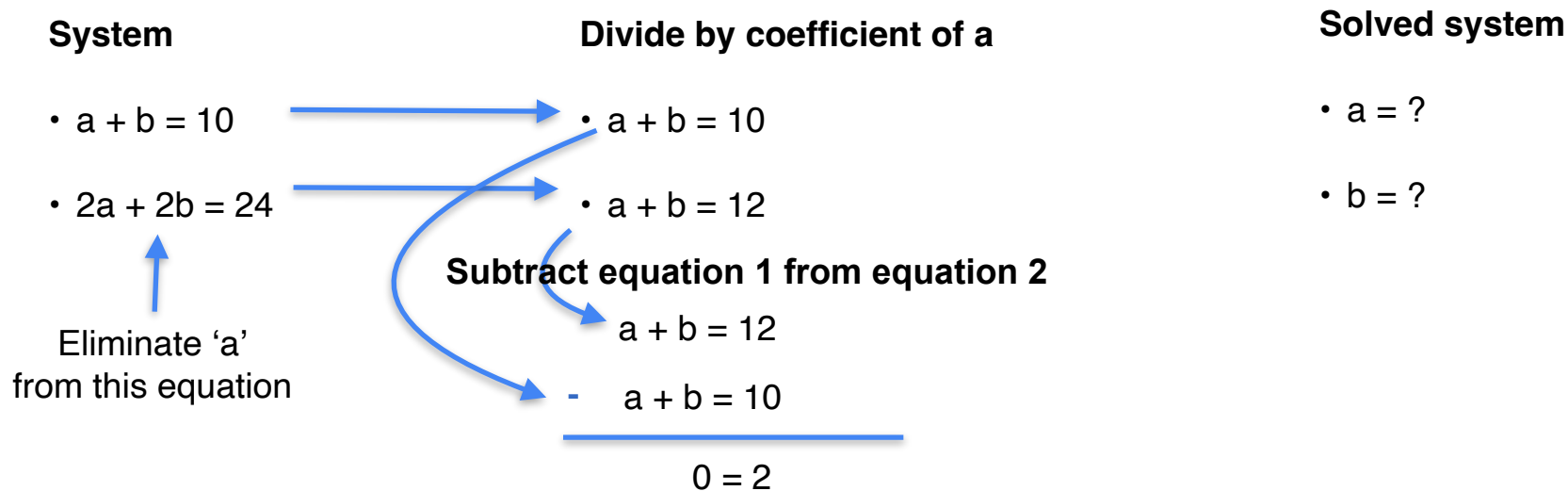




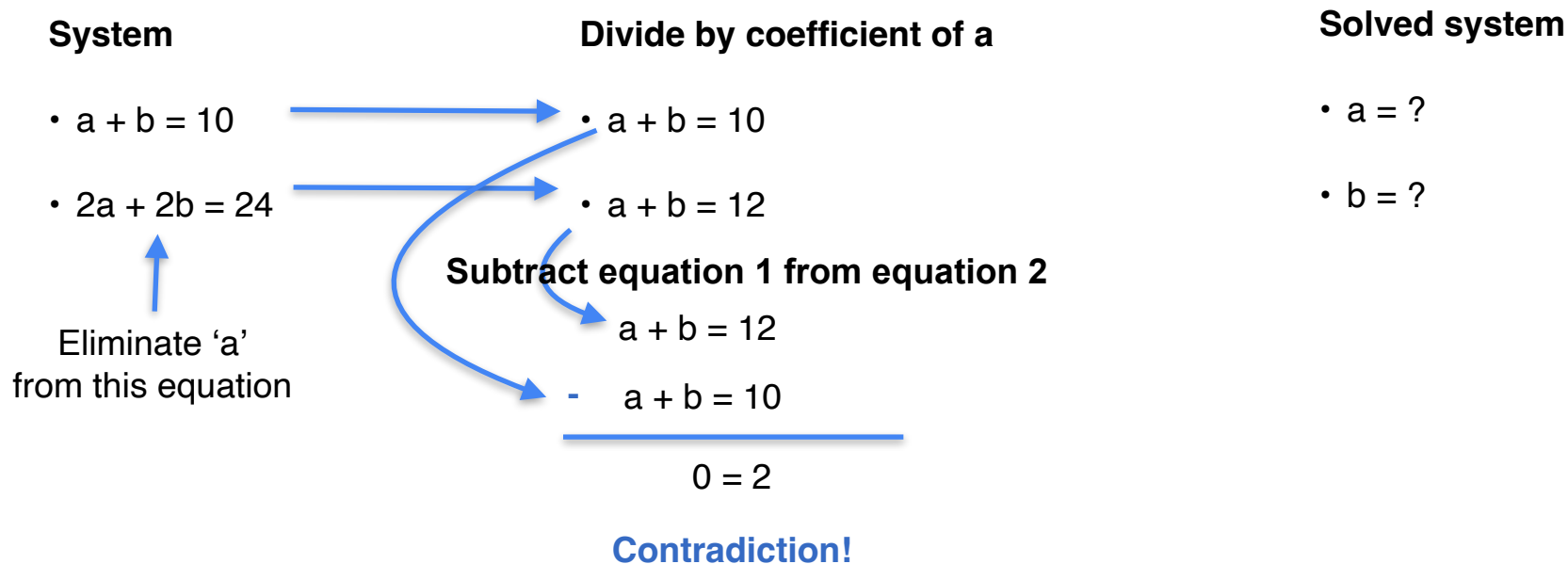
# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# Quiz

- Solve the following system of equations

## **System**

- $5a + b = 11$
- $10a + 2b = 22$

# Solution

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$

**Solution:** If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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# Solving System of Linear Equations

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**Solving system of equations  
with more variables**

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method


## System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by  
itself





# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

# Elimination method

## System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

- $a + b + 2c = 12$

- $a - b - 1/3 c = 1$

- $a - b/2 + 3c = 12$

Divide each  
row by the  
coefficient of 'a'

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

- $a + b + 2c = 12$

- $a - b - \frac{1}{3}c = 1$

- $a - \frac{b}{2} + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Isolated 'a'

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - \frac{1}{3}c = 1$
- $a - \frac{b}{2} + 3c = 12$

Use the first equation to remove 'a' from the others

- $a + b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$

Isolated 'a'

Solve this new system of 2 equations

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$



# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second  
equation to  
remove 'b' from  
the third

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second  
equation to  
remove 'b' from  
the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
  - $b + 7/6 c = 11/2$
  - $-11/6 c = -11/2$
- Isolated 'b'
- $c = 3$

# Elimination method

## System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

# Elimination method

## System


- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$



# Elimination method

## System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$    $b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

# Elimination method

## System



- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$    $b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System



- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System

- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

The solution is  
 $a = 4$   
 $b = 2$   
 $c = 3$



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# Solving System of Linear Equations

---

## **Matrix row reduction**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$



## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$



# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $1a + 0b = 3$
- $0a + 1b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

Reduced row echelon form

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $1a + 0b = 3$
- $0a + 1b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

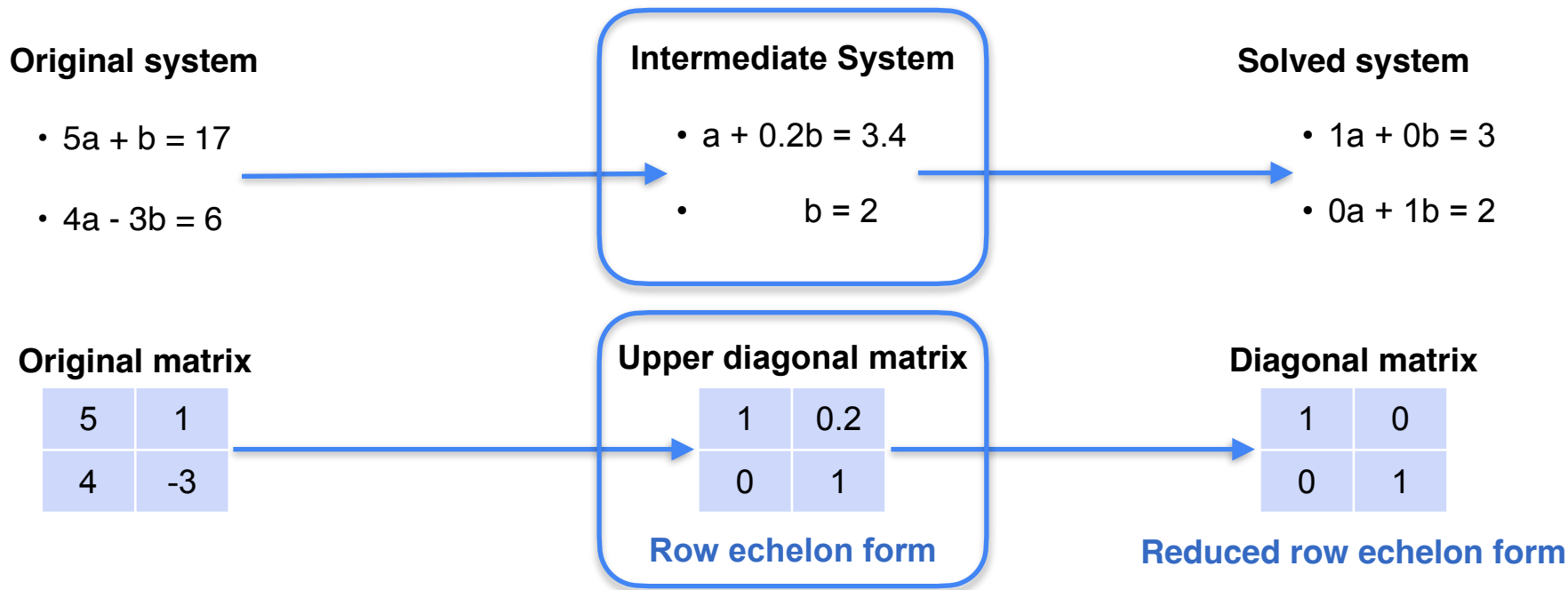
**Diagonal matrix**

1	0
0	1

Row echelon form

Reduced row echelon form

# Systems of equations to matrices



# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$



# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$



# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$

## Intermediate System

- $a + b = 10$
- $0a + 0b = 0$



## Original matrix

1	1
2	2

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

**Row echelon form**

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

**Row echelon form**

# Systems of equations to matrices

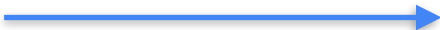
## Original system

- $5a + b = 11$
- $10a + 2b = 22$

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$



## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



## Original matrix

5	1
10	2



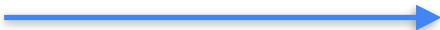
# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0



# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0

**Row echelon form**

# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0

**Row echelon form**

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

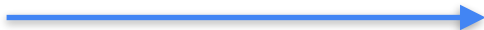
# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$



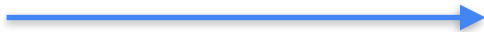
# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

## Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



## Original matrix

0	0
0	0

# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

**Row echelon form**



# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

**Row echelon form**



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# Solving System of Linear Equations

---

**Row operations that  
preserve singularity**

# Switching rows

5	1
4	3

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



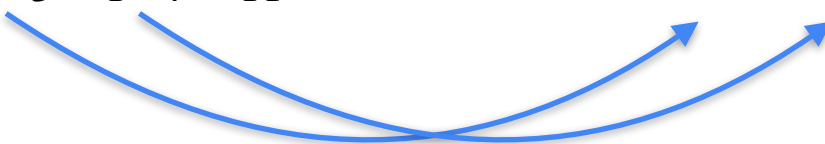
# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



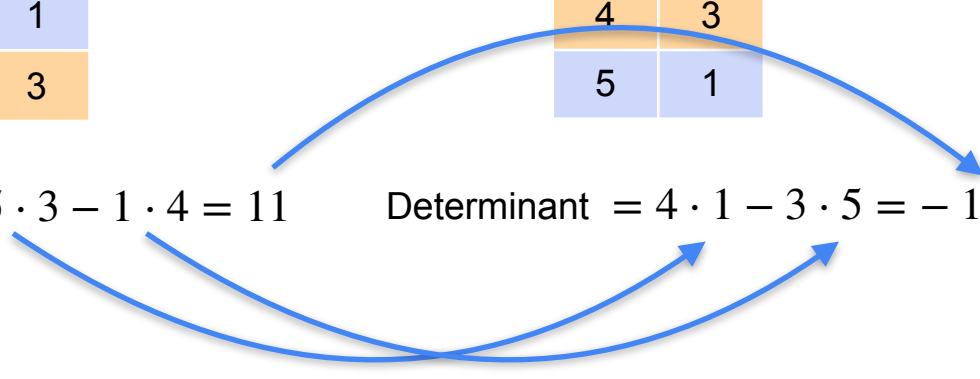


# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$


# Multiplying a row by a (non-zero) scalar

5	1
4	3

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

 $\times 10 =$

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$



# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

5	1
---	---

 $\times 10 =$ 

50	10
----	----

50	10
4	3

$$\text{Determinant} = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

5	1
---	---

 $\times 10 =$ 

50	10
----	----

50	10
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) \\ &= 10 \cdot 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$



# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

9	4
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

9	4
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

+	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>3</td></tr></table>	5	1	4	3
5	1				
4	3				
	<table><tr><td>9</td><td>4</td></tr></table>	9	4		
9	4				

9	4
4	3

$$\text{Determinant} = 9 \cdot 3 - 4 \cdot 4$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

+	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>3</td></tr></table>	5	1	4	3
5	1				
4	3				
	<table><tr><td>9</td><td>4</td></tr></table>	9	4		
9	4				

9	4
4	3

$$\begin{aligned}\text{Determinant} &= 9 \cdot 3 - 4 \cdot 4 \\ &= 11\end{aligned}$$



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# Solving System of Linear Equations

---

## **Rank of a matrix**

# Compressing Images - Reducing rank

# Compressing Images - Reducing rank



# Compressing Images - Reducing rank

Original (Rank 200)



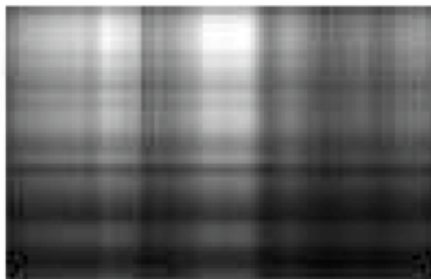


# Compressing Images - Reducing rank

Original (Rank 200)



Rank 1

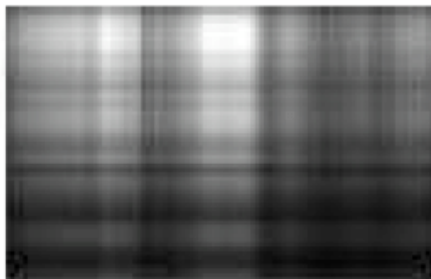


# Compressing Images - Reducing rank

Original (Rank 200)



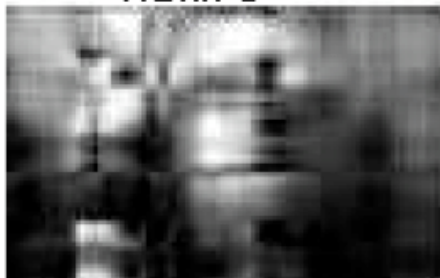
Rank 1



Rank 2



Rank 5

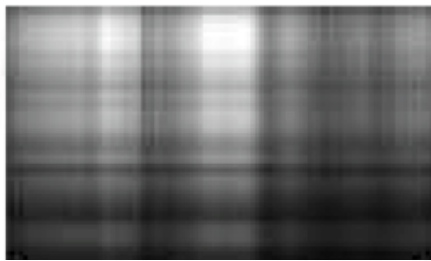


# Compressing Images - Reducing rank

Original (Rank 200)



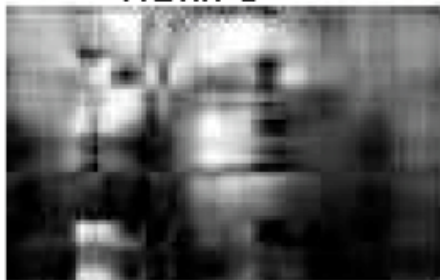
Rank 1



Rank 2



Rank 5



Rank 15

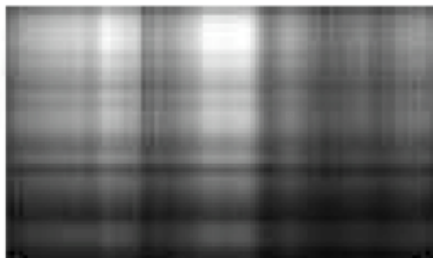


# Compressing Images - Reducing rank

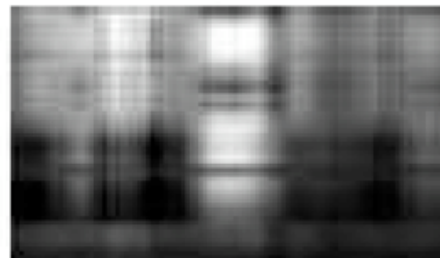
Original (Rank 200)



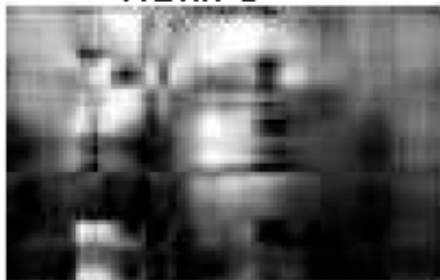
Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



# Systems of information



# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

## System 2



 The dog is **black**  
 The dog is **black**

# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**



## System 3

 The dog  
 The dog





# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**



## System 3

 The dog  
 The dog



Two sentences

# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**

## System 3

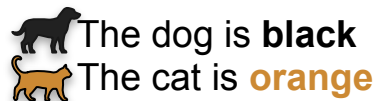
 The dog  
 The dog

**Two sentences**

**Two pieces of information**

# Systems of information

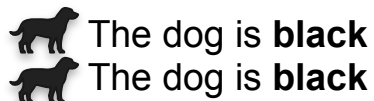
## System 1



Two sentences

Two pieces of information

## System 2



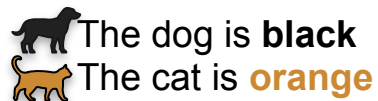
Two sentences

## System 3



# Systems of information

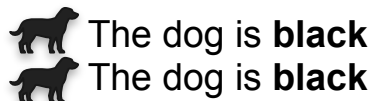
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

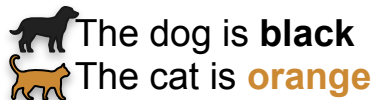
One piece of information

## System 3



# Systems of information

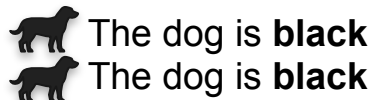
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

One piece of information

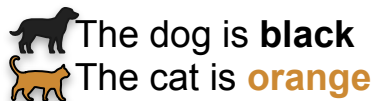
## System 3



Two sentences

# Systems of information

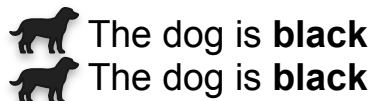
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

One piece of information

## System 3

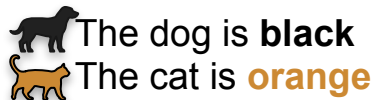


Two sentences

Zero pieces of information

# Systems of information

## System 1

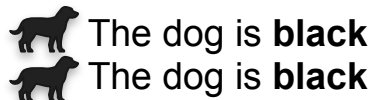


Two sentences

Two pieces of information

Rank = 2

## System 2



Two sentences

One piece of information

## System 3





Two sentences

Zero pieces of information

# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

## System 2

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

## System 3

 The dog  
 The dog

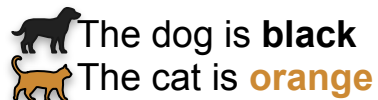
Two sentences

Zero pieces of information



# Systems of information

## System 1

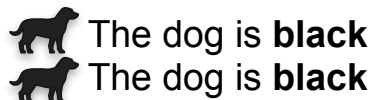


Two sentences

Two pieces of information

Rank = 2

## System 2



Two sentences

One piece of information

Rank = 1

## System 3



Two sentences

Zero pieces of information

Rank = 0

# Systems of equations

# Systems of equations

## System 1

$$a + b = 0$$







$$a + 2b = 0$$





# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 



## System 2



$$a + b = 0$$
 

$$2a + 2b = 0$$
 



# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 



## System 3



$$0a + 0b = 0$$

$$0a + 0b = 0$$



# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

## System 3


$$0a + 0b = 0$$


$$0a + 0b = 0$$

Two equations


# Systems of equations


## System 1

$$a + b = 0$$


$$a + 2b = 0$$


## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


## System 3

$$0a + 0b = 0$$



$$0a + 0b = 0$$



Two equations

Two pieces of information



# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations



Two pieces of information



Rank = 2



# Systems of equations

## System 1

$$a + b = 0$$
 



$$a + 2b = 0$$
 



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

Two equations


## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information


## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1


## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3


$$0a + 0b = 0$$


$$0a + 0b = 0$$

Two equations

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$


$$0a + 0b = 0$$


Two equations

Zero pieces of information

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

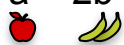
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

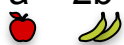
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0



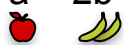
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

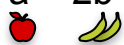
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1

Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

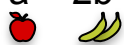
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

 	
0	0
0	0

$$0a + 0b = 0$$

Rank = 0

Two equations

Zero pieces of information

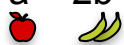
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1

Two equations

One piece of information



Rank = 1

## System 3

$$0a + 0b = 0$$



$$0a + 0b = 0$$

 	
0	0
0	0



Rank = 0

Two equations

Zero pieces of information



Rank = 0

# Rank and solutions to the system





1	1
1	2

**Rank = 2**



1	1
2	2



**Rank = 1**



0	0
0	0



**Rank = 0**

# Rank and solutions to the system





1	1
1	2

Rank = 2



1	1
2	2

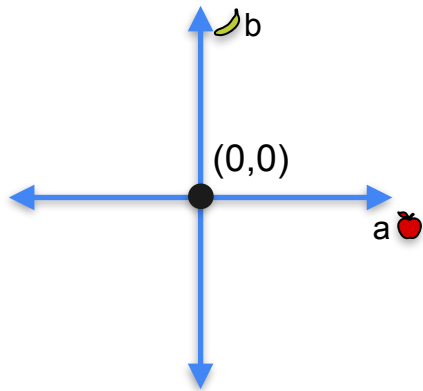
Rank = 1





0	0
0	0

Rank = 0

Dimension of solution space = 0





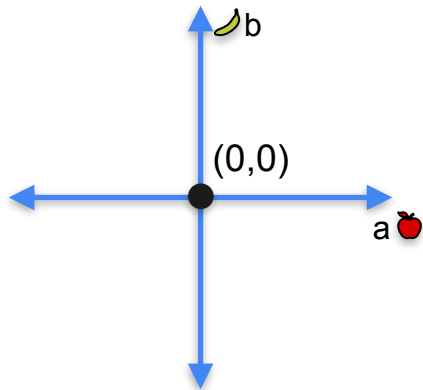
# Rank and solutions to the system



1	1
1	2

Rank = 2



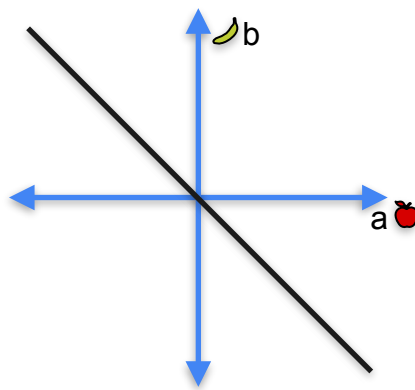
Dimension of solution space = 0



1	1
2	2

Rank = 1



Dimension of solution space = 1



0	0
0	0

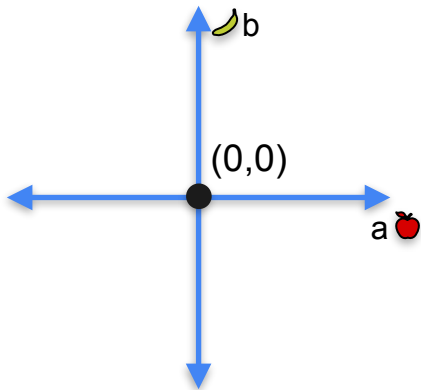
Rank = 0



# Rank and solutions to the system

	
1	1
1	2

Rank = 2

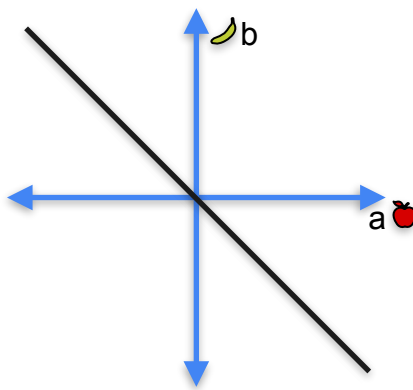
Dimension of solution space = 0





	
1	1
2	2

Rank = 1

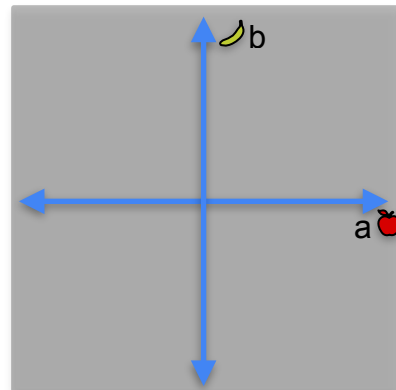
Dimension of solution space = 1



	
0	0
0	0



Rank = 0

Dimension of solution space = 2







# Rank of a matrix





1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**





0	0
0	0

**Rank = 0**

Dimension of solution space = 0    Dimension of solution space = 1    Dimension of solution space = 2



$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

# Rank and singularity





1	1
1	2

Rank = 2



1	1
2	2



Rank = 1



0	0
0	0



Rank = 0

# Rank and singularity





1	1
1	2

Rank = 2



1	1
2	2

Rank = 1





0	0
0	0

Rank = 0

**Non-singular**



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1



Singular



0	0
0	0

Rank = 0



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1

Singular





0	0
0	0

Rank = 0

Singular



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1

Singular



0	0
0	0

Rank = 0

Singular

# Quiz: Rank of a matrix

Determine the rank of the following two matrices

## Matrix 1

5	1
-1	3

## Matrix 2

2	-1
-6	3

# Solutions: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1:** Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

**Matrix 2:** Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3





DeepLearning.AI

# Solving System of Linear Equations

---

**Rank of a matrix:  
General case**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$



## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

## System 4


$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$


## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**

**3 Pieces of information**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

**Rank 3**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

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**3 Equations**  
**3 Pieces of information**

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1	1	1
1	2	1
1	1	2

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$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



# Rank for matrices

## System 1

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**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

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**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



# Rank for matrices

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$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

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$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

## Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**

## Rank 0

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**

## Rank 0

0	0	0
0	0	0
0	0	0

# Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



DeepLearning.AI

# Solving System of Linear Equations

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## Row echelon form

# Row echelon form of a matrix

# Row echelon form of a matrix

Original matrix

5	1
4	-3

# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1



5	1
10	2



# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

5	1
10	2



1	1
0	0

# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

5	1
10	2



1	1
0	0

0	0
0	0

# Row echelon form of a matrix

Original matrix

Row echelon form

5	1
4	-3



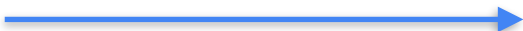
1	0.2
0	1

5	1
10	2



1	1
0	0

0	0
0	0



0	0
0	0

# Row echelon form

**Original matrix**

5	1
4	-3

# Row echelon form

**Original matrix**


5	1
4	-3

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

5	1
4	-3



Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

5	1
4	-3



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

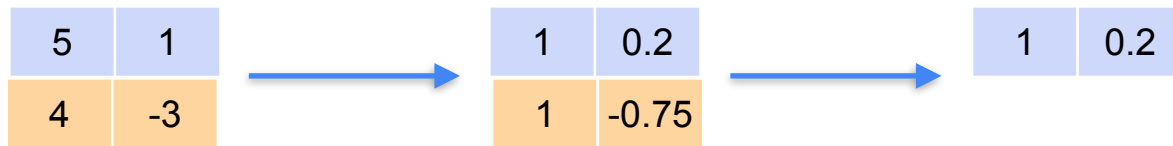
5	1		1	0.2
4	-3		1	-0.75

Divide each row by  
the leftmost coefficient



# Row echelon form

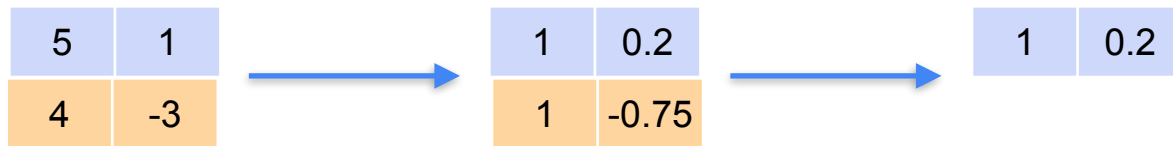
Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

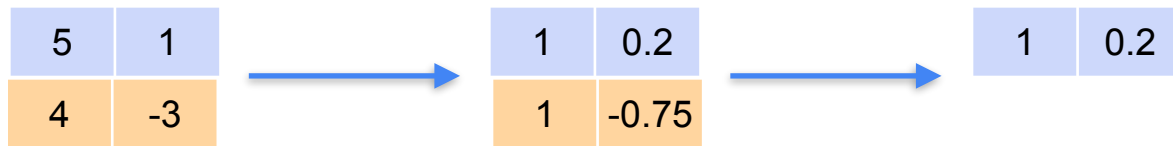


Divide each row by  
the leftmost coefficient

1	-0.75
---	-------

# Row echelon form

Original matrix

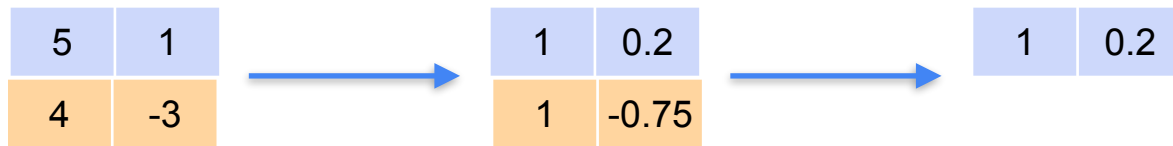


Divide each row by  
the leftmost coefficient

1	-0.75
1	0.2

# Row echelon form

Original matrix

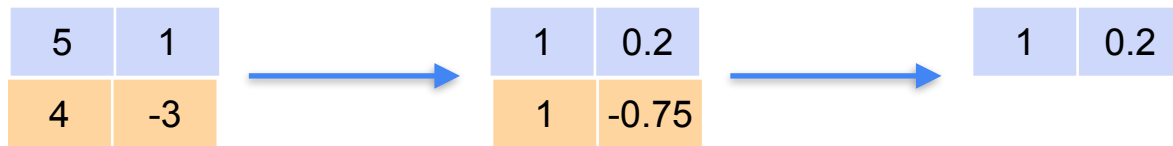


Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \end{array}$$

# Row echelon form

Original matrix

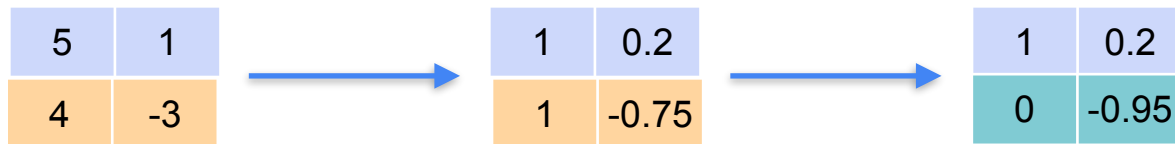


Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

# Row echelon form

Original matrix



Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

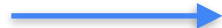
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient



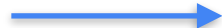
# Row echelon form

Original matrix

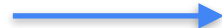
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

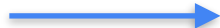
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
0	1

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form

Original matrix

5	1
4	-3

Divide each row by  
the leftmost coefficient

1	0.2
1	-0.75

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

Row echelon form

1	0.2
0	-0.95

1	0.2
0	1

# Row echelon form for singular matrices

**Original matrix**

5	1
10	2

# Row echelon form for singular matrices

**Original matrix**


5	1
10	2

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

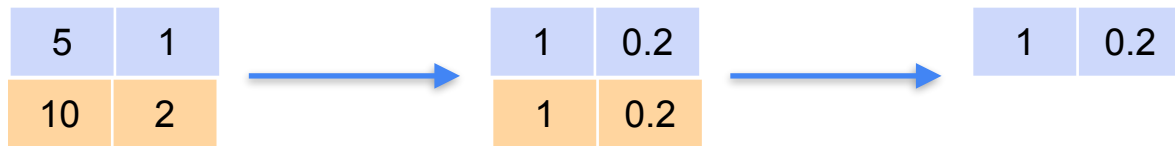
5	1		1	0.2
10	2		1	0.2

Divide each row by  
the leftmost coefficient



# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

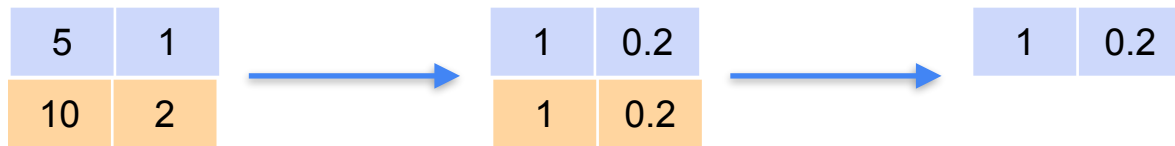


Divide each row by  
the leftmost coefficient

1	0.2
---	-----

# Row echelon form for singular matrices

Original matrix

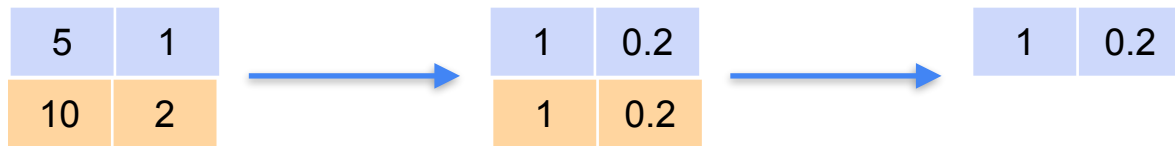


Divide each row by  
the leftmost coefficient

1	0.2
1	0.2

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

$$\begin{array}{r|cc} & 1 & 0.2 \\ - & 1 & 0.2 \\ \hline \end{array}$$

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

The diagram shows the row reduction process. It starts with the matrix after dividing the first row by its leftmost coefficient (5). A minus sign indicates the second row is subtracted from the first row. The result is a matrix with two rows of zeros.

1	0.2
---	-----

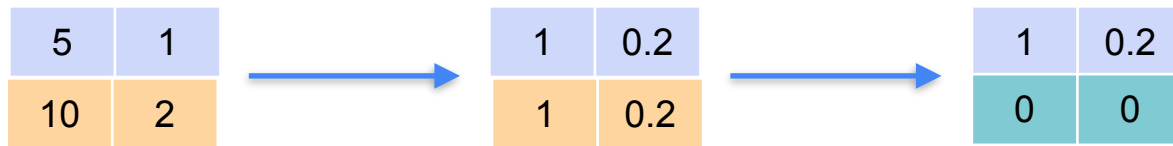
-	1	0.2
---	---	-----

---

0	0
---	---

# Row echelon form for singular matrices

Original matrix



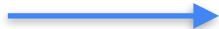
Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

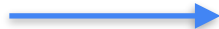
# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0

Divide each row by  
the leftmost coefficient

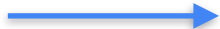
	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



Divide each row by  
the leftmost coefficient

	1	0.2
-	1	0.2
<hr/>		
	0	0

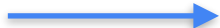
Divide the second row by  
the leftmost non-zero coefficient



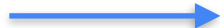
# Row echelon form for singular matrices

Original matrix

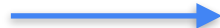
5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

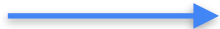
	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

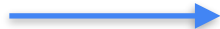
# Row echelon form for singular matrices

Original matrix

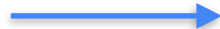
5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



1	0.2
?	?

Divide each row by  
the leftmost coefficient

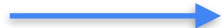
	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



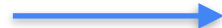
Divide each row by  
the leftmost coefficient

1	0.2
1	0.2



Row echelon form

1	0.2
0	0



Divide the second row by  
the leftmost non-zero coefficient

1	0.2
?	?

	1	0.2
-	1	0.2
<hr/>		
	0	0

# Row echelon form for singular matrices

**Original matrix**

0	0
0	0

# Row echelon form for singular matrices

**Original matrix**


0	0
0	0

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0
0	0



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0	→	?	?
0	0		?	?

Divide each row by  
the leftmost coefficient



# Row echelon form for singular matrices

Row echelon form

Original matrix

0	0	→	?	?
0	0		?	?

Divide each row by  
the leftmost coefficient

# Row echelon form, singularity, and rank

5	1
4	-3

5	1
10	2

0	0
0	0

# Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

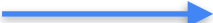
$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

0	0
0	0



0	0
0	0

# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

**Rank 2**  
2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

0	0	→	0	0
0	0		0	0

# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

**Rank 2**

2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

1 one in the diagonal

0	0	→	0	0
0	0		0	0



# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

0 ones in the diagonal

# Row echelon form, singularity, and rank

5	1
4	-3

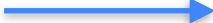


1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

**2** ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

**Rank 1**

**1** one in the diagonal

0	0
0	0

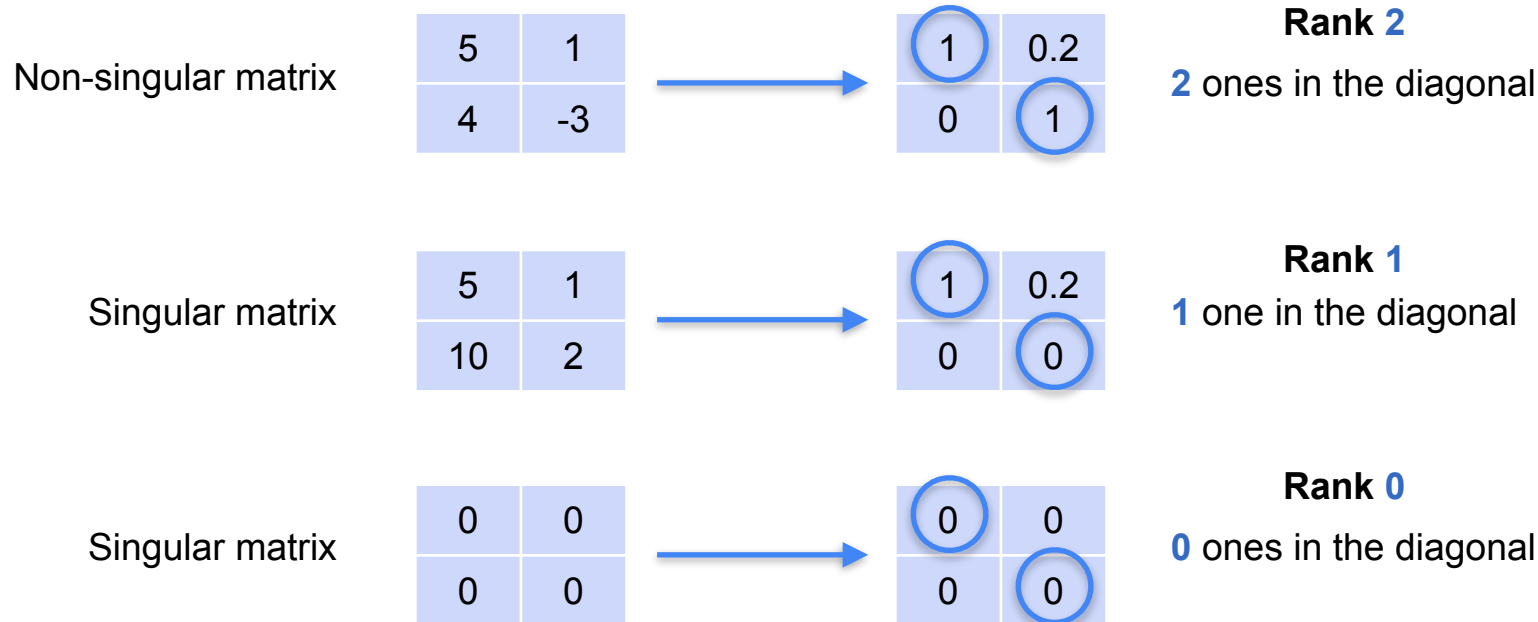


0	0
0	0

**Rank 0**

**0** ones in the diagonal

# Row echelon form, singularity, and rank





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# Solving System of Linear Equations

---

**Row echelon form:  
General case**

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Matrix

1	1	2
3	-3	-1
2	-1	6

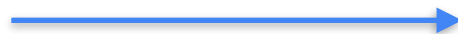
# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

## Matrix

1	1	2
3	-3	-1
2	-1	6



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Row echelon form matrix

1	1	2
0	-6	7
0	0	6

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

**Rank 5**

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots



# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

Subtract the first row  
from the second and  
the third ones

# Another example

Matrix				Row echelon form		
1	1	1	→	1	1	1
1	2	1		0	1	0
1	1	2		0	0	1

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3



1	1	1
0	0	1
0	0	2

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

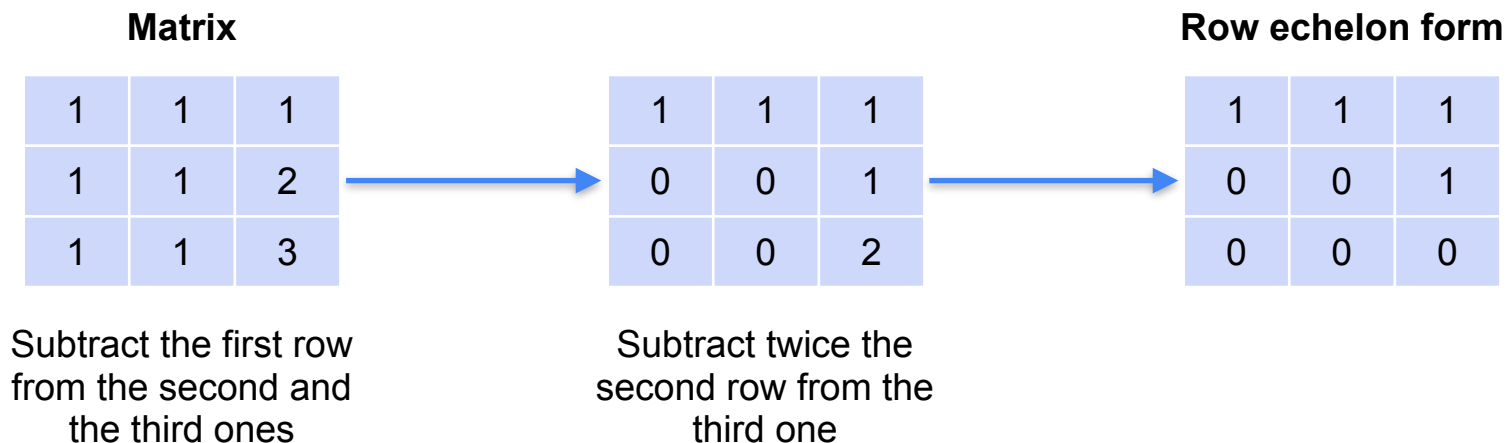
Subtract the first row  
from the second and  
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the  
second row from the  
third one

# What if the matrix is singular?





# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1	→	1	1	1
2	2	2		0	0	0
3	3	3		3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row



**Row echelon form**

1	1	1
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

**Number of pivots = 3**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Rank = 0**

## Row echelon forms

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



DeepLearning.AI

# Solving System of Linear Equations

---

## **Reduced row echelon form**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$



**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3



# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

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Row echelon form

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Row echelon form

Reduced row echelon form

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Row echelon form

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Reduced row echelon form

# Reduced row echelon form

Row echelon form

1	0.2
0	1

# Reduced row echelon form

Row echelon form





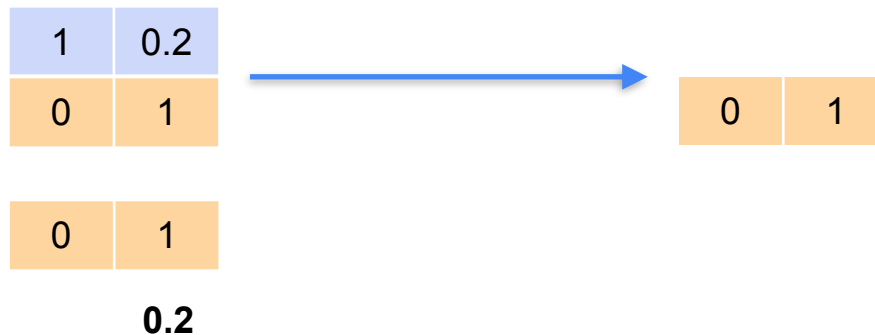
# Reduced row echelon form

Row echelon form



# Reduced row echelon form

Row echelon form



# Reduced row echelon form

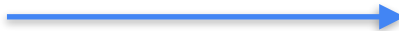
Row echelon form



# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

x

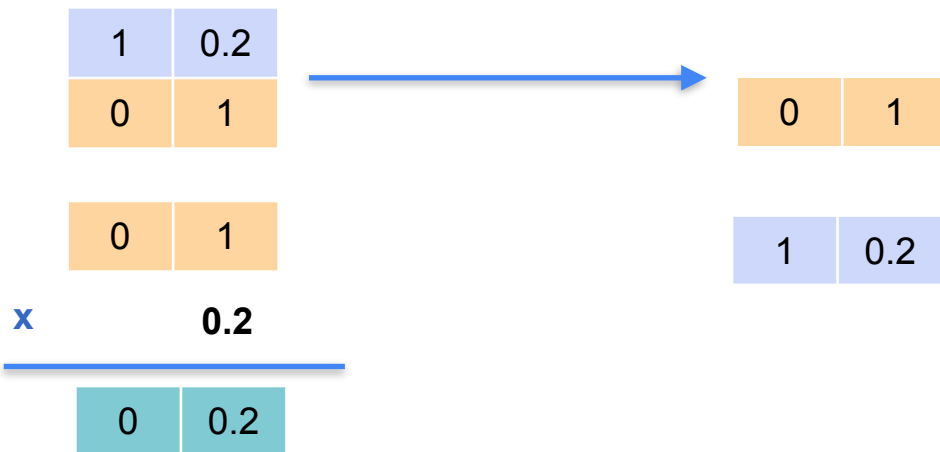
0.2

---

0	0.2
---	-----

# Reduced row echelon form

Row echelon form



# Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form.

**Initial Matrix (Row Echelon Form):**

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

**Row Operation:**  $\times$  0.2

**Intermediate Matrix:**

$$\begin{bmatrix} 0 & 1 \\ 0 & 0.2 \end{bmatrix}$$

**Row Operation:**  $-$

**Final Matrix (Reduced Row Echelon Form):**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form. It shows two matrices connected by a blue arrow, with the operations performed to reach the second matrix shown below each.

**Matrix 1 (Row Echelon Form):**

1	0.2
0	1

**Operation:**  $\times$  0.2

**Matrix 2 (Reduced Row Echelon Form):**

0	1
1	0

The operations shown below the matrices are:

**For Matrix 1:**

0	0.2
---	-----

**For Matrix 2:**

1	0.2
0	0.2

The final result of the row reduction is the matrix shown in the second row of the diagram:

1	0
0	1

# Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form.

**Initial Matrix (Row Echelon Form):**

1	0.2
0	1

**Transformation:**

$\times \quad 0.2$

**Resulting Matrix (Reduced Row Echelon Form):**

1	0
0	1

The transformation is shown as a subtraction of 0.2 times the second row from the first row, resulting in the second matrix.



# Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

x                      0.2

---

0	0.2
---	-----

Reduced row echelon form

1	0
0	1

1	0.2
0	0.2

-

---

1	0
---	---

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

● Is in row echelon form

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

**Rank 5**

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

# Reduced row echelon form

# Reduced row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0



# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

Reduced row  
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a  
pivot to 0

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one

# Reduced row echelon form

## Row echelon form

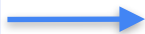
1	2	3	→	1	0	-5
0	1	4		0	1	4
0	0	1		0	0	1

Subtract 2 times the  
second row from the  
first one

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one

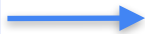
Add 5 times the third  
row to the first one

# Reduced row echelon form

## Row echelon form

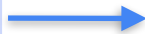
1	2	3
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third  
row to the first one



1	0	0
0	1	4
0	0	1

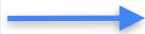


# Reduced row echelon form

## Row echelon form

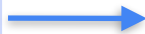
1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

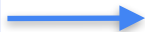
Subtract 4 times the third row from the second one

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one



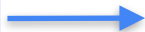
1	0	0
0	1	0
0	0	1

# Reduced row echelon form

**Row echelon form**

1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one



**Reduced row echelon form**

1	0	0
0	1	0
0	0	1



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# Solving System of Linear Equations

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## **Conclusion**