Equivalence of Some Common Linear Feature Extraction Techniques for Appearance-Based Object Recognition Tasks

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Abstract—Recently, a number of empirical studies have compared the performance of PCA and ICA as feature extraction methods in appearance-based object recognition systems, with mixed and seemingly contradictory results. In this paper, we briefly describe the connection between the two methods and argue that whitened PCA may yield identical results to ICA in some cases. Furthermore, we describe the specific situations in which ICA might significantly improve on PCA.

Index Terms—Computer vision, object recognition, principal component analysis, independent component analysis.

1 Introduction

Over the last few years, there has been increasing interest in appearance-based object recognition due to its successful results in noncontrolled scenes compared to classical model-based techniques [7], [8], [11]. Appearance-based recognition approaches are able to manage changes in illumination conditions, shape, pose and reflectance [24], and even to handle translation and partial occlusions [27].

First appearance-based systems found in the literature used Principal Component Analysis (PCA) for dimensionality reduction purposes [17], [24], [25], [27], [30], while recently the use of the independent component analysis (ICA) for feature extraction is preferred by some authors.[1] In fact, a number of empirical studies have claimed that ICA outperforms PCA as a feature extraction method in classification systems [4], [10], [12], [13], [22], [29], [31], [33], although [23], [32] state that both approaches perform equally, [28] suggests that the performance of ICA is very dependent on the data set, and [3], [14] claim that PCA is superior to ICA. The main goal of this short paper is to explain those seemingly contradictory results and to clarify under which circumstances ICA may outperform PCA.

2 Methods for Dimensionality Reduction

First of all, it is necessary to have a look at the subtle differences between PCA, Singular Value Decomposition (SVD), and whitening. Although they are intrinsically very close, whitening and PCA transform differ in one important aspect that can affect the performance of the classifier used in the recognition system.

1. Although ICA has been connected with sparse representations and edge detection [16], this paper only concerns its application to dimension reduction for subsequent classification.

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Manuscript received 30 Nov. 2005; revised 28 Apr. 2006; accepted 10 July 2006; published online 18 Jan. 2007.
Recommended for acceptance by H. Wechsler.
For information on obtaining reprints of this article, please send e-mail to: tpami@computer.org, and reference IEEECS Log Number TPAMI-0656-1105. Digital Object Identifier no. 10.1109/TPAMI.2007.1025.

2.1 PCA, SVD, and Whitening

In the original data matrix $\mathbf{X}$, each column contains the pixels values of one image vector, $\mathbf{x}$, and it is assumed that the data has been centered. The PCA transform of the data matrix $\mathbf{X}$ of size $m \times n$ is

$$\mathbf{Y} = \mathbf{U}^T\mathbf{X}, \tag{1}$$

where $\mathbf{U}$ is a $m \times m$ orthonormal matrix. The principal components (columns of $\mathbf{U}$) are found by recursively seeking out the directions of maximum data variance, under the constraint of orthogonality. The principal component vectors are exactly the eigenvectors of the covariance matrix $C_\mathbf{X} = \frac{1}{N}\mathbf{X}\mathbf{X}^T$ in decreasing order of corresponding eigenvalues. The largest eigenvalue equals the maximal variance, while the corresponding eigenvector determines the direction with the maximal variance. The transformation defined in (1) also gives uncorrelated components.

In (1), $\mathbf{Y}$ is the original data matrix projected on the PCA subspace defined by the eigenvectors, here, it is possible to reduce the dimension of the data just by selecting a subset of k eigenvectors from the total set ($\Re^m \rightarrow \Re^k$),

$$\widetilde{\mathbf{Y}} = \widetilde{\mathbf{U}}^T\mathbf{X}. \tag{2}$$

We can approach PCA from a slightly different point of view and consider that $\mathbf{U}$ is one of the matrices of the SVD of the data matrix $\mathbf{X}$,

$$\mathbf{X} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T, \tag{3}$$

where $\mathbf{U}$ and $\mathbf{V}$ are $m \times r$ and $n \times r$ matrices with orthonormal columns and $\mathbf{\Lambda}$ is a $r \times r$ diagonal matrix with the nonnegative singular values $\sigma_j, j = 1, \ldots, r$, arranged in nonincreasing order along the diagonal, and where r is the rank of $\mathbf{X}$.

From (3), it follows that $\mathbf{X}\mathbf{X}^T\mathbf{U} = \mathbf{U}\mathbf{\Lambda}\mathbf{\Lambda}^T$ and $\mathbf{X}^T\mathbf{X}\mathbf{V} = \mathbf{V}\mathbf{\Lambda}^T\mathbf{\Lambda}$, demonstrating that the columns of $\mathbf{U}$ are the eigenvectors of $\mathbf{X}\mathbf{X}^T$ and the columns of $\mathbf{V}$ are the eigenvectors of $\mathbf{X}^T\mathbf{X}$.

Note that $\mathbf{X}$ can be written as the sum of r rank-1 matrices,

$$\mathbf{X} = \sum_{j=1}^{r} \sigma_j \mathbf{u}_j \mathbf{v}_j^T. \tag{4}$$

This implies that the zero singular values may be ignored since they carry no information. Equation (5) also shows that it is possible to approximate $\mathbf{X}$ by just using the first k columns of $\mathbf{U}$ and $\mathbf{V}$,

$$\mathbf{X} \approx \sum_{j=1}^{k} \sigma_j \mathbf{u}_j \mathbf{v}_j^T = \widetilde{\mathbf{U}}\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{V}}^T. \tag{5}$$

A zero-mean random vector $\mathbf{z}$ is said to be *white* if its elements are uncorrelated and have unit variances, this obviously means that their covariance matrix is equal to the unit matrix $\mathbf{I}$. As whitening can be accomplished by decorrelation followed by scaling, the PCA technique can be used, and (6) defines the whitening transform of the original data matrix.

$$\widetilde{\mathbf{Z}} = \widetilde{\mathbf{\Lambda}}^{-1}\widetilde{\mathbf{U}}^T\mathbf{X} = \widetilde{\mathbf{V}}^T. \tag{6}$$

2.2 ICA

ICA [16] tries to explain the original data using statistically independent random vectors. The observed random data matrix $\mathbf{X}$ is modeled as

$$\mathbf{X} \approx \mathbf{A}\mathbf{S}, \tag{7}$$

where $\mathbf{S}$ is the matrix containing the statistically independent random vectors and $\mathbf{A}$ is the mixing matrix. We can also write (7) as:

$$\mathbf{S} = \mathbf{W}\mathbf{X}, \tag{8}$$

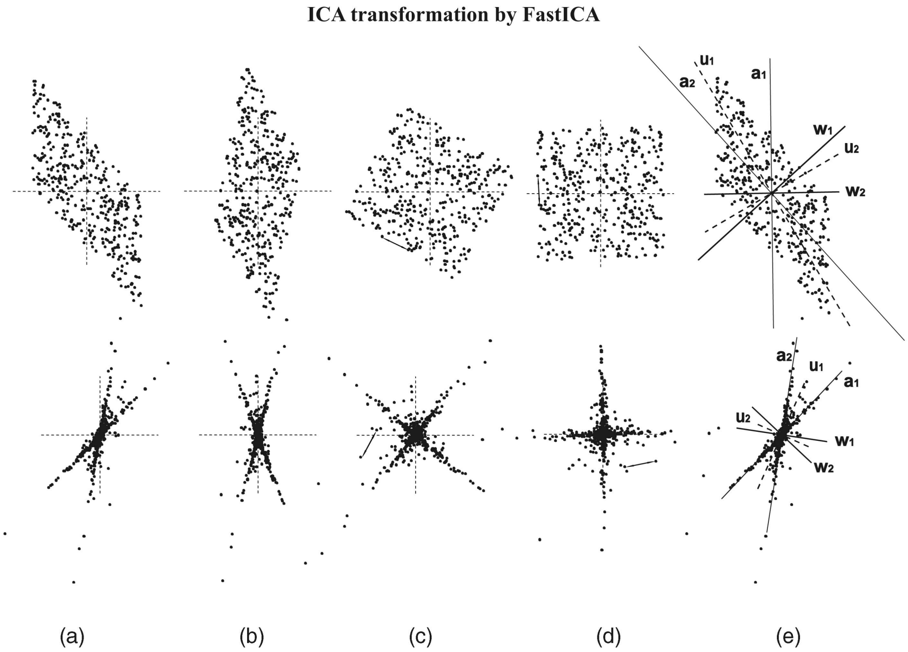


Fig. 1. Two artificial examples: (a) a sub-Gaussian data set (top row) and a super-Gaussian data set (bottom row), (b) both transformed by PCA, (c) whitening, and (d) using the ICA model by means of FastICA. (e) shows the original data set with the ICA ($\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{w}_1$, $\mathbf{w}_2$) and PCA directions ($\mathbf{u}_1$, $\mathbf{u}_2$). (a) Original. (b) Uncorrelated. (c) Whitened. (d) Independent. (e) All directions.

where $\mathbf{W}$ is equal to the pseudoinverse matrix of $\mathbf{A}$, that is, $\mathbf{W} = \mathbf{A}^{\dagger}$. Typically, in ICA algorithms, vectors $\mathbf{w}_i$ are sought such that the rows of $\mathbf{S}$ have maximally non-Gaussian distributions and are mutually (approximately) uncorrelated. A simple way to do this is to first whiten the data as in (6), and then seek orthogonal nonnormal projections ($\mathbf{R}$):

$$\mathbf{S} = \mathbf{R}^T \widetilde{\mathbf{\Lambda}}^{-1} \widetilde{\mathbf{U}}^T \mathbf{X} = \mathbf{R}^T \widetilde{\mathbf{Z}} \tag{9}$$

so, in (9), it is shown that in fact, in this case, ICA is a whitening operation followed by a rotation, and the ICA model can be also written as

$$\mathbf{X} \approx \widetilde{\mathbf{U}} \widetilde{\mathbf{\Lambda}} \widetilde{\mathbf{V}}^T = \underbrace{\widetilde{\mathbf{U}} \widetilde{\mathbf{\Lambda}} \mathbf{R}}_{\mathbf{A}} \underbrace{\mathbf{R}^T \widetilde{\mathbf{V}}^T}_{\mathbf{S}} = \mathbf{A}\mathbf{S}. \tag{10}$$

Here, we wish to emphasize that in the ICA model neither $\mathbf{A}$ nor $\mathbf{W}$ are constrained to be orthogonal. Rather, the constraint (exact [15] or approximate [5], depending on the choice of algorithm) is that the transform be decorrelating, meaning that the rows of $\mathbf{S}$ are (exactly or approximately) orthogonal. This is because strongly nonorthogonal rows imply strong linear correlations between the estimated components, which is not allowed since the goal was to get independent components, and independence entails uncorrelatedness. Thus, all ICA algorithms which output approximately uncorrelated components with approximately equal variances are essentially performing an orthogonal transform of the whitened data.

A number of popular ICA algorithms exist. These include FastICA [15], [16], Infomax [5], [18], Comon's algorithm [9], and KernelICA [2]. In the case of FastICA and KernelICA, the data is first whitened and, subsequently, an orthonormal separating matrix is sought, as in (9). In contrast, Infomax does not strictly enforce complete linear decorrelation. Note that this implies that in cases where higher-order dependencies in the data are strong relative to the second-order dependencies, Infomax may yield a decomposition which is not even approximately an orthogonal transform of the whitened data. Comon's algorithm may also give a significantly nonorthogonal transformation from the whitened data due to the different normalization employed, as described later in this section.

In Figs. 1 and 2, the results of transforming two-dimensional artificial data sets using PCA, whitening, and the ICA model (by means of FastICA (Fig. 1) and Extended Infomax [18] (Fig. 2)) are illustrated. In the top row of each figure, the original data set has a uniform distribution on a parallelogram (a sub-Gaussian data set); while the original data set in the bottom row has a sparse distribution (a super-Gaussian data set).

Fig. 1 has been drawn using FastICA, so it shows the ICA transformation as in (9). In the top row, the original data set has a uniform distribution on a parallelogram (Fig. 1a), but the components are not independent and it is possible to predict the value of one of them from the value of the other. The uncorrelated data set is shown in Fig. 1b, the direction with the maximal variance is the vertical axis, and the second principal axis is the horizontal one. The whitened data is shown in Fig. 1c, whitening gives the ICs only up to an orthogonal transformation. And, finally, the independent data set appears in Fig. 1d. The same process is shown in the bottom row, but in this case the original data set has a super-Gaussian distribution. Fig. 1e shows the original data sets with the ICA and PCA directions. There, we may appreciate that the constraint of orthogonality holds on PCA directions ($\mathbf{U}$) and does not hold on ICA directions ($\mathbf{W}$).

It should be noticed how whitening changes distances between points and, so, the distance between two points is not the same in the uncorrelated data and the whitened one. However, the distance between two points is equal in the whitened data and the independent one as the two are equivalent up to a rotation.

In order to show that (for this simple artificial data set) the ICs obtained by Infomax are also, approximately, an orthonormal transformation of the whitened data, Fig. 2 shows the same artificial data sets as Fig. 1 but the ICs have been obtained (Fig. 2b) by means of the Extended Infomax algorithm [18], which can estimate both sub- and super-Gaussian components. It is possible to see that the independent data (Fig. 2b) obtained from Infomax is, approximately, a rotated version of the whitened data (Fig. 2c, copied from Fig. 1c for comparison purposes). Note, however, that

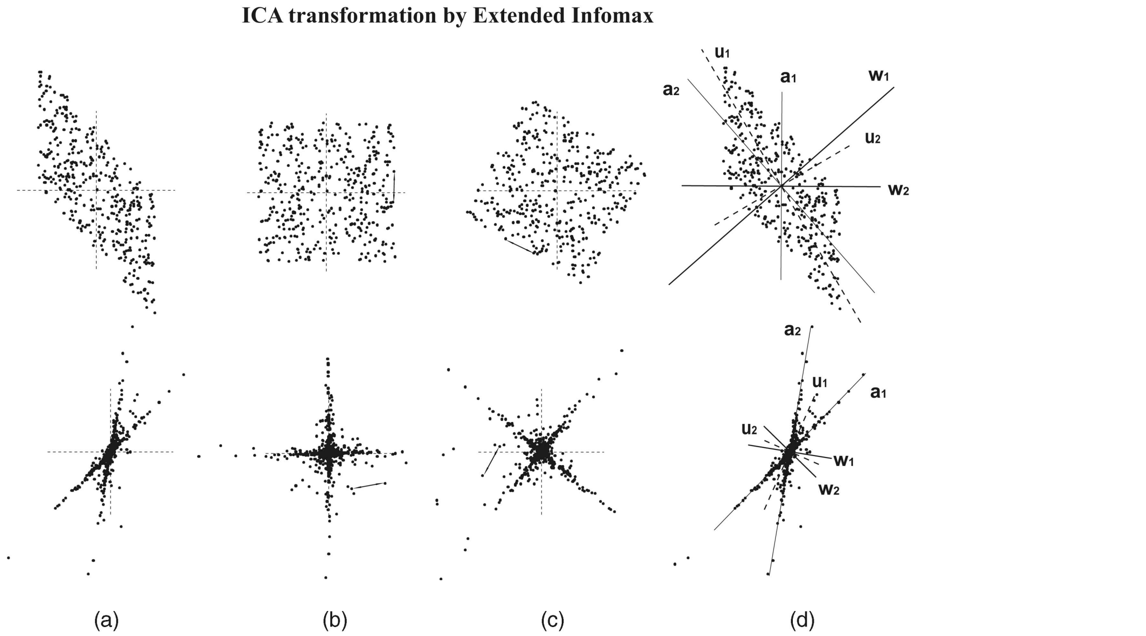


Fig. 2. Two artificial examples: (a) a sub-Gaussian data set (top row) and a super-Gaussian data set (bottom row), (b) both transformed using the ICA model by means of Extended Infomax. (c) The whitened data, copied from Fig. 1 for comparison. (d) shows the original data set with the ICA ($\mathbf{a_1}, \mathbf{a_2}$ and $\mathbf{w_1}, \mathbf{w_2}$) and PCA directions $(\mathbf{u_1}, \mathbf{u_2})$. (a) Original. (b) Independent. (c) Whitened. (d) All directions.

these results cannot always be extended to real-world data sets, where the results of Infomax may not be orthogonal from the whitened data [4]. This fact will be clearly shown on Section 6.

Although the above equivalence holds for the most widely used ICA algorithms [2], [5], [15], [16], [18], it does not quite hold for the algorithm of Comon [9]. This is because his algorithm renormalizes the components so that the basis vectors (columns of $\mathbf{A}$) have unit norm. Since this also changes the variance of the independent components (rows of $\mathbf{S}$), the transform is in this case *not* an approximate rotation from the whitened data. Hence, distances, and angles, may change. However, with a few notable exceptions [19], [20], [21], almost all [3], [4], [10], [12], [13], [14], [22], [23], [28], [29], [31], [32], [33] PCA/ICA comparisons for pattern recognition purposes have used FastICA or Infomax (or its extended version) to perform ICA.

3 Different Architectures: Independence in A or in S?

Previous papers [4], [10], [32] have described two different *architectures* for the ICA decomposition of an image set. The difference between these two is a simple choice of where we want the independence. Do we want the *basis images* (columns of $\mathbf{A}$) to be mutually independent, or do we seek a decomposition where the *coefficients* (rows of $\mathbf{S}$) are mutually independent? The former corresponds to architecture I, the latter to architecture II [4], [10].

In equations, the question is whether we write

$$\mathbf{X} \approx \widetilde{\mathbf{U}}\widetilde{\mathbf{\Lambda}}\mathbf{R}\mathbf{R}^T\widetilde{\mathbf{V}}^T,$$

and optimize the orthogonal matrix $\mathbf{R}$ to make $\mathbf{S} = \mathbf{R}^T\widetilde{\mathbf{V}}^T$ as independent and non-Gaussian as possible, or whether we write $\mathbf{X} \approx \widetilde{\mathbf{U}}\mathbf{R}\mathbf{R}^T\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{V}}^T$, and optimize $\mathbf{R}$ to make $\mathbf{A} = \widetilde{\mathbf{U}}\mathbf{R}$ as independent and non-Gaussian as possible. The concept about the two types of architectures is illustrated in Fig. 3.

In either case, the *independent components*, that is, the matrix having been optimized for independence, is always an orthogonal transformation from the *whitened* data given by SVD, at least in the case of FastICA. As we shall argue in the next section, this implies that if a rotationally invariant classifier is subsequently used, there is no point in performing the optimization of $\mathbf{R}$.

4 Rotation Invariant Classifiers

Most common classifiers are invariant to a rotation of the data space. This makes intuitive sense, if one does not have any preexisting knowledge on the structure of the data space, it seems reasonable to build a classifier which does not care about data rotations. Typical examples are all classifiers based on the Euclidean distances, or based on the angles, between data points [17], [24], [30]. As it was shown in (9), and further argued in Section 3, the independent components are the result of a simple rotation of the whitened data given by SVD. Hence, a rotation-invariant classifier will not do any better nor any worse when fed with the independent components than when fed with the whitened data. Thus, ICA gives you absolutely no advantage over SVD. Empirical verification of this claim is given in Section 6. Note

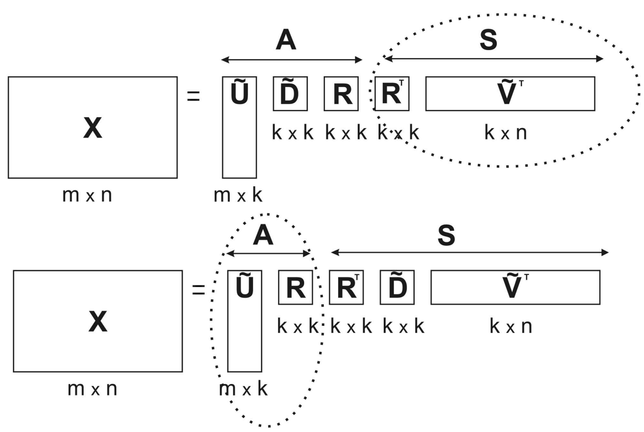
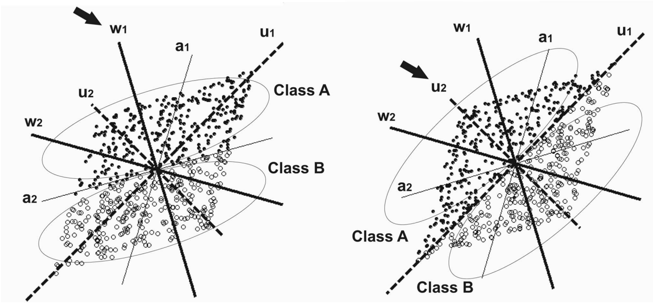


Fig. 3. Different architectures for the ICA decomposition of an image set: Do we want independence in A or in S?



(a) (b)

Fig. 4. The *best* selection of the features may be different depending on the classes of data set. In (a), the classes are perfectly separated using just the projection over the ICA direction $\mathbf{w}_1$, while in (b), the classes are better separated using the direction of the eigenvector $\mathbf{u}_2$.

also that, before us, Yang et al [32] came to the exact same conclusion based on experiments on the FERET face database.

However, there are many scenarios in which ICA can give you a different (and, hence, possibly better) result than SVD. First, if the used classifier is rotationally noninvariant, there may be advantages to performing the rotation ICA gives [6]. Second, if a feature selection step is employed and only a subset of all components are used for classification [4], the subspaces selected can differ, and hence, the classification results may differ as well. And, finally, when using Comon's algorithm, or in some cases when using Infomax, the found components are not necessarily a rotation of the whitened data and, hence, the equivalence can disappear.

In conclusion, when employing ICA it is important to use either feature selection or a rotation variant classifier, or both, since, otherwise, there is little justification for performing an expensive ICA optimization, instead of simply employing the SVD.

5 Selecting Subsets of Components

In the previous sections, we showed that, as typically applied, there is no reason to prefer ICA over whitened data. However, the performance of ICA and PCA may differ when a subset of components is used for classification. To illustrate this fact, an artificial data set similar to the sub-Gaussian data in Fig. 1 has been assigned to two different classes in two extreme examples in Fig. 4. In both plots, the directions of the eigenvectors, $\mathbf{U}$, and the ICA directions, $\mathbf{W}$ and $\mathbf{A}$, from the artificial data set are drawn. As both techniques are unsupervised, these directions are independent from the classes in the data set, so they are equal for both extreme examples. In Fig. 4a, it is easy to appreciate how the classes are perfectly separated using just the projection over the ICA direction $\mathbf{w}_1$, while in Fig. 4b, the classes are better separated using the direction of eigenvector $\mathbf{u}_2$. Therefore, in each example, a feature selection step may help to reduce the dimensionality and improve the classification. If no feature selection is carried out, ICA and whitened PCA perform exactly equally well on both data sets, provided a rotationally invariant classifier is used.

6 Experimental Results

Some experiments with real data have been carried out in order to test our hypothesis about the ICA/whitened PCA equivalence. Both the ORL face database [1] and the COIL-100 object database [26] have been used, and the results have been obtained both for FastICA and Infomax algorithms. The classifier used is a 1-NN with Euclidean distance. Due to space limitations, only a summary of the results obtained is shown in this paper; the full set of experiments and the Matlab code used are available at http://isa.umh.es/arvc/personal/suni/ica_pca/index.html.

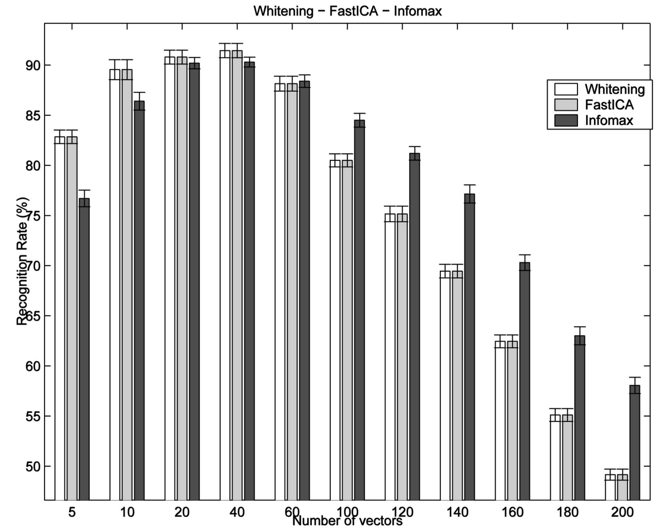


Fig. 5. ICA/whitened PCA comparison using the ORL face database using all components.

Fig. 5 (experiment 12 of the above mentioned URL) shows the results obtained by whitened PCA, FastICA and Infomax with the ORL face database. For a number of components ranging from 5 to 200, the average and standard deviation of 10 repetitions of the same experiment are shown. No feature selection is performed.

From these results, it becomes clear that FastICA and whitened PCA perform equally, and that Infomax shows some differences. These differences are more clear when the number of components used is either very low or very high. Particularly, when the number of components is very high, the recognition rates obtained by Infomax are clearly better than those of FastICA or whitened PCA. Although these good results are found when the number of components is far from the optimum in terms of recognition rates, they deserve a further study in order to find an explanation, which may be related to the nonorthogonality of the components returned by Infomax. Such study is left for further work, as it falls beyond the scope of this short paper.

Fig. 6 shows similar experiments to those of Fig. 5 but considering feature selection (experiment 21 of the above mentioned URL). In these figures, the original data (40 classes with 10 examples per class, so the size of $\mathbf{X}$ is $10{,}304 \times 400$) has been reduced to just 5, 10, and 20 components (5×400, 10×400, and

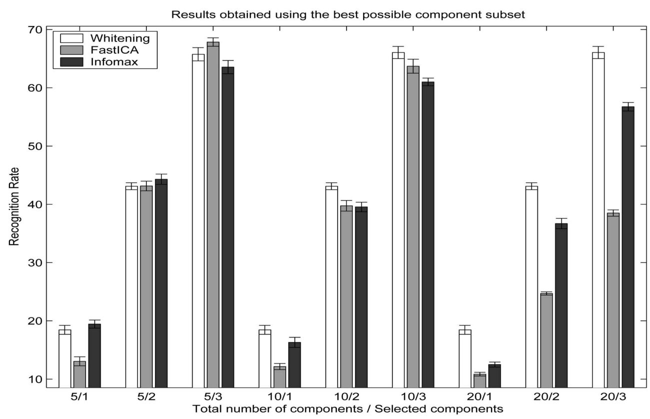


Fig. 6. ICA/whitened PCA comparison using the ORL database when an exhaustive feature selection process is carried out.

20 $\times$ 400) by means of whitening, Infomax, and FastICA. For each method, the best component is selected as the one which offers the best classification results when used alone. The same process is performed to find the best subset of two and three components using an exhaustive search or "brute force" method, so we have computed the recognition rates of all possible subsets of two and three components. In this way, the results show the recognition rates that could be obtained with each algorithm if a perfect selection of components is carried out. We have used a quite small number of components in order to be able to perform an exhaustive search. Such search quickly becomes computationally intractable when the number of components grows.

The results show clearly that a feature selection process may have a strong influence in the recognition rates obtained by each algorithm. FastICA and whitened PCA no longer perform equally; and the same applies to Infomax. Even though the results of the three algorithms differ to a great extent, there is no clear winner. As all of the algorithms are unsupervised, the results are highly dependent on the class distributions. This fact was previously illustrated in Section 5 with the artificial data sets from Fig. 4.

7 DISCUSSION

Visual appearance-based object recognition methods are usually based on feature extraction techniques such as PCA and ICA. In the present paper, it has been shown how ICA and PCA are closely connected and under which circumstances their perfomance is totally equivalent. Their performance may differ significantly if

1. a feature selection process is carried out,
2. a rotationally noninvariant classifier is used,
3. a renormalization such as that used in Comon's algorithm is performed, or
4. Infomax is used and the data yields independent components which are not close to an orthogonal transform from the whitened data.

ACKNOWLEDGMENTS

The authors wish to thank the anonymous reviewers for their insightful suggestions during the review process. Maria Asuncion Vicente would also like to thank C. Fernández for helpful discussions and collaboration. Patrik O. Hoyer was supported by the Academy of Finland, project #204826.

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