clasa a VI-a

1. Calculați:

$A=(\overline{ab}+\frac{\overline{abab}}{101}+\frac{\overline{ababab}}{10101}+\frac{\overline{abababab}}{1010101}+\dots+\frac{\overline{ababab\dots ab}}{\underbrace{1010\dots101}_{100 de 1}})$

b) Arată că $\frac{A}{1000a+10b}$ este natural

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Rezolvare

$\frac{\overline{abab}}{101}=\frac{\overline{ab}\cdot 100+\overline{ab}}{101}=\frac{\overline{ab}(100+1)}{101}=\frac{\overline{ab}\cdot 101}{101}=\overline{ab}$

$\frac{\overline{ababab}}{10101}=\frac{\overline{ab}\cdot 10^4+\overline{ab}\cdot 10^2+\overline{ab}}{10101}=\frac{\overline{ab}(10^4+10^2+1)}{10^4+10^2+1}=\overline{ab}$

$\frac{\overline{ababab\dots ab}}{10101\dots 101}=\frac{\overline{ab}\cdot 10^{98}+\overline{ab}\cdot 10^{96}+\dots+\overline{ab}}{10^{98}+10^{96}+\dots+10+1}=$

$=\frac{\overline{ab}(10^{98}+10^{96}+\dots+1)}{10^{98}+10^{96}+\dots+1}=\overline{ab}$

$A=\underbrace{\overline{ab}+\overline{ab}+\dots+\overline{ab}}_{100\ ori}=100\overline{ab}$

b) $\frac{A}{1000a+10b}=\frac{100\overline{ab}}{100(10a+b)}=\frac{10a+b}{10a+b}=1\in\mathbb{N}$

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(2) $A = 2020^{100} - 2019 \cdot 2020^{99} - 2019 \cdot 2020^{98} - \ldots - 2019 \cdot 2020$

$B = \frac{1}{1+2} + \frac{1}{2+3} + \ldots + \frac{1}{1+2+\ldots+2019}$

Să se calculeze $\left(B + \frac{2}{A}\right)^{2020}$ Petrescu Octavi.. Petrescu Mih...

Rez

$A = 2020^{99}(\underbrace{2020-2019}_{1}) - 2019 \cdot 2020^{98} - \ldots - 2019 \cdot 2020^{2} - 2019 \cdot 2020$

$= \underbrace{2020^{99} - 2019 \cdot 2020^{98}}_{} - \ldots - 2019 \cdot 2020^{2} - 2019 \cdot 2020 =$

$= 2020^{98}(2020-2019) - \ldots - 2019 \cdot 2020^{2} - 2019 \cdot 2020 =$

$= 2020^{98} - 2020^{97} - \ldots - 2019 \cdot 2020^{2} - 2019 \cdot 2020 =$

— —

$= 2020^{2} - 2019 \cdot 2020 = 2020(2020-2019) = \underline{2020}$

$B = \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots + \frac{1}{1+2+3+\ldots+2019} = \frac{1}{\frac{2\cdot 3}{2}} + \frac{1}{\frac{3\cdot 4}{2}} + \ldots + \frac{1}{\frac{2019\cdot 2020}{2}} =$

$= \frac{2}{2\cdot 3} + \frac{2}{3\cdot 4} + \ldots + \frac{2}{2019\cdot 2020} = 2\cdot\left[\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{2019\cdot 2020}\right] =$

$= 2\left(\frac{3-2}{2\cdot 3} + \frac{4-3}{3\cdot 4} + \ldots + \frac{2020-2019}{2019\cdot 2020}\right) = 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots + \frac{1}{2019} - \frac{1}{2020}\right)$

$= 2\left(\frac{1}{2} - \frac{1}{2020}\right) = 2\left(\frac{1010}{2020} - \frac{1}{2020}\right) = \frac{2 \cdot 1009}{2020} = \frac{1009}{1010}$

$B + \frac{2}{A} = \frac{1009}{1010} + \frac{2}{2020} = \frac{1009}{1010} + \frac{1}{1010} = \frac{1010}{1010} = 1$

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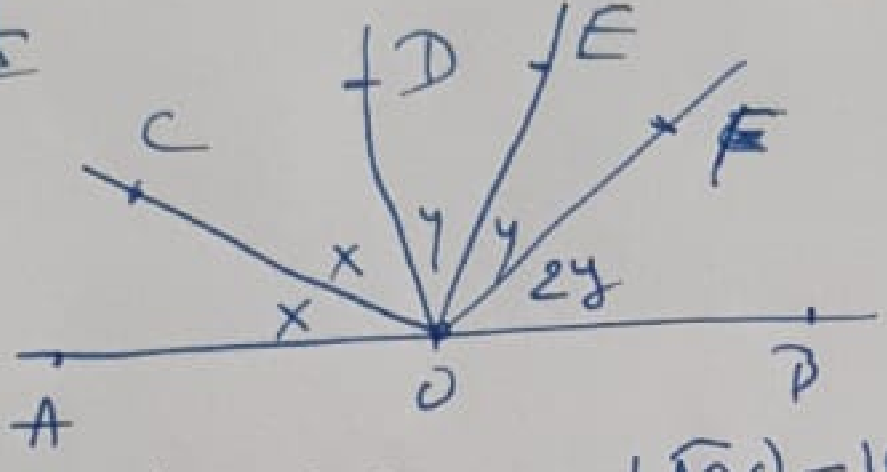
(3) Se consideră unghiul alungit $\widehat{AOB}$, C, D, E, F puncte în această ordine de aceeași parte față de dreapta AO (de la stânga spre dreapta). [OC este bisectoarea ∢AOD, [OE – este bisectoarea ∢DOF, [OF este bisectoarea unghiului $\widehat{DOB}$. Dacă {m($\widehat{AOC}$), m(m($\widehat{DOE}$))} sunt direct proporționale cu 5 și 2 să se afle măsurile unghiurilor $\widehat{AOE}$, $\widehat{BOF}$

Potrean Octavia. Tofrean Mira

Rez



[OC – bis ∢AOD ⇒ m($\widehat{AOC}$) = m($\widehat{COD}$) $\overset{not}{=}$ x

[OE bis ∢DOF ⇒ m($\widehat{DOE}$) = m($\widehat{EOF}$) $\overset{not}{=}$ y

[OF – bis ∢DOB ⇒ m($\widehat{DOF}$) = m($\widehat{FOB}$) = 2y

m($\widehat{AOC}$) + m($\widehat{COD}$) + m($\widehat{DOE}$) + m($\widehat{EOF}$) + m($\widehat{FOB}$) = 180°

$x + x + y + y + 2y = 180°$

$2x + 4y = 180°$

$x + 2y = 90°$

(x, y) d.p. $(5, 2)$ ⇒ $\frac{x}{5} = \frac{y}{2} = k, k \neq 0$ ⇒ $x = 5k$, $y = 2k$

$5k + 2 \cdot 2k = 90°$

$9k = 90°$ ⇒ $k = 10°$ ⇒ $x = 5 \cdot 10° = 50°$, $y = 2 \cdot 10° = 20°$

m($\widehat{AOE}$) = 50° + 50° + 20° = 120°

m($\widehat{BOF}$) = 2 · 20° = 40°

m($\widehat{DOB}$) = 4 · 20° = 80°