Finite Difference

FD/CD/BD Representation

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FD/CD/BD Representation using Taylor Series

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In[819]:=

```
ClearAll[Evaluate[Context[]<>"*"]]
```

Taylor Series

Functions Definition

In[1191]:=

```
TS[f_,S_,s_,order_,n_]:=Normal[Series[f[Subscript[x,S]],{Subscript[x,S],Subscript[x,s],order

DT[f_,S_,s_,order_,n_]:=SeriesCoefficient[TS[f,S,s,order,n]+O[Δx]^(order+1),1]
```

Examples

Forward

In[1194]:=

```
f[x_{i+1}]==TS[f,i+1,i,5,Δx]
```

Out[1194]=

$$f[x_{1+i}] == f[x_i] + \Delta x\, f'[x_i] + \frac{1}{2}\Delta x^2 f''[x_i] + \frac{1}{6}\Delta x^3 f^{(3)}[x_i] + \frac{1}{24}\Delta x^4 f^{(4)}[x_i] + \frac{1}{120}\Delta x^5 f^{(5)}[x_i]$$

Backward

In[1195]:=

```
f[x_{i-1}]==TS[f,i-1,i,5,-Δx]
```

Out[1195]=

$$f[x_{-1+i}] == f[x_i] - \Delta x\, f'[x_i] + \frac{1}{2}\Delta x^2 f''[x_i] - \frac{1}{6}\Delta x^3 f^{(3)}[x_i] + \frac{1}{24}\Delta x^4 f^{(4)}[x_i] - \frac{1}{120}\Delta x^5 f^{(5)}[x_i]$$

Derivative Term

In[1152]:=

```
DT[f,i+1,i,2,Δx]
```

Out[1152]=

$$f'[x_i]$$

Taylor Series Plot

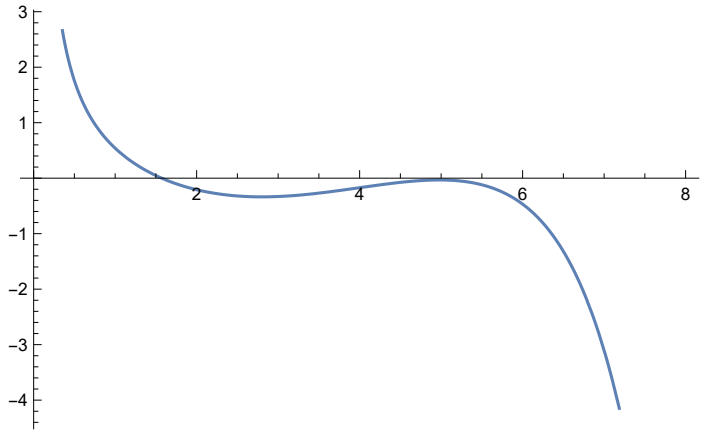
In[1211]:=

```
Series[Cos[x]/x,{x,0,10}]
Plot[Evaluate[Normal[1/x - x/2 + x^3/24 - x^5/720 + x^7/40320 - x^9/3628800 + O[x]^11]],{x,0,8}]
```

Out[1211]=

$$\frac{1}{x} - \frac{x}{2} + \frac{x^3}{24} - \frac{x^5}{720} + \frac{x^7}{40\,320} - \frac{x^9}{3\,628\,800} + O[x]^{11}$$

Out[1212]=



First Order Difference

Forward Difference

Functions Definition

```
FD[f_,S_,s_,order_,n_]:=CoefficientList[(-TS[f,S,s,order,n]+f[Subscript[x,S]]),Δx][[1]]/Δx+Sim
```

Examples

In[1091]:=

```
f'[x_i]==FD[f,i+1,i,3,Δx]
```

Out[1091]=

$$f'[x_i] == \frac{-f[x_i] + f[x_{1+i}]}{\Delta x} - \frac{1}{2} f''[x_i]\,\Delta x - \frac{1}{6} f^{(3)}[x_i]\,\Delta x^2 + O[\Delta x]^3$$

Backward Difference

Functions Definition

```
BD[f_,S_,s_,order_,n_]:=CoefficientList[(TS[f, S, s, order, n] - f[Subscript[x, S]]), Δx][[1]]
   DT[f, S, s, order, n]] + O[Δx]^order
```

Examples

In[1114]:=
```
f'[xi]==BD[f,i-1,i,3,-Δx]
```

Out[1114]=
$$f'[x_i] == \frac{-f[x_{-1+i}] + f[x_i]}{\Delta x} + \frac{1}{2} f''[x_i]\,\Delta x - \frac{1}{6} f^{(3)}[x_i]\,\Delta x^2 + 0[\Delta x]^3$$

Central Difference

Functions Definition

```
CD[f_,SF_,SB_,s_,order_,n_]:=((f[Subscript[x,SF]]-f[Subscript[x,SB]])-(TS[f,SF,s,order,n]-TS
```

Examples

In[1118]:=
```
f'[xi]==CD[f,i+1,i-1,i,3,Δx]
```

Out[1118]=
$$f'[x_i] == \frac{-f[x_{-1+i}] + f[x_{1+i}]}{2\,\Delta x} - \frac{1}{6} f^{(3)}[x_i]\,\Delta x^2 + 0[\Delta x]^4$$

Second Order Difference

Forward Difference

In[1170]:=
```
FDS[f_,S_,s_,order_,n_]:=Simplify[((f'[Subscript[x, S+1]]-2*f'[Subscript[x, S]])-(TS[f,S+1,s
```

Examples

In[1174]:=
```
f''[xi]==FDS[f,i+1,i,2,Δx]
```

Out[1174]=
$$f''[x_i] == \frac{f[x_i] - 2\,f'[x_{1+i}] + f'[x_{2+i}]}{\Delta x^2} + 0[\Delta x]^1$$

Backward Difference

In[1172]:=
```
BDS[f_,S_,s_,order_,n_]:=Simplify[((f'[Subscript[x, S-1]]-2*f'[Subscript[x, S]])-(TS[f,S-1,s
```

Examples

In[1175]:=

```
f''[xi]==BDS[f,i-1,i,2,-Δx]
```

Out[1175]=

$$f''[x_i] == \frac{f[x_i] + f'[x_{-2+i}] - 2\,f'[x_{-1+i}]}{\Delta x^2} + O[\Delta x]^1$$

Central Difference

In[1188]:=

```
CDS[f_,SF_,SB_,s_,order_,n_]:=Simplify[((f'[Subscript[x, SF]]+f'[Subscript[x, SB]])-(TS[f,SF
```

Examples

In[1189]:=

```
CDS[f,i+1,i-1,i,2,Δx]
```

Out[1189]=

$$\frac{-2\,f[x_i] + f'[x_{-1+i}] + f'[x_{1+i}]}{\Delta x^2} + O[\Delta x]^2$$