$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=Q$$

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial U}\frac{\partial U}{\partial x}+\frac{\partial G}{\partial U}\frac{\partial U}{\partial y}=Q \quad \rightarrow \quad \frac{\partial U}{\partial t}+[A]\frac{\partial U}{\partial x}+[B]\frac{\partial U}{\partial y}=Q$$

$$\frac{\partial U}{\partial V}\frac{\partial V}{\partial t}+\frac{\partial F}{\partial U}\frac{\partial U}{\partial V}\frac{\partial V}{\partial x}+\frac{\partial G}{\partial U}\frac{\partial U}{\partial V}\frac{\partial V}{\partial y}=Q \quad \rightarrow \quad [T]\frac{\partial V}{\partial t}+[A][T]\frac{\partial V}{\partial x}+[B][T]\frac{\partial V}{\partial y}=Q$$

$$\frac{\partial U}{\partial V}\frac{\partial V}{\partial t}+\frac{\partial F}{\partial V}\frac{\partial V}{\partial x}+\frac{\partial G}{\partial V}\frac{\partial V}{\partial y}=Q$$

$$[T]\frac{\partial V}{\partial t}+\frac{\partial F}{\partial V}\frac{\partial V}{\partial x}+\frac{\partial G}{\partial V}\frac{\partial V}{\partial y}=Q$$

$$\frac{\partial V}{\partial t}+[T]^{-1}\frac{\partial F}{\partial V}\frac{\partial V}{\partial x}+[T]^{-1}\frac{\partial G}{\partial V}\frac{\partial V}{\partial y}=[T]^{-1}Q$$

$$\frac{\partial V}{\partial t}+[A]'\frac{\partial V}{\partial x}+[B]'\frac{\partial V}{\partial y}=Q'$$

$$[A]=\frac{\partial F}{\partial U}=\frac{\partial F}{\partial V}\frac{\partial V}{\partial U}=\frac{\partial F}{\partial V}[T]^{-1}$$

$$[B]=\frac{\partial G}{\partial U}=\frac{\partial G}{\partial V}\frac{\partial V}{\partial U}=\frac{\partial G}{\partial V}[T]^{-1}$$

$$[A]'=[T]^{-1}\frac{\partial F}{\partial V}=[T]^{-1}[A][T]$$

$$[B]'=[T]^{-1}\frac{\partial G}{\partial V}=[T]^{-1}[B][T]$$

Let

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E \end{bmatrix},$$

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} \rho u \\ P + \rho u^2 \\ \rho u v \\ Eu + Pu \end{bmatrix},$$

$$G = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix} = \begin{bmatrix} \rho v \\ \rho u v \\ P + \rho v^2 \\ Ev + Pv \end{bmatrix},$$

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \rho \\ u \\ v \\ P \end{bmatrix},$$

where

$$P = \frac{\mathrm{R}}{c_v}\left(E - \frac{1}{2}\rho(u^2 + v^2)\right) = (\gamma - 1)\left(E - \frac{1}{2}\rho(u^2 + v^2)\right) = \beta\left(E - \frac{1}{2}\rho(u^2 + v^2)\right)$$

$$\rightarrow E = \frac{P}{\beta} + \frac{1}{2}\rho(u^2 + v^2) = \frac{V_4}{\beta} + \frac{1}{2}V_1(V_2^2 + V_3^2)$$

$$U = \begin{bmatrix} V_1 \\ V_1 V_2 \\ V_1 V_3 \\ \frac{V_4}{\beta} + \frac{1}{2}V_1(V_2^2 + V_3^2) \end{bmatrix}$$

$$F = \begin{bmatrix} V_1 V_2 \\ V_4 + V_1 V_2^2 \\ V_1 V_2 V_3 \\ \left(\frac{V_4}{\beta} + \frac{1}{2}V_1(V_2^2 + V_3^2)\right)V_2 + V_2 V_4 \end{bmatrix}$$

$$G = \begin{bmatrix} V_1 V_3 \\ V_1 V_2 V_3 \\ V_4 + V_1 V_3^2 \\ \left(\frac{V_4}{\beta} + \frac{1}{2}V_1(V_2^2 + V_3^2)\right)V_3 + V_3 V_4 \end{bmatrix}$$

$Get\ [T]$:

$$[T]=\frac{\partial U}{\partial V}=\begin{bmatrix}\frac{\partial U_1}{\partial V_1} & \frac{\partial U_1}{\partial V_2} & \frac{\partial U_1}{\partial V_3} & \frac{\partial U_1}{\partial V_4}\\ \frac{\partial U_2}{\partial V_1} & \frac{\partial U_2}{\partial V_2} & \frac{\partial U_2}{\partial V_3} & \frac{\partial U_2}{\partial V_4}\\ \frac{\partial U_3}{\partial V_1} & \frac{\partial U_3}{\partial V_2} & \frac{\partial U_3}{\partial V_3} & \frac{\partial U_3}{\partial V_4}\\ \frac{\partial U_4}{\partial V_1} & \frac{\partial U_4}{\partial V_2} & \frac{\partial U_4}{\partial V_3} & \frac{\partial U_4}{\partial V_4}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ V_2 & V_1 & 0 & 0\\ V_3 & 0 & V_1 & 0\\ \frac{1}{2}(V_2^2+V_3^2) & V_1V_2 & V_1V_3 & \frac{1}{\beta}\end{bmatrix}$$

$$[T]=\begin{bmatrix}1 & 0 & 0 & 0\\ u & \rho & 0 & 0\\ v & 0 & \rho & 0\\ \frac{1}{2}(u^2+v^2) & \rho u & \rho v & \frac{1}{\beta}\end{bmatrix}$$

$Get\ \frac{\partial F}{\partial V}$:

$$F = \begin{bmatrix} V_1V_2 \\ V_4 + V_1V_2^2 \\ V_1V_2V_3 \\ \left(\frac{V_4}{\beta} + \frac{1}{2}V_1(V_2^2 + V_3^2)\right)V_2 + V_2V_4 \end{bmatrix} = \begin{bmatrix} V_1V_2 \\ V_4 + V_1V_2^2 \\ V_1V_2V_3 \\ \left(\frac{V_4}{\beta}V_2 + \frac{1}{2}V_1(V_2^3 + V_2V_3^2)\right) + V_2V_4 \end{bmatrix}$$

$$\frac{\partial F}{\partial V} = \begin{bmatrix} \frac{\partial F_1}{\partial V_1} & \frac{\partial F_1}{\partial V_2} & \frac{\partial F_1}{\partial V_3} & \frac{\partial F_1}{\partial V_4} \\ \frac{\partial F_2}{\partial V_1} & \frac{\partial F_2}{\partial V_2} & \frac{\partial F_2}{\partial V_3} & \frac{\partial F_2}{\partial V_4} \\ \frac{\partial F_3}{\partial V_1} & \frac{\partial F_3}{\partial V_2} & \frac{\partial F_3}{\partial V_3} & \frac{\partial F_3}{\partial V_4} \\ \frac{\partial F_4}{\partial V_1} & \frac{\partial F_4}{\partial V_2} & \frac{\partial F_4}{\partial V_3} & \frac{\partial F_4}{\partial V_4} \end{bmatrix}$$

$$= \begin{bmatrix} V_2 & V_1 & 0 & 0 \\ V_2^2 & 2V_1V_2 & 0 & 1 \\ V_2V_3 & V_1V_3 & V_1V_2 & 0 \\ \frac{1}{2}(V_2^2 + V_3^2)V_2 & \frac{V_4}{\beta} + \frac{1}{2}V_1(3V_2^2 + V_3^2) + V_4 & V_1(V_2V_3) & \frac{V_2}{\beta} + V_2 \end{bmatrix}$$

$$= \begin{bmatrix} u & \rho & 0 & 0 \\ u^2 & 2\rho u & 0 & 1 \\ uv & \rho v & \rho u & 0 \\ \frac{1}{2}(u^2 + v^2)u & \frac{P}{\beta} + \frac{1}{2}\rho(3u^2 + v^2) + P & \rho(uv) & \frac{u}{\beta} + u \end{bmatrix}$$

$$= \begin{bmatrix} u & \rho & 0 & 0 \\ u^2 & 2\rho u & 0 & 1 \\ uv & \rho v & \rho u & 0 \\ \frac{1}{2}(u^2 + v^2)u & \frac{(1+\beta)}{\beta}P + \frac{1}{2}\rho(3u^2 + v^2) & \rho uv & \frac{(1+\beta)}{\beta}u \end{bmatrix}$$

$$Get\ [A] = \frac{\partial F}{\partial V}[T]^{-1} \quad then \quad [A]' = [T]^{-1}[A][T]$$

$$Or\ directly\ get\ [A]' = [T]^{-1}\frac{\partial F}{\partial V}$$

Do the same procedure for $Y\ direction$

$$get\ \frac{\partial G}{\partial V}$$

$$then\ Get\ [B] = \frac{\partial G}{\partial V}[T]^{-1} \quad then \quad [A]' = [T]^{-1}[B][T]$$

$$Or\ directly\ get\ [B]' = [T]^{-1}\frac{\partial G}{\partial V}$$