Chapter 1: First-Order Differential Equations

Definitions

Differential equation: *a function that contains at least of its own derivatives*

Ordinary differential equation: *the unknown function depends on one unknown variable*

General solution: *the solution to a differential equation with unknown initial values*

Particular solution: *the solution to a differential equation with known initial values*

Singular solution: *occurs in constant functions* $y(x) \equiv c$ *and nonlinear ODEs*

Initial value problem (IVP): *a type of problem often involving particular solutions*

Order of a differential equation: *the order of the highest derivative*

General form of a differential equation: F *is a real-valued function of* $n + 2$ *variables*

$$F\left(x, y, y', y'', \ldots, y^{(n)}\right) = 0$$

Normal form of a differential equation: *solving for the highest order derivative*

$$y^{(n)} = G(x, y, y', y'', \ldots, y^{(n-1)})$$

Linear differential equations: *based on the trend of order of derivatives, not independent variable*

$$F_0(x)y^{(n)} + F_1(x)y^{(n-1)} + \cdots + F_{n-1}(x)y' + F_n(x)y = G(x)$$

where $F_n(x)$ *does* <u>*not*</u> *have to be linear*

Slope field: *collection of lines segments that represent the slope of the tangent line at each selected point*

Existence and Uniqueness Theorem

- Used in IVPs
- Take $\frac{dy}{dx} = f(x, y), y(a) = b$

(a) Existence
  - If f is continuous on some open rectangle R, then at least one solution exists in the x-subinterval I containing $x = a$

(b) Uniqueness
  - If f and f_y are continuous on R, then the IVP has some unique solution in the open x-subinterval I containing $x = a$

Differential Equations and Solutions

General solution for logistic equation (1)

$$\frac{dP}{dt} = kP(t) \tag{1a}$$

$$P(t) = Ae^{kt} \tag{1b}$$

Method of integrating factors (2)

$$y' + P(x)y = Q(x) \tag{2a}$$

$$\rho(x) = e^{\int P(x)\,dx} \tag{2b}$$

$$\rho(x)(y' + P(x)y) = \rho(x)Q(x) \tag{2c}$$

$$\rho'(x)y + \rho(x)y' = \rho(x)Q(x) \tag{2d}$$

$$(\rho(x)y)' = \rho(x)Q(x) \tag{2e}$$

$$\rho(x)y = \int \rho(x)Q(x)\,dx \tag{2f}$$

$$y = \frac{1}{\rho(x)}\int \rho(x)Q(x)\,dx + \frac{C}{\rho(x)} \tag{2g}$$