EQUATIONS:

Distance d between two points P_1 and P_2…

$$d\big((x_1, y_1, z_1), (x_2, y_2, z_2)\big) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation of a sphere with radius r and center (h, k, l)…

$$r^2 = (x - h)^2 + (y - k)^2 + (z - l)^2$$

Dot product by matrix…

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Dot product by geometry…

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \cos\theta \, \|\vec{b}\|$$

Test for orthogonality…

$$\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$$

Scalar projection…

$$comp_{\vec{a}}\vec{b} = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|}$$

Vector projection…

$$proj_{\vec{a}}\vec{b} = \left(comp_{\vec{a}}\vec{b}\right)\left(\frac{\vec{a}}{\|\vec{a}\|}\right) = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2}\vec{a}$$

Cross product by matrix…

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{\imath} \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Cross product by geometry…

$$\vec{a} \times \vec{b} = \|\vec{a}\| \sin\theta \, \|\vec{b}\|$$

Test for parallel…

$$\vec{a} \times \vec{b} = 0$$

Area of a parallelogram with sides $\vec{a}$ and $\vec{b}$…

$$A = \|\vec{a} \times \vec{b}\|$$

Lines...

$$\vec{r} = \vec{r}_0 + t\vec{v}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + t\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x_0 + ta \\ y_0 + tb \\ z_0 + tc \end{bmatrix}$$

"line = point + t*direction"

Planes with normal vector $\vec{n} = \langle a, b, c \rangle$ ...

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Limits…

$$\lim_{t \to a} \vec{r}(t) = \begin{bmatrix} \lim\limits_{t \to a} f(t) \\ \lim\limits_{t \to a} g(t) \\ \lim\limits_{t \to a} h(t) \end{bmatrix}$$

Derivatives..

$$\vec{r}'(t) = \begin{bmatrix} f'(t) \\ h'(t) \\ g'(t) \end{bmatrix}$$

Indefinite integral…

$$\int \vec{r}(t)dt = \begin{bmatrix} F(t) \\ G(t) \\ H(t) \end{bmatrix} + \vec{c}$$

Definite integral…

$$\int_{t_i}^{t_f} \vec{r}(t)dt = \begin{bmatrix} \int_{t_i}^{t_f} f(t)dt \\ \int_{t_i}^{t_f} g(t)dt \\ \int_{t_i}^{t_f} h(t)dt \end{bmatrix}$$

Unit tangent vector at $t = t_0$…

$$\frac{\vec{r}'(t_0)}{\|\vec{r}'(t_0)\|} = \frac{\vec{v}(t_0)}{\|\vec{v}(t_0)\|}$$

Definite arclength…

$$L = \int_{t_i}^{t_f} \|\vec{r}\,'(t)\|\, dt = \int_{t_i}^{t_f} \sqrt{(f'(t)^2) + \left(g'(t)\right)^2 + \left(h'(t)\right)^2}\, dt$$

Arclength function…

$$s(t) = \int_a^t s(u) dt$$

Unit tangent vector…

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$$

Unit normal vector…

$$\vec{N}(t) = \frac{\vec{T}'(t)}{\left\|\vec{T}'(t)\right\|}$$

Unit binormal vector…

$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

Curvature…

$$\kappa = \left\|\frac{d\vec{T}}{ds}\right\| = \frac{\left\|\vec{T}'(t)\right\|}{\|\vec{r}\,'(t)\|} = \frac{\|\vec{r}\,'(t) \times \vec{r}\,''(t)\|}{\|\vec{r}\,'(t)\|^3}$$

Position…

$$\vec{r}(t) = \int \vec{v}(t) dt$$

Velocity…

$$\vec{v}(t) = \frac{d\vec{x}}{dt} = \int \vec{a}(t) dt$$

Acceleration…

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$