Chapter 2: Vectors: Force & Position

Basic Concepts (2.1)

Performing vector operations

Law of sines

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Law of cosines

$$A = \sqrt{B^2 + C^2 - 2BC\cos\alpha}$$

$$B = \sqrt{A^2 + C^2 - 2AC\cos\beta}$$

$$C = \sqrt{A^2 + B^2 - 2AB\cos\gamma}$$

Cartesian Representation of Vectors in 2D (2.2)

Unit vector notation

$$\hat{u} = \frac{\vec{v}}{|\vec{v}|} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$$

General vector notation

$$\vec{v} = \vec{v}_x + \vec{v}_y = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\vec{v} = v(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) = v\left(\cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath}\right)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x}$$

Vector transformations

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = v_t\hat{t} + v_n\hat{n}$$

To x and y…

$$v_x = v_t\cos\phi - v_n\sin\phi$$

$$v_y = v_t\sin\phi + v_n\cos\phi$$

To t and n…

$$v_t = v_x \cos\phi + v_y \sin\phi$$

$$v_n = -v_x \sin\phi + v_y \cos\phi$$

Cartesian Representation of Vectors in 3D (2.3)

General vector notation

$$\vec{v} = \vec{v}_x + \vec{v}_y + \vec{v}_z = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\vec{F} = F\frac{\vec{r}}{r}$$

Direction cosines

$$\vec{v} = v(\cos\theta_x\,\hat{\imath} + \cos\theta_y\,\hat{\jmath} + \cos\theta_z\,\hat{k})$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\cos\theta_x = \frac{v_x}{v};\ \cos\theta_y = \frac{v_y}{v};\ \cos\theta_z = \frac{v_z}{v}$$

Vector Dot Product (2.4)

Base definition

$$\vec{A}\cdot\vec{B} = A\cos\theta\, B$$

Properties

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

$$s(\vec{A}\cdot\vec{B}) = (s\vec{A})\cdot\vec{B} = \vec{A}\cdot(s\vec{B})$$

$$(\vec{A}+\vec{B})\cdot\vec{C} = (\vec{A}\cdot\vec{C}) + (\vec{B}\cdot\vec{C})$$

Cartesian coordinates

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\cdot\vec{B} = \begin{bmatrix}A_x\\A_y\\A_z\end{bmatrix}\cdot\begin{bmatrix}B_x\\B_y\\B_z\end{bmatrix}$$

Component of a vector in a particular direction

$$F_{parallel} = F_{||} = \vec{F} \cdot \frac{\vec{r}}{r}$$

$$F^2 = F_{\perp}^2 + F_{||}^2$$

$$\vec{F} = \vec{F}_{\perp} + \vec{F}_{||}$$

Vector Cross Product (2.5)

Base definition

$$\left|\vec{A} \times \vec{B}\right| = A \sin\theta \, B$$

Properties

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$s\left(\vec{A} \times \vec{B}\right) = \left(s\vec{A}\right) \times \vec{B} = \vec{A} \times (s\vec{B})$$

$$\left(\vec{A} + \vec{B}\right) \times \vec{C} = \left(\vec{A} \times \vec{C}\right) + \left(\vec{B} \times \vec{C}\right)$$

Cartesian coordinates

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Scalar triple product

$$\left(\vec{A} \times \vec{B}\right) \cdot \vec{C} = \begin{vmatrix} C_x & C_y & C_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$