## Chapter 8: Matrix Exponential Methods

## **Definitions**

Fundamental matrix: takes the form

$$\Phi(t) = \begin{bmatrix} \vec{x}_1 & \cdots & \vec{x}_n \end{bmatrix}$$

Nonhomogeneous linear system: takes the form

$$\vec{x}' = A\vec{x} + \vec{f}(t)$$

## **Differential Equations and Solutions**

Solution to homogenous systems (1)

$$\vec{x}' = A\vec{x} \tag{1a}$$

$$\vec{x} = c_1 \vec{x}_1 + \dots + c_n \vec{x}_n \tag{1bi}$$

$$\vec{x} = \Phi \vec{c} \tag{1bii}$$

Solution to nonhomogeneous systems (2)

$$\vec{x}' = A\vec{x} + \vec{f}(t) \tag{2a}$$

$$\vec{x}_p = guess \ based \ on \ \vec{f}(t)$$
 (2bi)

$$\vec{x}_p = \Phi \int \Phi^{-1} \vec{f} \, dt \tag{2bii}$$

$$\vec{x} = \vec{x}_c + \vec{x}_p \tag{2c}$$