Chapter 2: Mathematical Models and Numerical Methods

Definitions

Autonomous first-order differential equation: *when the independent variable does not appear explicitly. Often takes the form,*

$$\frac{dy}{dx} = f(y)$$

Equilibrium solution: *critical points; values of* $y = c$ *such that* $f(y) = 0$

Local error: *the error in each step of Euler's method between the actual* y *value and* y_{n+1}

Cumulative error: *the amount in which the final* y_n *value differs from the actual solution curve*

Differential Equations and Solutions

General population model (3)

$$\frac{dP}{dt} = \big(\beta(t) - \delta(t)\big)P(t) \tag{3a}$$

$$\frac{dP}{dt} = (\beta - \delta)P \tag{3b}$$

Logistic equation with birth rate β and death rate δ (4)

$$\beta(t) = \beta_0 - \beta_1 P(t) \tag{4a}$$

$$\frac{dP}{dt} = (\beta_0 - \beta_1 P - \delta_0)P \tag{4b}$$

$$\frac{dP}{dt} = (\beta_0 - \delta_0)P - \beta_1 P^2 \tag{4c}$$

Take $a = \beta_0 - \delta_0, b = \beta_1 : a, b > 0$ satisfies a logistic equation

Logistic equation with carrying capacity M (5)

$$\frac{dP}{dt} = kP(M - P) \tag{5a}$$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) \tag{5b}$$

$$M = \lim_{t \to \infty} P(t) \tag{5c}$$

Determining the stability of solutions (6)

Stable…

$$y(x) \to c \tag{6a}$$

Unstable…

$$y(x) \nrightarrow c \tag{6b}$$

Metastable…

$y(x)$ moves towards and away from c

Euler's method (7)

$$x_n = x_0 + nh \tag{7ai}$$

$$y_{n+1} = y_n + hf(x_n, y_n) \tag{7aii}$$

$$u_{n+1} = y_n + hf(x_n, y_n) \tag{7bi}$$

$$y_{n+1} = y_n + h\frac{f(x_n, y_n) + f(x_{n+1}, u_{n+1})}{2} \tag{7bii}$$

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