Chapter 6: Eigenvalues & Eigenvectors

Definitions

General forms…

$$A\vec{v} = \lambda\vec{v}$$

$$A - \lambda I_n = \begin{bmatrix} a_{11} - \lambda & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} - \lambda \end{bmatrix}$$

Characteristic polynomial: *takes the form*

$$p(\lambda) = \det(A - \lambda I_n)$$

Eigenvalue: λ; *characteristic value found from the roots of* $p(\lambda)\backslash$

Eigenvector: $\vec{v}$; *vector associated with eigenvalue*

$$(A - \lambda I_n)\vec{x} = \vec{0}$$

Eigenspace: *subspace of* $\mathbb{R}^n$ *satisfying,*

$$E_\lambda(A) = \{\vec{v} \in \mathbb{R}^n | A\vec{v} = \lambda\vec{v}\}$$

Geometric multiplicity: *the dimension of the eigenspace of a matrix*

$$p = \dim E_\lambda(A) = \dim[\text{null}\,(A - \lambda I_n)]$$

$$1 \leq p \leq k$$

Diagonalization of matrices

- Take a $n \times n$ matrix with n linearly independent eigenvectors
- Let $P = [\vec{v}_1 \quad \cdots \quad \vec{v}_n]$

$$P^{-1}AP = D$$

- Where $D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$

Notes based on Dr. Kaitlyn Phillipson, Math Department, University of Wisconsin-Madison