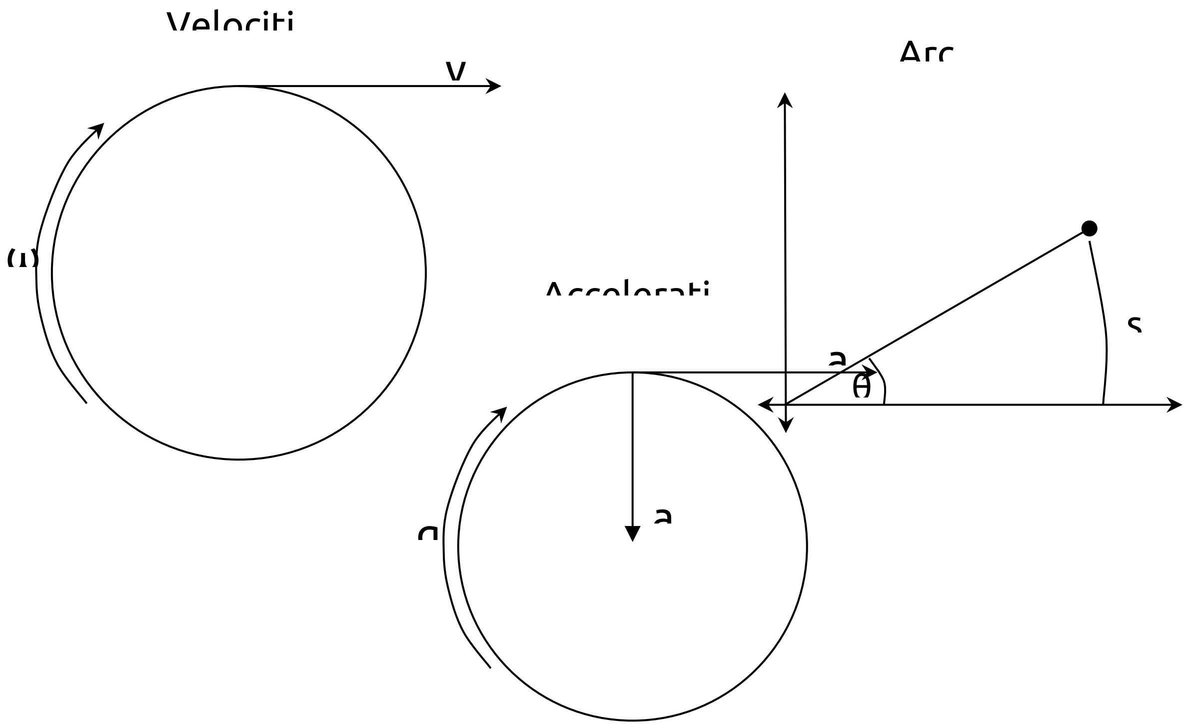
# Unit 7 → Rotational Motion

## Rotational Motion

- Rotational Motion
  - The motion of objects that spin about an axis
  - Variables are assigned new types of equations using properties of rotation such as the radius
- Arc Length
  - Distance the object has traveled around its circular path
  - Formula
    - $s = \theta r = \Delta x$
      - $s = arc\ length\ (m)$
      - $r = radius\ (m)$
      - $\theta = angular\ position\ (rad)$
  - The arc length around one full circle is the circumference
- Sign Convention
  - Counterclockwise is positive (+)
  - Clockwise is negative (-)
- Angular Position
  - Represented by $\theta$ and $\Delta\theta$, measured in radians or $rad$
  - Rotational equivalent to $x$ and $\Delta x$, which are measured in meters
- Angular Velocity
  - Rate at which angular position changes, measured in $rad/s$
    - Uniform circular motion = constant angular velocity
  - Represented by lowercase omega, $\omega$
  - Formula
    - $\omega = \frac{\Delta\theta}{\Delta t}$
  - Rotational equivalent to $v_x$, which is measured in $m/s$
  - May also be called angular speed or angular frequency
- Angular Acceleration
  - Rate at which angular velocity changes
  - Represented by lowercase alpha, $\alpha$
  - Formula
    - $\alpha = \frac{\Delta\omega}{\Delta t}$
  - Rotational equivalent to $a_x$, which is $m/s^2$

- Converting Equations
    - $\omega = \omega_0 + \alpha t$
        - Was $v_x = v_{x_0} + at$
    - $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
        - Was $\Delta x = v_{x_0} t + \frac{1}{2} a t^2$
    - $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$
        - Was $v_x^2 = v_{x_0}^2 + 2a_x\Delta x$
- Velocities
    - Angular speed ($\omega$)
    - Tangential velocity ($v_t$)
        - Tangent to the circle
        - $v_t = \omega r$
- Accelerations
    - Angular acceleration ($\alpha$)
    - Tangential acceleration ($a_t$)
        - Tangent to the circle
        - $a_t = \alpha r$
    - Centripetal acceleration ($a_c$)
        - $a_c = \frac{v_t^2}{r}$

# Concepts of Rotational Motion



## Rotational Forces

- Moment of Inertia
    - An object rotating wants to stay rotating and an object not rotating wants to stay not rotating unless acted upon by an unbalanced torque
    - The resistance to change in rotation
        - Stubbornness
    - Depends on…
        - Mass
        - Axis of rotation
    - Greater the radius, greater the moment of inertia
    - Equations
        - NOT IN THE REFERENCE TABLE
        - $I_{collection\ of\ particles} = \Sigma m_i r_i^2$
        - $I_{collection\ of\ particles} = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + ...$
        - $I = moment\ of\ inertia\ (kg \cdot m^2)$
- Torque
    - A rotational force
    - Depends on…
        - Magnitude of force
        - Distance from pivot
        - Angle of force
    - Equations
        - ON the reference table
        - $\tau = r_{\perp} F = rF sin\theta$
            - $\tau = torque\ (N \cdot m)$
            - $r = distance\ from\ pivot\ (m)$
    - The Set Up

$$\tau_{net} = \tau_{net}$$
$$I\alpha = \Sigma\tau$$
$$I\alpha = \tau_1 + \tau_2 + \tau_3 + ...$$

- Center of gravity
    - $\tau_{net} = 0N$ when the pivot point is the center of gravity
    - BALANCED
    - Equations
        - $x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + ...}{m_1 + m_2 + m_3 + ...}$
        - $y_{cg} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3 + ...}{m_1 + m_2 + m_3 + ...}$

- Constraints due to ropes and pulleys
    - Nonslipping rope
    - $v_{obj} = \omega R$
        - Rim speed
    - $a_{obj} = \alpha R$
        - Rim acceleration

## Rotational Momentum

- Angular momentum
    - $L = angular\ momentum\ (kg \cdot m^2/s)$
    - Equations
        - $L = I\omega$
        - $\Delta L = \tau \Delta t$
        - $L_{total\ before} = L_{total\ after}$
- Conservation of angular momentum
    - Relationships
        - When radius decreases, moment of inertia increases
        - When moment of interia decreases, angular momentum decreases
    - Zero total momentum
- Transer of angular momentum
    - $L = r_{\perp} p = prsin\theta = mvrsin\theta$
        - Relationship between linear and angular momentum

## Rotational Energy

- Rotational kinetic energy
    - $K_{rot} = rotational\ kinetic\ energy\ (J)$
    - Equations
        - $K_{rot} = \frac{1}{2} I\omega^2$
        - $E_T = E_T \rightarrow U_g = K + K_{rot}$