Unit 4 → Energy

Understanding Energy

- Total energy of a system is represented by: E_{total}
    - Sum of all energies in a system
    - $E_{total} = K + U_g + U_s + Q$
- Energy transfer
    - Energy of one kind can be transformed into another
    - Exchange of energy between a system and its environment
    - TYPES
        - Work
            - Mechanical transfer of energy (pushing and pulling)
        - Heat
            - Nonmechanical transfer of energy (temperature difference)
- Law of conservation of energy
    - Total energy of an isolated system is conserved
        - In an isolated system, there is no way of transferring energy in and out of the system
    - $\Delta E = W$

Work

- Forces
  - An external force occurs when work is done from outside of a system
  - An internal force occurs from forces within an object
  - Greatest force is done when force points in the same direction as displacement
  - $F \rightarrow F_{\parallel}$ and $F_{\perp}$
    - $F_{\parallel}$
      - $F_{\parallel}$ can increase kinetic energy
      - $F_{\parallel} = cos\theta$
- Relationship between work and displacement
  - For a change in energy to occur, there must be a displacement
  - Larger the displacement, greater the work done
  - $d \propto W$ or $\Delta x \propto W$
  - If force is constant, force will point in the same direction as displacement
- Relationship between work and force
  - Stronger the force, the greater the work done
  - $F \propto W$
- Equations and units
  - $W = Fd$ or $W = F\Delta x$
  - $W = F_{\parallel}\, d = Fdcos\theta$
  - $Newtons \cdot meters = N \cdot m = Joules = J$
  - Joules (J) is the unit used for ALL forms of energy
  - Sign of W is determined by the angle between force and displacement
  - $W = \Delta E_{total} = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{th}$
  - $W = \Delta E$
    - Expand to all forms when ΔE is equal to different types of energy
- Systems with NO work
  - Systems that undergo NO displacement
  - A force is perpendicular to the displacement
  - Part of an object with a force undergoes no displacement

Kinetic Energy

- Understanding kinetic energy
  - Depsnds on velocity of an object squared
  - Must always be zero or positive
  - NOT a vector, although velocity is
- An object's energy in motion
- Equation and units
  - $W = \Delta K$
    - $W = \Delta K = K - K_0 = K_{final} - K_{initial}$
  - J (Joules)
    - $kg \cdot \frac{m^2}{s^2} = kg \cdot m \cdot \frac{m}{s^2} = N \cdot m = J$
  - $\Delta K = \frac{1}{2} m \Delta v^2$
  - Proving the formula using manipulation

$$v^2 = v_0^2 + 2a\Delta x$$

Substitute $a = \frac{F}{m}$

$$v^2 = v_0^2 + \frac{2F\Delta x}{m}$$

Substitute $F\Delta x = W$

$$v^2 = v_0^2 + \frac{2W}{m}$$

$$W = \frac{1}{2} m (v^2 - v_0^2)$$

$$K_{final} = \frac{1}{2} m v_{final}^2 \qquad K_0 = \frac{1}{2} m v_0^2$$

$$\Delta K = \frac{1}{2} m \Delta v^2$$

If it starts from rest,

$$K = \frac{1}{2} m v^2$$

Potential Energy

- An object's stored energy
- Forces
  - Conservative forces
    - Interactive forces that store useful energy
    - Gravity and elastic forces
      - Mechanical energy is only conserved when conservative forces act upon it
  - Nonconservative forces
    - Forces where energy is not stored
    - Friction
- Gravitational potential energy
  - Gravitational potential energy depends on height of an object, not path taken to the position
    - As an object is thrown up, ΔU_g increases because height increases
      - At it's highest point, it will start to decrease
  - Equations
    - $\Delta U_g = mg\Delta y$
    - $W = \Delta U_g$
    - Proof of formula

$$U_g = U_{g_0} + W$$

Substitute $W = Fd = mg\Delta y$

$$U_g = U_{g_0} + mg\Delta y$$

$$\Delta U_g = mg\Delta y$$

- Elastic potential energy
  - Energy stored in compressed or extended springs
  - Hooke's Law
    - $|F_s| = -k\Delta x$
  - Equations
    - $\Delta U_s = \frac{1}{2}k\Delta x^2$
    - $W = \Delta U_s$
    - Proof of formula

$$W = F_s\Delta x$$

Substitute $F_s = k\Delta x$

$$W = (k\Delta x) \cdot \Delta x$$

$$W = k\Delta x^2$$

Substitute $W = \Delta U_s$ and halve the equation because some of the energy goes to the spring

$$\Delta U_s = \frac{1}{2}k\Delta x^2$$

Conservation of Energy

- Total energy of a system equals the energy transferred to or from systems of work
- Energy of an isolated system is conserved
- Mechanical energy

- Sum of potential and kinetic energy of a system
- Conserved if the isolated system DOES NOT have friction

- Equations
  - $W = \Delta K + \Delta U_g + \Delta U_s + Q$
  - $E_{total} = U_g + K + Q + U_s$
  - $W_{fricion} = Q = F_f d = F_d \Delta x$

Heat

- *HEAT IS THE SAME THING AS THERMAL ENERGY*
- Sum of all microscopic potential and kinetic energies
  - Atoms move fast → higher temperature → higher kinetic energy
  - Further away from equilibrium → higher potential energy
- Describes the energy lost
- Internal energy describes the energy inside of a system
  - Friction is a type of internal energy
- Force is the force of kinetic friction
  - $F = F_f$
    - F is a force on the box
    - F_f is the frictional force
  - Box is at a constant speed
- Equations
  - $Q = \Delta E_{th} = F_k \Delta x$
  - $W = W_f = Q = \Delta E_{th}$
  - Proof of formula

$$W = F\Delta x$$
Substitute $F = F_k$
$$W = F_f \Delta x$$
Substitute $W = Q$
$$Q = F_f \Delta x$$
Q can futher be expanded into more components
$$Q = \mu F_N \Delta x$$
When dealing along the *y-axis*, F_N *may* be substituted for F_g
$$Q = \mu F_g \Delta x$$
$$Q = \mu m g \Delta x$$

Power

- Rate at which energy is transferred
- Equations
  - $P = \frac{\Delta E}{\Delta t}$
  - W (watts)
    - $\frac{J}{s}$ (Joules per second)
  - Proof of formula

To find the power of each type of energy use the general formula,

$$P = \frac{\Delta E}{\Delta t}$$

FInd P_W by substituting $\Delta E = W$

$$P_W = \frac{W}{\Delta t}$$

Substitute $W = F\Delta x$

$$P_W = \frac{F\Delta x}{\Delta t}$$

$$P_W = F\frac{\Delta x}{\Delta t}$$

Substitute $\frac{\Delta x}{\Delta t} = v$

$$P_W = Fv$$

Find P_K by substituting $\Delta E = \Delta K$

$$P_K = \frac{\Delta K}{\Delta t}$$

Substitute $\Delta K = \frac{1}{2}m\Delta v^2$

$$P_K = \frac{\frac{1}{2}m\Delta v^2}{\Delta t}$$

$$P_K = \frac{m\Delta v^2}{2\Delta t}$$

Find P_g by substituting $\Delta E = \Delta U_g$

$$P_g = \frac{\Delta U_g}{\Delta t}$$

Substitute $U_g = mg\Delta y$

$$P_g = \frac{mg\Delta y}{\Delta t}$$

$$P_g = mg\frac{\Delta y}{\Delta t}$$

Substitute $\frac{\Delta y}{\Delta t} = v$

$$P_g = mgv$$

Find P_s by substituting $\Delta E = \Delta U_s$

$$P_s = \frac{\Delta U_s}{\Delta t}$$

Substitute $\Delta U_s = \frac{1}{2}k\Delta x^2$

$$P_s = \frac{\frac{1}{2}k\Delta x^2}{\Delta t}$$

$$P_s = \frac{k\Delta x^2}{2\Delta t}$$

“Skeletons”

- Kinetic Energy
  - $K = \frac{1}{2}mv^2$
  - $K(1)()()^2$
  - $\Delta K(1)()\,()^2 - ()_0^2$
- Potential Energy
  - Gravitational Potential Energy
    - $U_g = mg\Delta y$
    - $U_g()()()$
  - Elastic Potential Energy
    - $U_s = \frac{1}{2}k\Delta x^2$
    - $U_s(1)()()^2$
    - $\Delta U_g(1)()\,()^2 - ()_0^2$
- Work
  - $W = F\Delta x$
  - $W()()$
- Heat
  - $Q = F_f\Delta x$
  - $Q()()$

Applying Formulas

- Linear Motion
  - Prove $v_y^2 = v_{y_0}^2 + 2a_y\Delta y$

$$\Delta K = \Delta U_g$$
$$\frac{1}{2}m\Delta v_y^2 = mg\Delta y$$
Substitute $g = a_y$
$$\frac{1}{2}m\Delta v_y^2 = ma_y\Delta y$$
Cancel m
$$\frac{1}{2}\Delta v_y^2 = a_y\Delta y$$
Solve for v_y^2
$$\Delta v_y^2 = 2a_y\Delta y$$
Expand $\Delta v_y^2 = (v_y^2 - v_{y_0}^2)$
$$v_y^2 - v_{y_0}^2 = 2a_y\Delta y$$
$$v_y^2 = v_{y_0}^2 + 2a_y\Delta y$$

  - Why does this work?
    - The masses cancel each other out, therefore indicating mass is not a determining factor in the equation
    - It describe the velocity and acceleration over a certain distance

Energy Quiz

Multiple Choice

1) A
2) C
3) B
4) A
5) B
6) C
7) A
8) D
9) B
10) D
11) A
12) D
13) A
14) D
15) A

Energy Test

Multiple Choice

1) A
2) E
3) B
4) E
5) C
6) C
7) E
8) D
9) A
10) A
11) A
12) E
13) B
14) B
15) C
16) B
17) E
18) C
19) E
20) C

Short Response

21) $550{,}000\,W$
22) 0.60
23) $96m$
24)
25)