Unit 3 → Circular Motion

Horizontal Circular Paths

- Uniform Circular Motion
    - Constant, consistent, evenly applied *speed*
    - Uniform
        - Consistent, constantly, evenly applied
- Circular Motion
    - Objects are moving in a curved path
    - How can we find the speed of an object as it moves through a circle?
    - Reference Table
        - $\underline{v} = \frac{d}{t} = \frac{circumference}{period} = \frac{2\pi r}{T}$
        - $T = period =$ time for one circle(s)
        - As r increases, so does v
- Tangential Veloctity
    - The direction of velocity is constantly changing
    - The direction is *tangent* to the circle
    - Speed is constant
    - Velocity is *not* constant
        - Change in direction therefore there is acceleration
- Centripetal Acceleration
    - Reference Table
        - $a_c = \frac{v^2}{r}$
            - $a_c =$ centripetal acceleration (m/s^2)
            - $v =$ speed (m/s)
            - $r =$ radius (m)
        - $F_{net} = F_{net\ circular} = F_c = ma_c = \frac{mv^2}{r}$
    - The force that cauces an object to move toward the inside of a circle
        - Centripetal force is circular net force
        - Centripetal force causes centripetal accleration
        - Net force produces centripetal acceleration of circular motion
        - F_{net} produces a_c of circular motion

- Horizontal Circular Motion
    - Flat tabletop
    - Car on a curve
    - Record player
    - Force of gravity DOES NOT directly play a role
    - Particular Cases
        - Make towards the center of the circle positive (+)
        - Flat curve

$$F_c = F_F$$

$$F_{net_x} = F_{net_x}$$

$$F_{c_x} = F_{c_x}$$

$$ma_c = \Sigma F_x$$

$$ma_c = F_F$$

$$ma_c = \mu F_N$$

$$F_{net_y} = F_{net_y}$$

$$ma_y = \Sigma F_y$$

$$0N = F_N + -F_g$$

$$F_g = F_N$$

$$ma_c = \mu F_g$$

$$a_c = \mu g$$

$$\frac{v^2}{r} = \mu g$$

- Conical pendulum
  - Object on a string
    F_c is a component of the tension force (F_T)

$$F_{net_x} = F_{net_x}$$
$$F_{c_x} = F_{c_x}$$
$$ma_c = \Sigma F_x$$
$$ma_c = F_{T_x}$$
$$F_{T_x} = F_{T_Y} tan\theta$$
$$ma_c = F_{T_y} tan\theta$$
$$ma_c = F_g tan\theta$$
$$ma_c = mgtan\theta$$
$$a_c = gtan\theta$$
$$\frac{v^2}{r} = tan\theta$$

- Banked curve
  - F_c is a component of F_N
  - Proper banking angle indicates no friction
  - SAME SITUATION AS CONICAL PENDULUMS

$$F_{net_x} = F_{net_x}$$
$$F_{c_x} = F_{c_x}$$
$$ma_c = F_{N_x}$$
$$F_{net_y} = F_{net_y}$$
$$ma_y = \Sigma F_y$$
$$0N = F_{N_y} +\text{-}F_g$$
$$F_g = F_{N_y}$$
$$F_{N_x} = F_{N_y} tan\theta$$
$$ma_c = F_g tan\theta$$
$$\frac{mv^2}{r} = mgtan\theta$$
$$\frac{v^2}{r} = gtan\theta$$

Vertical Circular Paths

- Vertical Circular Motion
  - Examples
    - Rollercoaster loop
    - Driving over a bump
    - Walking
  - Force of gravity ***DOES*** play a *direct* role
- Particular Cases
  - Towards the cener of the circle is positive (+)
  - Bottom of a curve

    $F_{net_y} = F_{net_y}$

    $F_{c_y} = F_{c_y}$

    $ma_c = \Sigma F_y$

    $ma_c = F_N + -F_g$
  - Top of a curve

    $F_{net_y} = F_{net_y}$

    $F_{c_y} = F_{c_y}$

    $ma_c = \Sigma F_y$

    $ma_c = F_g + -F_N$
  - Top of a curve (upside-down)

    $F_{net_y} = F_{net_y}$

    $F_{c_y} = F_{c_y}$

    $ma_c = \Sigma F_y$

    $ma_c = F_N + F_g$
- Critical Speed
  - Slowest speed at which an object can complete a circle
  - At the top of a curve
    - $F_N = 0N$
      - $F_N = F_g$

$$\frac{mv^2}{r} = F_N + F_g$$

$$\frac{mv^2}{r} = F_g$$

$$\frac{mv^2}{r} = mg$$

$$\frac{v^2}{r} = g$$

Force of Gravity

- Long Rangeforce
    - No need for contact
    - Extends to infinity
    - Always attractive
- $F_g = mg$
    - Force of attraction between Earth and another object *on* Earth
        - $F_g = m_{object}\, g_{Earth}$
        - Weight
- $F_g = G\frac{m_1 m_2}{r^2}$
    - Force of attraction between 2 objects
        - m_1 and m_2
        - G is the Universal Gravitation Constant
            - $G = 6.67 \times 10^{-11}\ N \cdot \frac{m^2}{kg^2}$
- Acceleration due to gravity
    - $|F_g| = F_g$
    - $\frac{G m_P m_o}{r^2} = m_o g$
        - $g = \frac{G m_P}{r^2}$
            - m_P represents the mass of a planet
            - m_o represent the mass of an object
- The Skeleton
    - Equation
        - $F_g = \frac{G m_1 m_2}{r^2}$
    - Skeleton
        - $\frac{(1)(\ )(\ )}{(\ )^2} F_g$

Orbits

- Orbits
    - Vertical circular motion
    - Objects are in free-fall
        - $F_c = F_g$
        - $a_c = g$
    - Period (T) is the time for one full revolution
- Period of an orbit

$$a_c = g$$

$$\frac{v^2}{r} = G\frac{m_P}{r^2}$$

$$\frac{(\frac{2\pi r}{T})^2}{r} = G\frac{m_P}{r^2}$$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_P}}$$

Circular Motion Quiz

Multiple Choice

1) C
2) D
3) C
4) C
5) A
6) A
7) B
8) D
9) D
10) A

Short Response

11) *9000N*
12) *0.21N*
13) *51.34*°
14) *788N*
15) *1,741.5N*

Circular Motion Test

Multiple Choice

1) C
2) D
3) D
4) A
5) E
6) E
7) D
8) D
9) E
10) B
11) A
12) B
13) B
14) D
15) C
16) B
17) E
18) A
19) B

Short Response

20) $2.35 \cdot 10^{-3} N$
21) $6.418 \cdot 10^{-8}\ m/s^2$
22) $2{,}294.41\ km$
23) $1.44 \cdot 10^{7}\ m$
24) $5.77\ m/s$
25) $1.9 \cdot 10^{21}\ kg$