



GPU Teaching Kit
Accelerated Computing



西南石油大学 计算机科学学院
SCHOOL OF COMPUTER SCIENCE, SOUTHWEST PETROLEUM UNIVERSITY



Module 5.2 – Parallel Computation Patterns (Reduction)

Parallel Reduction

Objective

- **To learn the parallel reduction pattern**
 - An important class of parallel computation
 - **Work efficiency** analysis
 - **Resource efficiency** analysis

“Partition and Summarize”

- A commonly used strategy for processing large input data sets
 - There is no required order of processing elements in a data set (associative and commutative)
 - Partition the data set into smaller chunks
 - Have each thread to process a chunk
 - Use a **reduction tree** to summarize the results from each chunk into the final answer
- E.G., **Google** and **Hadoop MapReduce** frameworks support this strategy
- We will focus on **the reduction tree step** for now

Reduction enables other techniques

- Reduction is also needed to clean up after some commonly used **parallelizing transformations**
- Privatization
 - Multiple threads write into an output location
 - Replicate(复制) the output location so that each thread has a private output location (privatization)
 - Use a **reduction tree** to combine the values of private locations into the original output location

What is a reduction computation?

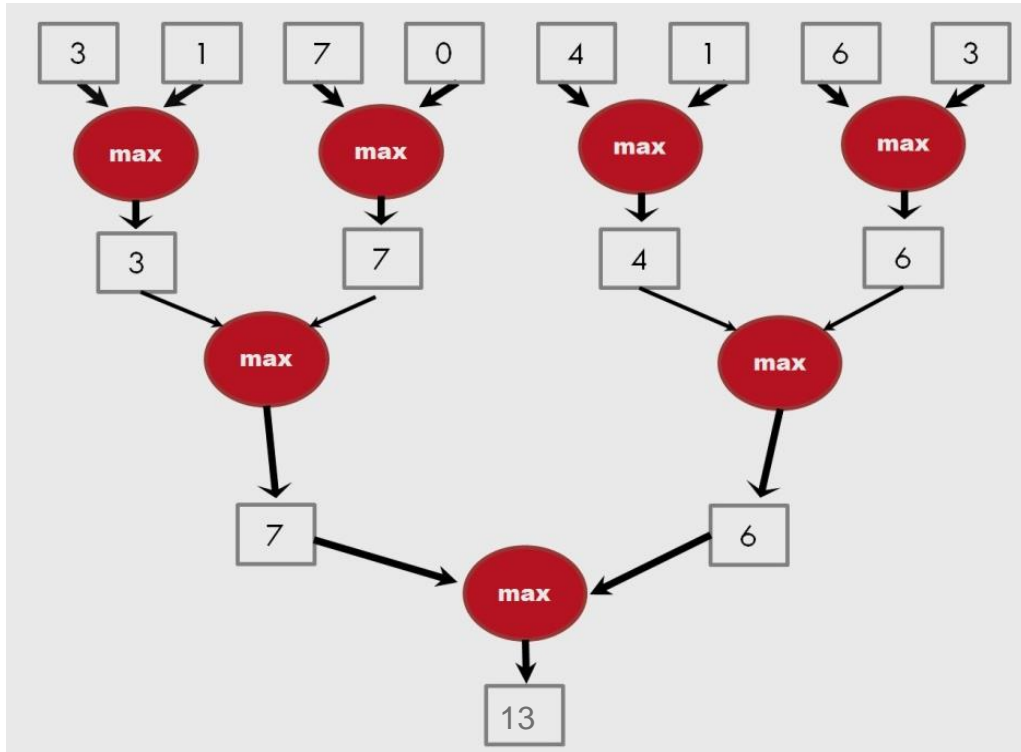
- Summarize a set of input values into one value using a “reduction operation”
 - Max
 - Min
 - Sum
 - Product
- Often used with a user defined reduction operation function as long as the operation:
 - Is associative and commutative
 - Has a well-defined(明确的) identity value (e.g., 0 for sum)
 - For example, the user may supply a custom “max” function for 3D coordinate data sets **where the magnitude for the each coordinate data tuple is the distance from the origin.**

An example of “collective operation(集合操作)”

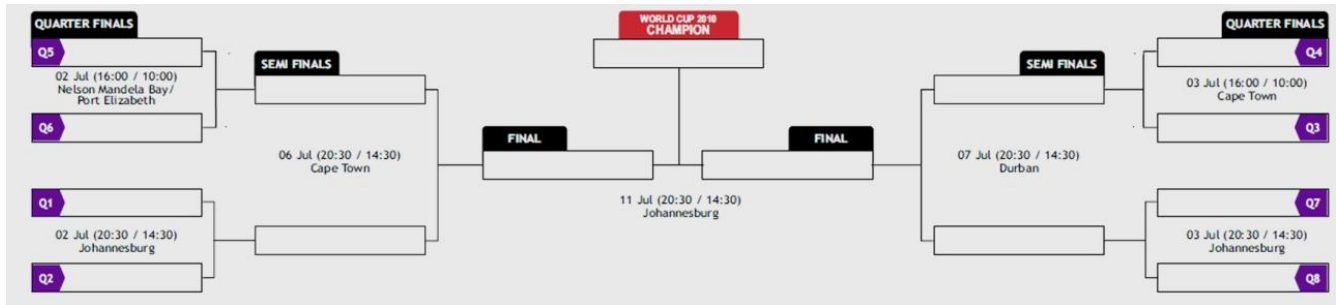
An Efficient Sequential Reduction $O(N)$

- Initialize the result as an identity value for the reduction operation
 - Smallest possible value for max reduction
 - Largest possible value for min reduction
 - 0 for sum reduction
 - 1 for product reduction
- Iterate through the input and perform the reduction operation **between the result value and the current input value**
 - N reduction operations performed for N input values
 - Each input value is only visited once – an $O(N)$ algorithm
 - This is a computationally efficient algorithm.

A parallel reduction tree algorithm performs $N-1$ operations in $\log(N)$ steps



A tournament is a reduction tree with “max” operation



A Quick Analysis

- For N input values, the reduction tree performs
 - $(1/2)N + (1/4)N + (1/8)N + \dots (1)N = (1 - (1/N))N = \textbf{N-1 operations}$
 - In **Log (N) steps** – 1,048,576(2^{20}) input values take 20 steps
 - Assuming that we have enough execution resources
 - **Average Parallelism (N-1)/Log(N)**
 - For N = 1,048,576, average parallelism is approximately 52,000
 - However, peak resource requirement is 500,000
 - This is not **resource efficient**
- This is a **work-efficient** parallel algorithm
 - The amount of work done is comparable to the an efficient sequential algorithm
 - Many parallel algorithms are not work efficient



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Module 9 – Parallel Computation Patterns (Reduction)

Lecture 9.2 - A Basic Reduction Kernel

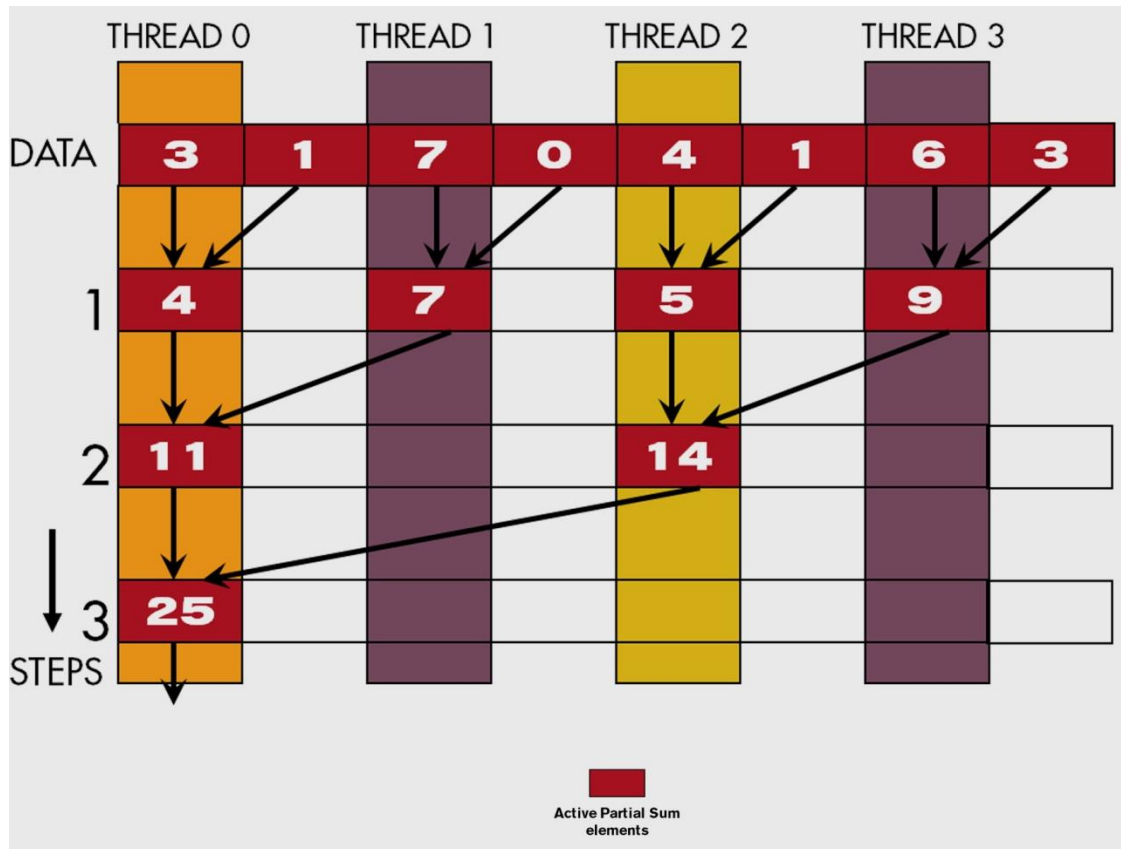
Objective

- **To learn to write a basic reduction kernel**
 - Thread to data mapping
 - Turning off threads
 - Control divergence

Parallel Sum Reduction

- Parallel implementation
 - Each thread adds two values in each step
 - Recursively halve # of threads
 - Takes $\log(n)$ steps for n elements, requires $n/2$ threads
- Assume an in-place(就地) reduction using shared memory
 - The original vector is in device global memory
 - The shared memory is used to hold a partial sum vector
 - Initially, the partial sum vector is simply the original vector
 - Each step brings the partial sum vector closer to the sum
 - The final sum will be in element 0 of the partial sum vector
 - Reduces global memory traffic due to reading and writing partial sum values
 - Thread block size limits n to be less than or equal to 2,048

A Parallel Sum Reduction Example



A Naive Thread to Data Mapping

- Each thread is responsible for an even-index location of the partial sum vector (location of responsibility)
- After each step, half of the threads are no longer needed
- One of the inputs is always from the location of responsibility
- In each step, one of the inputs comes from an increasing distance away

A Simple Thread Block Design

- Each thread block takes **2*BlockDim.x** input elements
- Each thread loads **2 elements** into shared memory

```
__shared__ float partialSum[2*BLOCK_SIZE];  
  
unsigned int tx = threadIdx.x;  
unsigned int start = 2*blockIdx.x*blockDim.x;  
partialSum[tx] = input[start + tx];  
partialSum[blockDim+tx] = input[start + blockDim.x+tx];
```

The Reduction Steps

```
for (unsigned int stride = 1;
     stride <= blockDim.x;  stride *= 2)
{
    __syncthreads();
    if (tx % stride == 0)
        partialSum[2*tx] += partialSum[2*tx+stride];
}
```

Why do we need `__syncthreads()`?

Barrier Synchronization

- `__syncthreads()` is needed to ensure that all elements of each version of partial sums have been generated before we proceed to the next step

Back to the Global Picture

- At the end of the kernel, **Thread 0** in each block writes the sum of the thread block in `partialSum[0]` into a vector indexed by the **blockIdx.x**
- There can be a large number of such sums if the original vector is very large
 - The host code may iterate and launch another kernel
- If there are only a small number of sums, the host can simply transfer the data back and add them together
- Alternatively, Thread 0 of each block could use **atomic operations** to accumulate into a global sum variable.



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Module 9 – Parallel Computation Patterns (Reduction)

Lecture 9.3 - A Better Reduction Kernel

Objective

- **To learn to write a better reduction kernel**
 - Improved resource efficiency
 - Improved thread to data mapping
 - Reduced control divergence

Some Observations on the naïve reduction kernel

- In each iteration, two control flow paths will be **sequentially traversed for each warp**
 - Threads that perform addition and threads that do not
 - Threads that do not perform addition still consume execution resources
- **Half or fewer of threads** will be executing after the first step
 - **All odd-index threads** are disabled after first step
 - After the 5th step, entire warps in each block will fail the `if` test, **poor resource utilization but no divergence**
 - This can go on for a while, up to 6 more steps (stride = 32, 64, 128, 256, 512, 1024), where **each active warp** only has **one productive thread** until all warps in a block retire

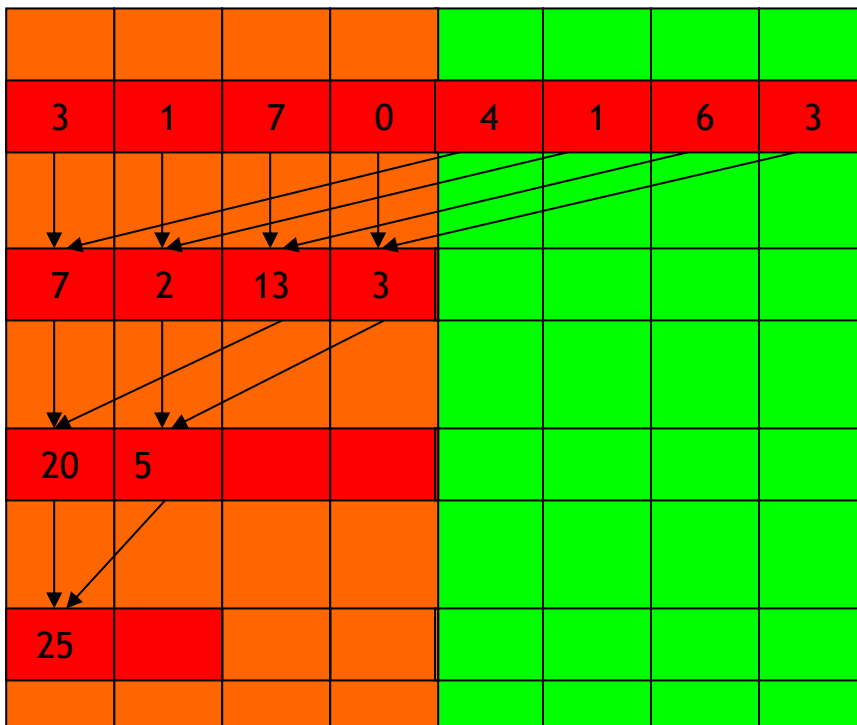
```
for (unsigned int stride = 1;
     stride <= blockDim.x;  stride *= 2)
{
    __syncthreads();
    if (t % stride == 0)
        partialSum[2*t] += partialSum[2*t+stride];
}
```

Thread Index Usage Matters

- In some algorithms, one can shift the index usage to improve the divergence behavior
 - Commutative and associative operators
- Always compact the partial sums into the front locations in the partialSum[] array
- Keep the active threads consecutive

An Example of 4 threads

Thread 0 Thread 1 Thread 2 Thread 3



A Better Reduction Kernel

```
for (unsigned int stride = blockDim.x;  
    stride > 0;  stride /= 2)  
{  
    __syncthreads();  
    if (t < stride)  
        partialSum[t] += partialSum[t+stride];  
}
```


A Quick Analysis

- For a 1024 thread block
 - **No divergence in the first 5 steps**
 - 1024, 512, 256, 128, 64, 32 consecutive threads are active in each step
 - All threads in each warp either all active or all inactive
 - **The final 5 steps will still have divergence**
 - 16, 8, 4, 2, 1 consecutive threads are active in each step
 - Fewer than 32 threads in each warp



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