GPU Teaching Kit

Accelerated Computing

Module 10 – Parallel Computation Patterns (scan)

Lecture 10.1 - Prefix Sum

Objective

- **To master parallel scan prefix sum(前缀和) algorithms**
  - Frequently used for **parallel work assignment** and **resource allocation**
  - A key primitive(原语) in many parallel algorithms to convert serial computation into parallel computation
  - A foundational **parallel computation pattern**
  - Work efficiency in parallel code/algorithms

- Reading –Mark Harris, Parallel Prefix Sum with CUDA
  - http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Inclusive Scan (Prefix-Sum) Definition

Definition: *The* scan *operation takes* ***a binary associative operator*** ***(二元结合操作符)*** $\oplus$ (pronounced as circle plus), *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then scan operation on the array would return

[3 1 7 0 4 1 6 3],
[3 4 11 11 15 16 22 25].

An Inclusive Scan Application Example

- Assume that we have a 100-inch sandwich to feed 10 people
- We know how much each person wants in inches
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut the sandwich quickly?
- How much will be left?

- **Method 1**: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.

- **Method 2**: calculate prefix sum:
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences(递归) from sequential(串行) Into parallel
  - **Sequential** :

    ```
    for(j=1;j<n;j++)
    out[j] = out[j-1] + f(j);
    ```

  - **Parallel**:

    ```
    for  all(j) { temp[j] = f(j) };
         scan(out, temp);
    ```

- Useful for many parallel algorithms:

Other Applications

- Assigning camping spots
- Assigning Farmer's Market spaces
- Allocating memory to parallel threads
  - different threads may consume different sizes of memory. you need to know the total amount of memory required (pre-allocated in advance) and determine the memory location each thread will access.
- Allocating memory buffer space for communication channels
- …

An Inclusive Sequential Addition Scan

Given a sequence $[x_0, x_1, x_2, ...]$

Calculate output $[y_0, y_1, y_2, ...]$

Such that $y_0 = x_0$

$y_1 = x_0 + x_1$

$y_2 = x_0 + x_1 + x_2$

...

Using a recursive definition

$y_i = y_{i-1} + x_i$

A Work Efficient C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++)
      y[i] = y [i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements - O(N)!

Only slightly more expensive than sequential reduction.

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$

“Parallel programming is easy as long as you do not care about performance.”

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Module 10 – Parallel Computation Patterns (scan)

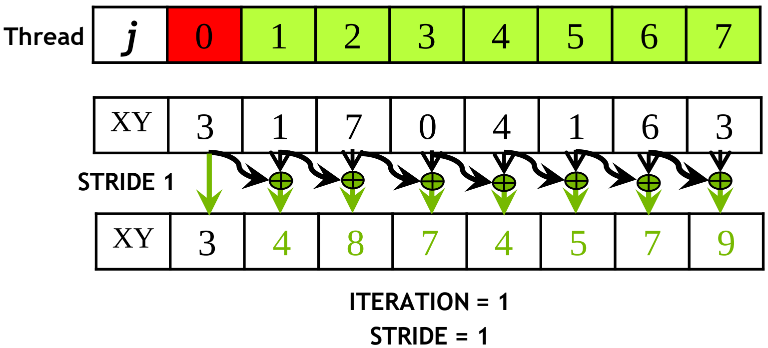
Lecture 10.2 - A Work-inefficient Scan Kernel

Objective

- To learn to write and analyze a **high-performance scan kernel**
  - **Interleaved reduction trees**
  - Thread index to data mapping
  - Barrier Synchronization
  - Work efficiency analysis

A Better Parallel Scan Algorithm

1. Read input from device global memory to shared memory
2. Iterate **$\log_2$(n)** times; **stride** from 1 to n-1: double stride each iteration

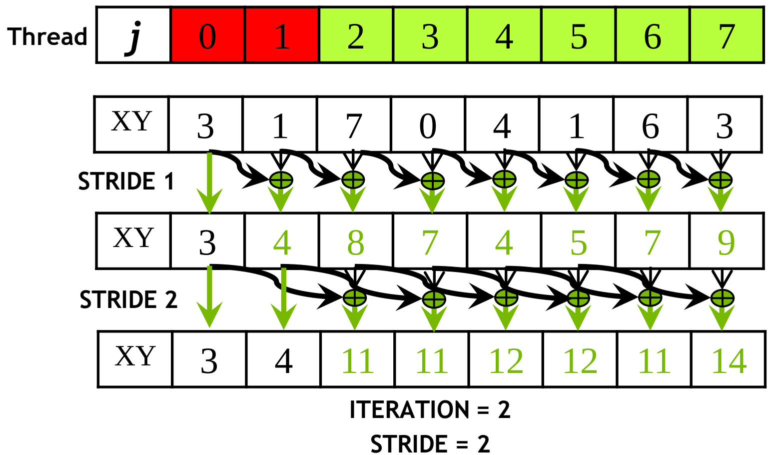


- Active threads *stride* to n-1 (n-stride threads)
- **Thread *j*** adds elements ***j*** and ***j-stride*** from shared memory and writes result into element ***j*** in shared memory
- Requires barrier synchronization, once **before read** and once **before write**

XY[j] += XY[j-stride]

A Better Parallel Scan Algorithm

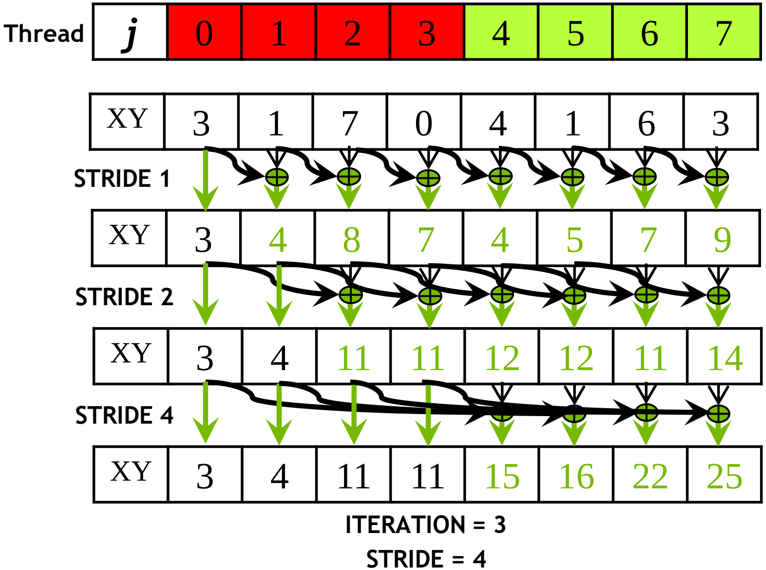
1. Read input from device to shared memory
2. Iterate **$log_2(n)$** times; **stride** from 1 to n-1: double stride each iteration.



XY[j] += XY[j-stride]

A Better Parallel Scan Algorithm

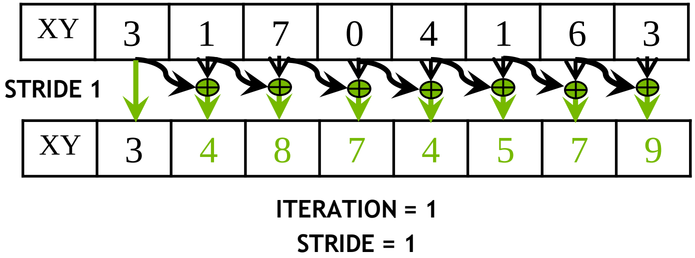
1. Read input from device to shared memory
2. Iterate $\log_2(n)$ times; **stride** from 1 to n-1: double stride each iteration
3. Write output from shared memory to device memory



XY[j] += XY[j-stride]

Handling Dependencies

- During every iteration, each thread can overwrite the input of another thread
  - **Barrier synchronization** to ensure all inputs have been properly generated
  - **All threads secure** input operand that can be overwritten by another thread
  - **Barrier synchronization** is required to ensure that all threads have secured their inputs
  - All threads perform addition and write output



A Work-Inefficient Inclusive Scan Kernel

```
__global__ void work_inefficient_scan_kernel(float *X, float *Y, int InputSize){
    __shared__ float XY[SECTION_SIZE];
    int i= blockIdx.x * blockDim.x + threadIdx.x;
    if (i < InputSize) {
        XY[threadIdx.x] = X[i];
     }
        // the code below performs iterative scan on XY
        for (unsigned int stride = 1; stride <= threadIdx.x; stride *= 2) {
            __syncthreads();
            float in1 = XY[threadIdx.x - stride];
            __syncthreads();
            XY[threadIdx.x] += in1;
        }
      __ syncthreads();
      If (i < InputSize) {
         Y[i] = XY[threadIdx.x];}
}
```

Work Efficiency Considerations

- This Scan executes $\mathbf{log_2(N)}$ parallel iterations
- M= $\mathbf{log_2(N), 2^M=N;}$
- The iterations do $(N-2^0) + (N-2^1) + \dots + (N-2^k) + \dots + (N-2^{M-1})$ adds each
- **<u>Total Adds</u>**: $M*N-(2^0+\dots+ 2^{M-1}) = M*N-(N-1)=N(M-1)+1$
- $=N * \mathbf{log_2(N)} - (N-1) \rightarrow O(N* \mathbf{log_2(N)})$ work

- **This scan algorithm is not work efficient**
  - Sequential scan algorithm: O(N)
  - This parallel scan algorithm: $O(N* \mathbf{log_2(N)})$
  - For 1024 elements:
    - Sequential scan algorithm is 1024
    - parallel scan algorithm is 10*1024
- A parallel algorithm can be slower than a sequential one when execution resources are <u>saturated(饱和)</u> from low work efficiency

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Lecture 10.3 – Parallel Computation Patterns (scan)

A Work-Efficient Parallel Scan Kernel

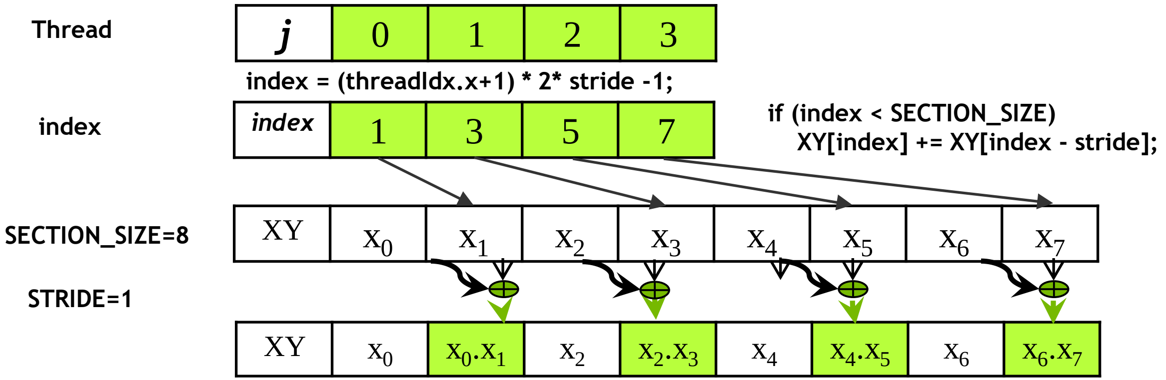
Objective

- **To learn to write a work-efficient scan kernel**
  - Two-phased balanced tree traversal(遍历)
  - Aggressive re-use of intermediate results
  - Reducing control divergence with more complex thread index to data index mapping

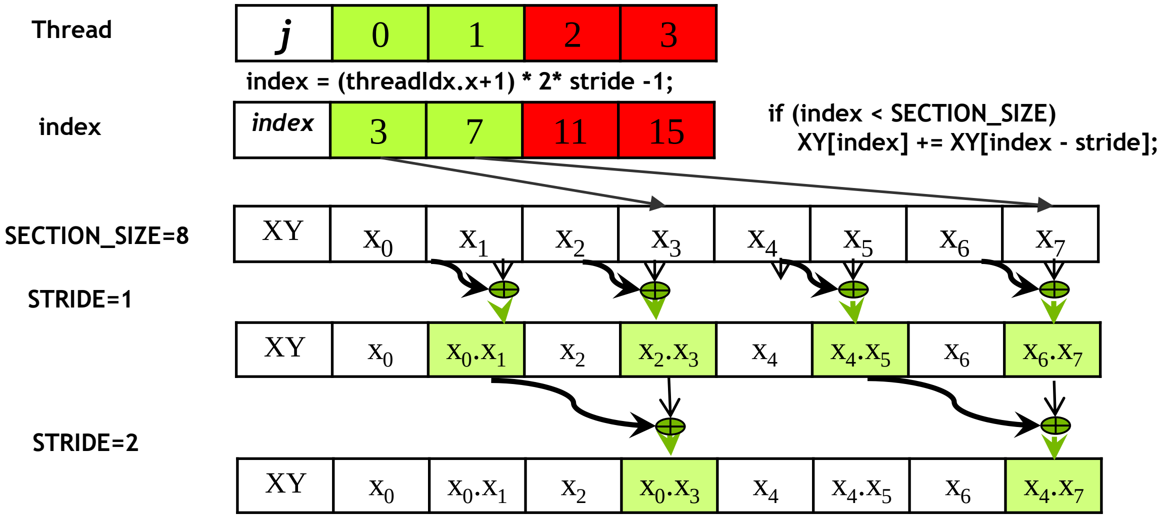
Improving Efficiency

- *Balanced Trees*
  - Form **a balanced binary tree** on the input data and sweep it to and from the root
  - Tree is **not an actual data structure**, but a concept to determine what each thread does at each step
- For scan:
  - Traverse down **from leaves to the root** building partial sums at internal nodes in the tree
    - The root holds the sum of all leaves
  - Traverse **back up the tree** building the output from the partial sums

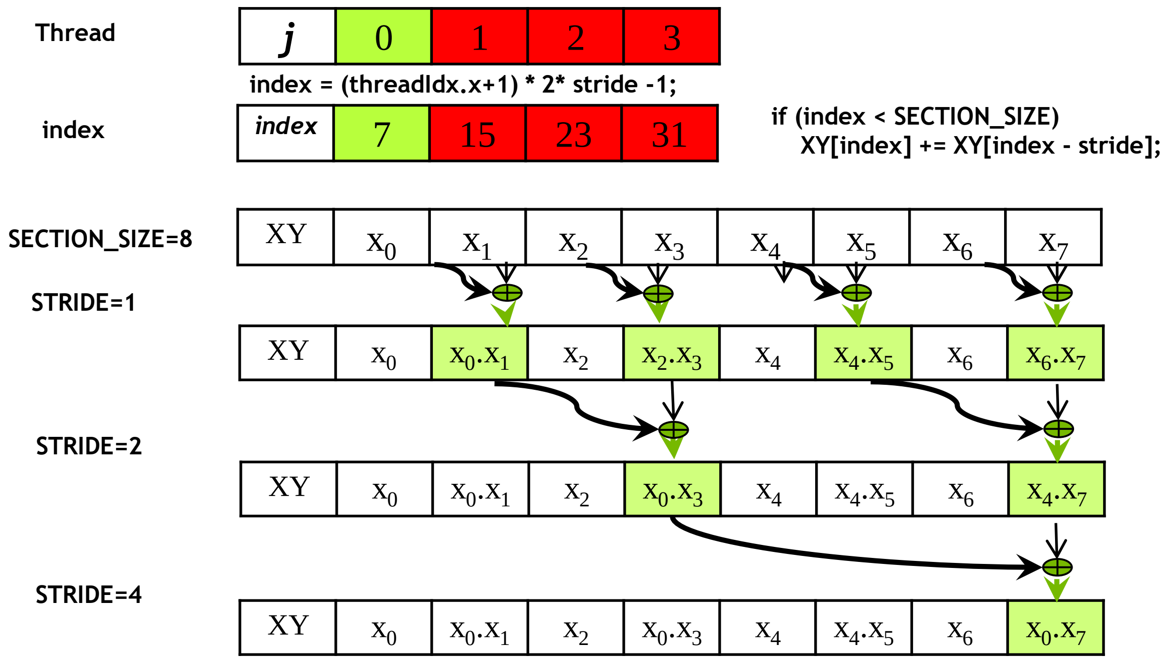
Parallel Scan - Reduction Phase



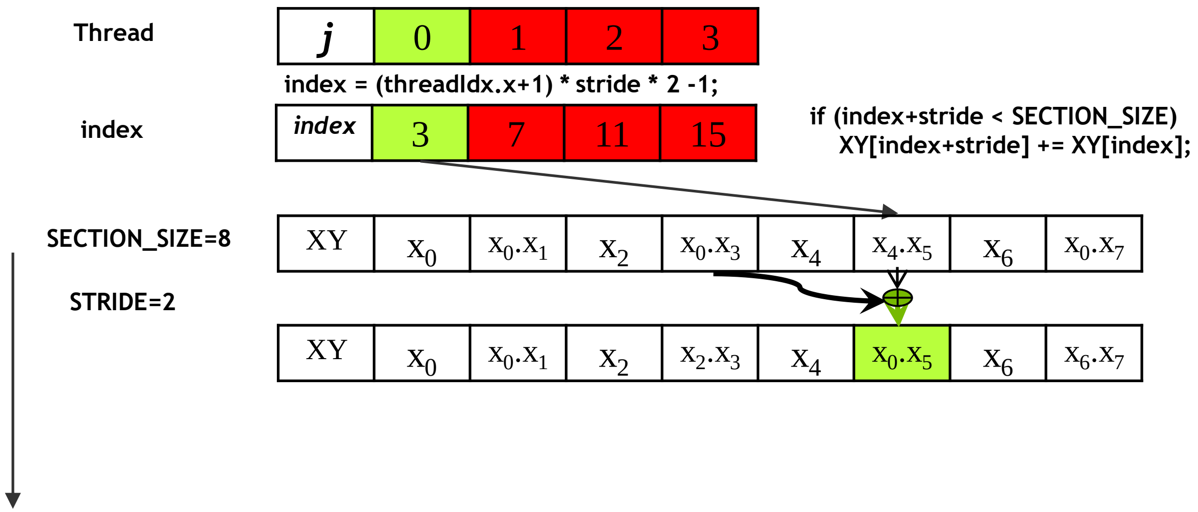
Parallel Scan - Reduction Phase (cont.)



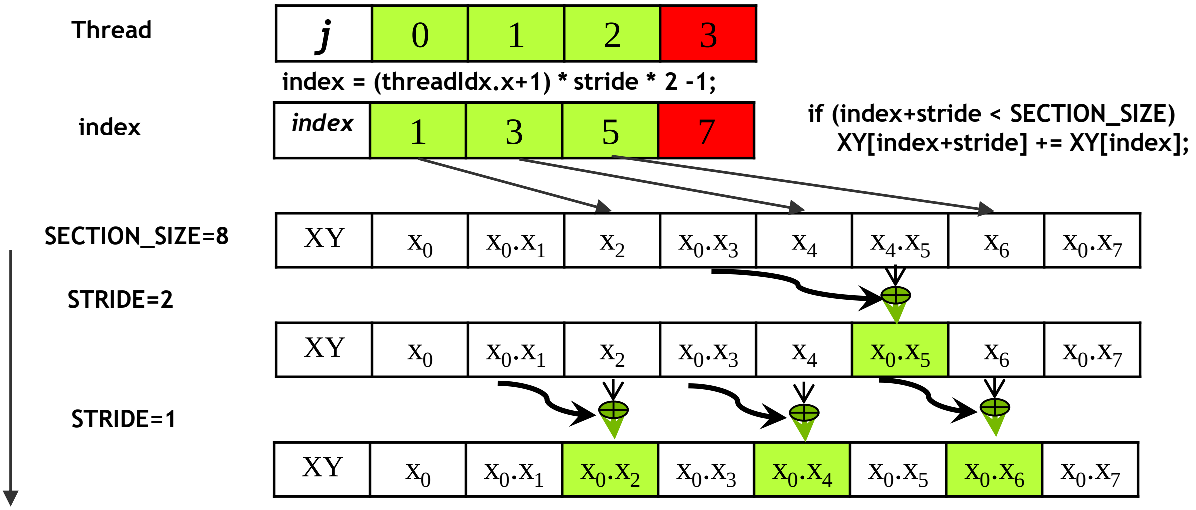
Parallel Scan - Reduction Phase (cont.)



Parallel Scan - Reverse Phase



Parallel Scan - Reverse Phase (cont.)



A work-efficient kernel for an inclusive scan

```
__global__ void work_efficient_scan_kernel(float *X, float *Y, int InputSize){
    __shared__ float XY[SECTION_SIZE];
    int i= blockIdx.x * blockDim.x + threadIdx.x;
    if (i < InputSize) {
        XY[threadIdx.x] = X[i];
     }

    for (unsigned int stride = 1; stride < blockDim.x; stride *= 2) {
        __syncthreads();
        int index = (threadIdx.x+1) * 2* stride -1;
        if (index < SECTION_SIZE) {
            XY[index] += XY[index - stride];
        }
    }
```

A work-efficient kernel for an inclusive scan(con.t)

```
…

    for (int stride = blockDim.x /2;  stride  >  0;  stride  /=  2)  {
        __syncthreads();
        int index = (threadIdx.x+1) * stride * 2 -1;
        if (index + stride < SECTION_SIZE) {
            XY[index + stride] += XY[index];
        }
    }
    __syncthreads();
    Y[i] = XY[threadIdx.x]
…
}
```

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Module 10.4 – Parallel Computation Patterns (scan)

More on Parallel Scan

Objective

- **To learn more about parallel scan**
  - Analysis of the work efficient kernel
  - Exclusive scan
  - Handling very large input vectors

Work Analysis of the Work Efficient Kernel

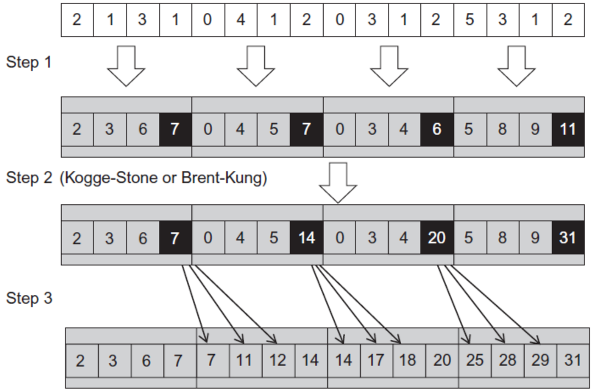
- The work efficient kernel executes **$\log_2(N)$** parallel iterations in the reduction step
  - The iterations do N/2, N/4,..1 adds
  - Total adds: (N-1) → O(N) work
- It executes **$\log_2(N)$ -1** parallel iterations in the post-reduction reverse step
  - The iterations do 2-1, 4-1, …. N/2-1 adds
  - Total adds: (N-2) – (**$\log_2(N)$** -1) → O(N) work
- Both phases perform up to no more than 2x(N-1) adds
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

Some Tradeoffs

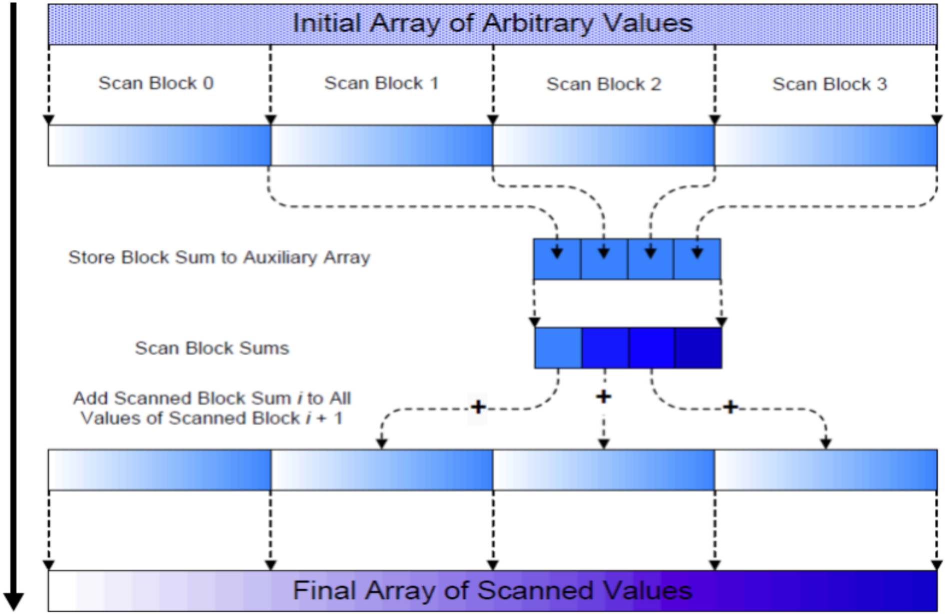
- The work efficient scan kernel is normally more desirable
  - Better Energy efficiency
  - Less execution resource requirement
- **However**, **the work inefficient kernel** could be better for absolute performance due to its single-phase nature (forward phase only)
  - There is sufficient execution resource

Handling Large Input Vectors

– Have each section of **2*blockDim.x** elements assigned to a block
  – Perform parallel scan on each section
– Have each block write the sum of its section into a Sum[] array indexed by blockIdx.x
– Run the scan kernel on the Sum[] array
– Add the scanned Sum[] array values to **all the elements of corresponding sections**



Overall Flow of Complete Scan



Exclusive Scan Definition

Definition: ***The* exclusive scan** *operation takes a binary associative operator* $\oplus$, *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation

on the array [3 1 7 0 4 1 6 3],

would return [0 3 4 11 11 15 16 22].

Why Use Exclusive Scan?

– To find the beginning address of allocated buffers

– Inclusive and exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

A Simple Exclusive Scan Kernel

- Adapt an inclusive, work inefficient scan kernel
- Block 0:
  - Thread 0 loads 0 into **XY[0]**
  - Other threads load **X[threadIdx.x-1]** into **XY[threadIdx.x]**
- All other blocks:
  - All thread load **X[blockIdx.x*blockDim.x+threadIdx.x-1]** into **XY[threadIdex.x]**
- Similar adaption for work efficient scan kernel but ensure that each thread loads two elements
  - Only one zero should be loaded
  - All elements should be shifted to the right by only one position

Read the Harris article (Parallel Prefix Sum with CUDA) for a more intellectually interesting approach to exclusive scan kernel implementation.

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