

GPU Teaching Kit

Accelerated Computing



Module 4.1 – Memory and Data Locality

CUDA Memories

Objective

- To learn to effectively use the CUDA memory types in a parallel program
 - Importance of memory access efficiency
 - Registers, shared memory, global memory
 - Scope and lifetime

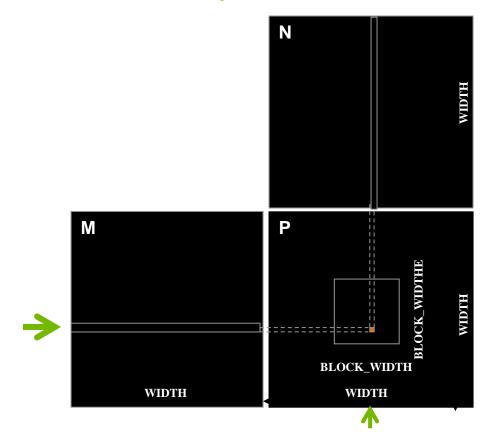
Review: Image Blur Kernel.

```
// Get the average of the surrounding 2xBLUR SIZE x 2xBLUR SIZE box
for(int blurRow = -BLUR SIZE; blurRow < BLUR SIZE+1; ++blurRow) {
  for(int blurCol = -BLUR SIZE; blurCol < BLUR SIZE+1; ++blurCol) {</pre>
    int curRow = Row + blurRow;
    int curCol = Col + blurCol;
    // Verify we have a valid image pixel
    if(curRow > -1 && curRow < h && curCol > -1 && curCol < w) {
      pixVal += in[curRow * w + curCol];
      pixels++; // Keep track of number of pixels in the accumulated total
// Write our new pixel value out
out[Row * w + Col] = (unsigned char)(pixVal / pixels);
```

How about performance on a GPU

- All threads access global memory for their input matrix elements
 - One memory accesses (4 bytes) per floating-point addition
 - 4B/s of memory bandwidth/FLOPS
- Assume a GPU with
 - Peak floating-point rate 1,500 GFLOPS with 200 GB/s DRAM bandwidth
 - 4*1,500 = 6,000 GB/s required to achieve peak FLOPS rating
 - The 200 GB/s memory bandwidth limits the execution at 50 GFL OPS
- This limits the execution rate to 3.3% (1/30) of the peak floating-point execution rate of the device!
- Need to drastically cut down memory accesses to get close to the1,500 GFLOPS

Example – Matrix Multiplication





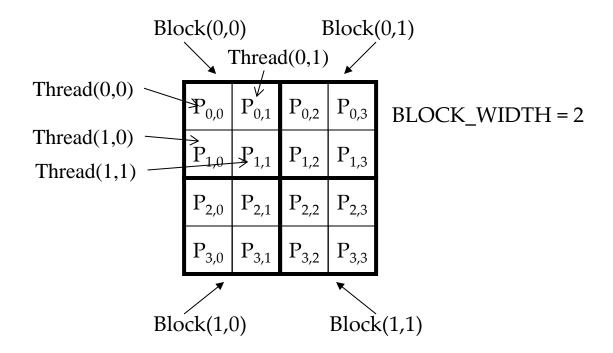
A Basic Matrix Multiplication

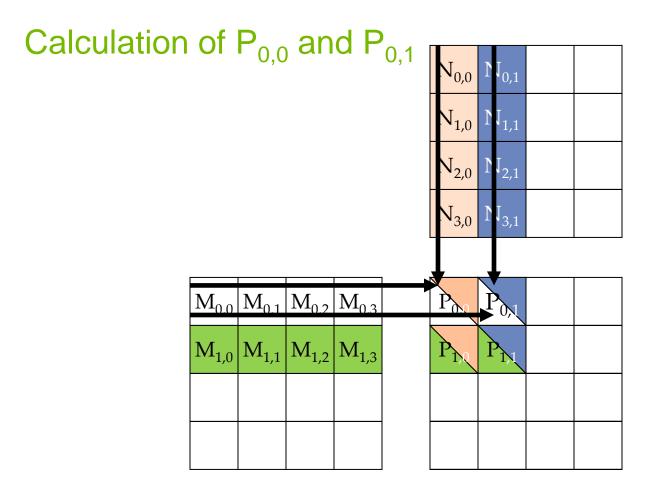
```
qlobal void MatrixMulKernel(float* M, float* N, float* P, int Width) {
// Calculate the row index of the P element and M
int Row = blockIdx.y*blockDim.y+threadIdx.y;
// Calculate the column index of P and N
int Col = blockIdx.x*blockDim.x+threadIdx.x;
if ((Row < Width) && (Col < Width)) {
  float Pvalue = 0;
  // each thread computes one element of the block sub-matrix
  for (int k = 0; k < Width; ++k) {
    Pvalue += M[Row*Width+k]*N[k*Width+Col];
  P[Row*Width+Col] = Pvalue;
```

Example – Matrix Multiplication

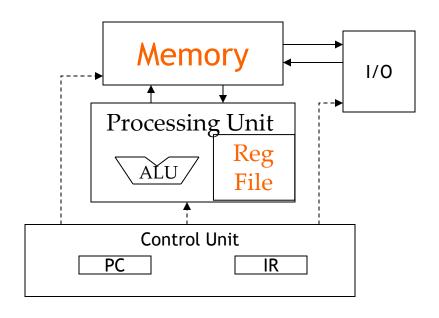
```
qlobal void MatrixMulKernel(float* M, float* N, float* P, int Width) {
// Calculate the row index of the P element and M
int Row = blockIdx.y*blockDim.y+threadIdx.y;
// Calculate the column index of P and N
int Col = blockIdx.x*blockDim.x+threadIdx.x;
if ((Row < Width) && (Col < Width)) {
  float Pvalue = 0;
  // each thread computes one element of the block sub-matrix
  for (int k = 0; k < Width; ++k) {
    Pvalue += M[Row*Width+k]*N[k*Width+Col];
  P[Row*Width+Col] = Pvalue;
```

A Toy Example: Thread to P Data Mapping

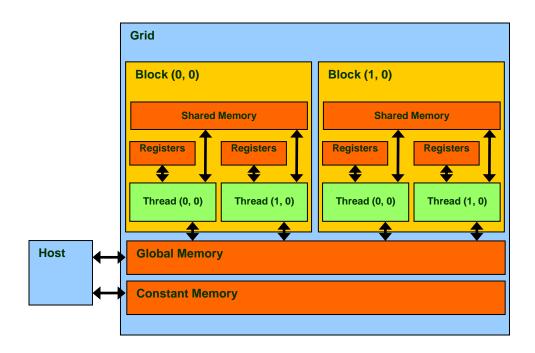




Memory and Registers in the Von-Neumann Model



Programmer View of CUDA Memories



Declaring CUDA Variables

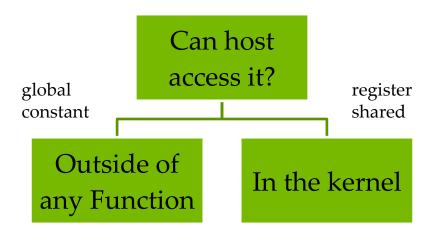
| Variable declaration | Memory | Scope | Lifetime |
|---------------------------------|----------|--------|-------------|
| int LocalVar; | register | thread | thread |
| deviceshared int SharedVar; | shared | block | block |
| device int GlobalVar; | global | grid | application |
| deviceconstant int ConstantVar; | constant | grid | application |

- __device__ is optional when used with __shared__, or __constant__
- Automatic variables reside in a register
 - Except per-thread arrays that reside in global memory

Example: Shared Memory Variable Declaration

```
void blurKernel(unsigned char * in, unsigned char * out, int w, int h)
{
    __shared__ float ds_in[TILE_WIDTH][TILE_WIDTH];
    ...
}
```

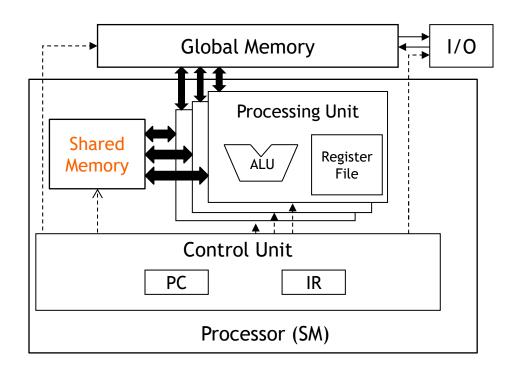
Where to Declare Variables?



Shared Memory in CUDA

- A special type of memory whose contents are explicitly(显式地) defined and used in the kernel source code
 - One in each Streaming Multiprocessor (SM)
 - Accessed at much higher speed (in both latency and throughput) than global memory
 - Scope (作用域) of access and sharing thread blocks
 - Lifetime (生命周期) thread block, contents will disappear after the corresponding thread finishes terminates execution
 - Accessed by memory load/store instructions
 - A form of scratchpad memory (快速缓存存储器) in computer architecture

Hardware View of CUDA Memories





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Module 4.2 – Memory and Data Locality

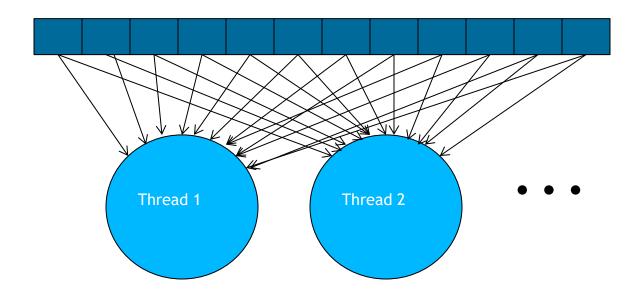
Tiled Parallel Algorithms

Objective

- To understand the motivation and ideas for tiled(分块) parallel algorithms
 - Reducing the limiting effect of memory bandwidth on parallel kernel performance
 - Tiled algorithms and barrier synchronization(栅栏同步)

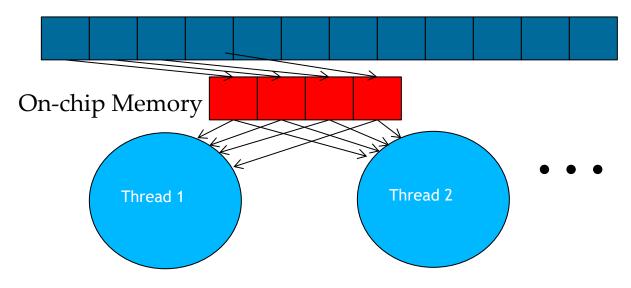
Global Memory Access Pattern of the Basic Matrix Multiplication Kernel

Global Memory



Tiling/Blocking - Basic Idea

Global Memory

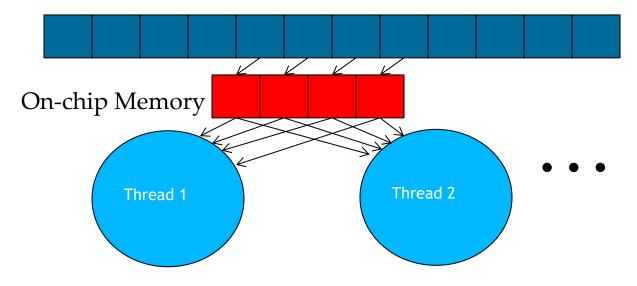


Divide the global memory content into tiles

Focus the computation of threads on one or a small number of tiles at each point in time

Tiling/Blocking - Basic Idea

Global Memory



Basic Concept of Tiling

- In a congested(拥挤的) traffic system, significant
 (显著的) reduction of vehicles can greatly improve
 (改善) the delay seen by all vehicles
 - Carpooling(拼车) for commuters(通勤者)
 - Tiling for global memory accesses
 - drivers = threads accessing their memory data operands
 - cars = memory access requests



Some Computations are More Challenging to Tile

- Some carpools may be easier than others
 - Car pool <u>participants(参与者)</u> need to have similar work schedule
 - Some <u>vehicles(交通工具)</u> may be more suitable for carpooling
- Similar challenges exist in tiling





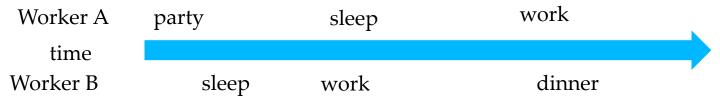
Carpools need synchronization.

Good: when people have similar schedule(日程)



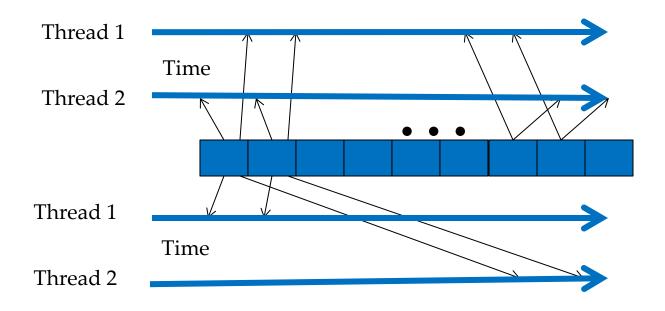
Carpools need synchronization.

Bad: when people have very different schedule



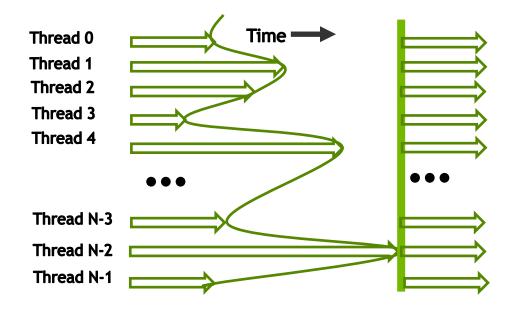
Same with Tiling

Good: when threads have similar access timing



Bad: when threads have very different timing

Barrier Synchronization for Tiling



Outline of Tiling Technique

- Identify <u>a tile of global memory contents</u> that are accessed by multiple threads
- Load the tile from global memory into <u>on-chip</u> <u>memory</u>
- Use <u>barrier synchronization</u> to make sure that all threads are ready to start the <u>phase(阶段)</u>
- Have the multiple threads to access their data from the on-chip memory
- Use <u>barrier synchronization</u> to make sure that all threads have completed the current phase
- Move on to the next tile



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Module 4.3 - Memory Model and Locality

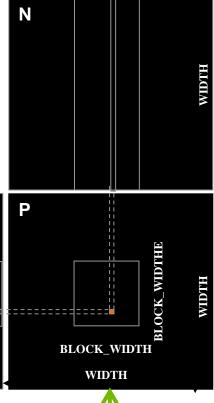
Tiled Matrix Multiplication

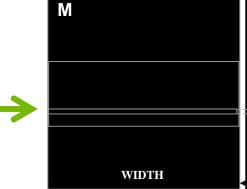
Objective

- To understand the design of a tiled parallel algorithm for matrix multiplication
 - Loading a tile
 - Phased execution(分阶段执行)
 - Barrier Synchronization

Matrix Multiplication

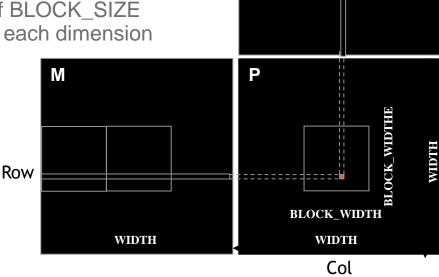
- Data access pattern
 - Each thread a row of M and a column of N
 - Each thread block a strip of M and a strip of N





Tiled Matrix Multiplication

- Break up the execution of each thread into phases
- so that the data accesses by the thread block in each phase are focused on one tile of M and one tile of N
- The tile is of BLOCK SIZE elements in each dimension

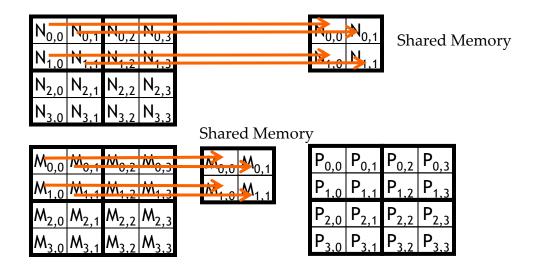


N

Loading a Tile

- All threads in a block participate(参与)
 - Each thread loads one M element and one N element in tiled code
 - Why?

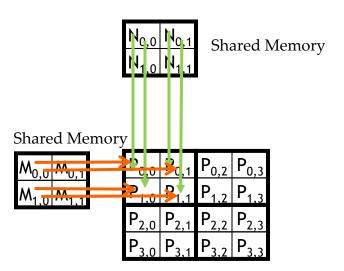
Phase 0 Load for Block (0,0)



Phase 0 Use for Block (0,0) (iteration 0)

| $N_{0,0}$ | N _{0,1} | $N_{0,2}$ | $N_{0,3}$ |
|------------------|------------------|-----------|------------------|
| $N_{1,0}$ | N _{1,1} | | N _{1,3} |
| N _{2,0} | N _{2,1} | $N_{2,2}$ | |
| N _{3,0} | $N_{3,1}$ | $N_{3,2}$ | $N_{3,3}$ |

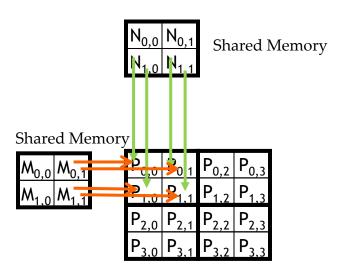
| $M_{0,0}$ | $M_{0,1}$ | $M_{0,2}$ | $M_{0,3}$ |
|-----------|-----------|-----------|-----------|
| $M_{1,0}$ | $M_{1,1}$ | $M_{1,2}$ | $M_{1,3}$ |
| $M_{2,0}$ | $M_{2,1}$ | $M_{2,2}$ | $M_{2,3}$ |
| $M_{3,0}$ | $M_{3,1}$ | $M_{3,2}$ | $M_{3,3}$ |



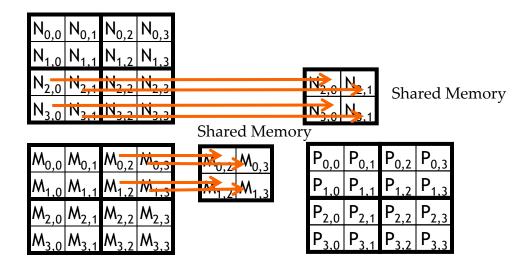
Phase 0 Use for Block (0,0) (iteration 1)

| $N_{0,0}$ | N _{0,1} | $N_{0,2}$ | $N_{0,3}$ |
|------------------|------------------|-----------|------------------|
| $N_{1,0}$ | N _{1,1} | $N_{1,2}$ | N _{1,3} |
| N _{2,0} | N _{2,1} | $N_{2,2}$ | |
| $N_{3,0}$ | N _{3,1} | $N_{3,2}$ | N _{3,3} |

| $M_{0,0}$ | $M_{0,1}$ | $M_{0,2}$ | $M_{0,3}$ |
|-----------|-----------|-----------|-----------|
| $M_{1,0}$ | $M_{1,1}$ | $M_{1,2}$ | $M_{1,3}$ |
| $M_{2,0}$ | $M_{2,1}$ | $M_{2,2}$ | $M_{2,3}$ |
| $M_{3,0}$ | $M_{3,1}$ | $M_{3,2}$ | $M_{3,3}$ |



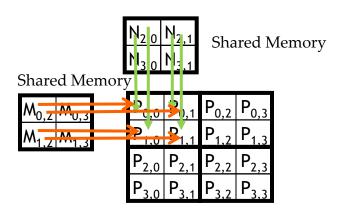
Phase 1 Load for Block (0,0)



Phase 1 Use for Block (0,0) (iteration 0)

| $N_{0,0}$ | N _{0,1} | $N_{0,2}$ | $N_{0,3}$ |
|------------------|------------------|-----------|------------------|
| | N _{1,1} | | N _{1,3} |
| $N_{2,0}$ | | $N_{2,2}$ | |
| N _{3,0} | | $N_{3,2}$ | |

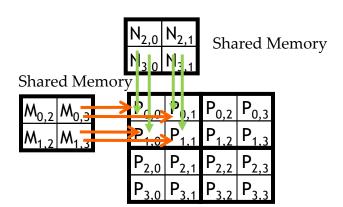
| $M_{0,0}$ | $M_{0,1}$ | $M_{0,2}$ | $M_{0,3}$ |
|-----------|-----------|-----------|-----------|
| $M_{1,0}$ | $M_{1,1}$ | $M_{1,2}$ | $M_{1,3}$ |
| $M_{2,0}$ | $M_{2,1}$ | $M_{2,2}$ | $M_{2,3}$ |
| | $M_{3,1}$ | | |



Phase 1 Use for Block (0,0) (iteration 1)

| $N_{0,0}$ | N _{0,1} | $N_{0,2}$ | $N_{0,3}$ |
|------------------|------------------|-----------|------------------|
| $N_{1,0}$ | N _{1,1} | $N_{1,2}$ | N _{1,3} |
| N _{2,0} | N _{2,1} | $N_{2,2}$ | |
| $N_{3,0}$ | N _{3,1} | $N_{3,2}$ | N _{3,3} |

| $M_{0,0}$ | $M_{0,1}$ | $M_{0,2}$ | $M_{0,3}$ |
|----------------------|-----------|-----------|-----------|
| $M_{1,0}$ | $M_{1,1}$ | $M_{1,2}$ | $M_{1,3}$ |
| $M_{2,0}$ | $M_{2,1}$ | $M_{2,2}$ | $M_{2,3}$ |
| $\overline{M}_{3,0}$ | $M_{3,1}$ | $M_{3,2}$ | $M_{3,3}$ |



Execution Phases of Toy Example

| | Phase 0 | | Phase 1 | | | |
|-----------------------|--|--|---|--|------------------------------------|--|
| thread _{0,0} | $M_{0,0}$ \downarrow $Mds_{0,0}$ | $N_{0,0}$ \downarrow $Nds_{0,0}$ | $\begin{array}{c} PValue_{0,0} += \\ Mds_{0,0}*Nds_{0,0} + \\ Mds_{0,1}*Nds_{1,0} \end{array}$ | $\mathbf{M}_{0,2}$ \downarrow $\mathbf{M}ds_{0,0}$ | $N_{2,0}$ \downarrow $Nds_{0,0}$ | $\begin{array}{l} PValue_{0,0} += \\ Mds_{0,0}*Nds_{0,0} + \\ Mds_{0,1}*Nds_{1,0} \end{array}$ |
| thread _{0,1} | $M_{0,1}$ \downarrow $Mds_{0,1}$ | $\begin{matrix} \mathbf{N_{0,1}} \\ \downarrow \\ \mathbf{Nds_{1,0}} \end{matrix}$ | $\begin{array}{c} PValue_{0,1} += \\ Mds_{0,0}*Nds_{0,1} + \\ Mds_{0,1}*Nds_{1,1} \end{array}$ | $\mathbf{M}_{0,3}$ \downarrow $\mathbf{M}ds_{0,1}$ | $N_{2,1}$ \downarrow $Nds_{0,1}$ | $PValue_{0,1} += \\ Mds_{0,0}*Nds_{0,1} + \\ Mds_{0,1}*Nds_{1,1}$ |
| thread _{1,0} | $\begin{matrix} \mathbf{M_{1,0}} \\ \downarrow \\ \mathbf{Mds_{1,0}} \end{matrix}$ | $N_{1,0}$ \downarrow $Nds_{1,0}$ | $PValue_{1,0} += \\ Mds_{1,0}*Nds_{0,0} + \\ Mds_{1,1}*Nds_{1,0}$ | $\mathbf{M}_{1,2}$ \downarrow $\mathbf{M}ds_{1,0}$ | $N_{3,0}$ \downarrow $Nds_{1,0}$ | $PValue_{1,0} += \\ Mds_{1,0}*Nds_{0,0} + \\ Mds_{1,1}*Nds_{1,0}$ |
| thread _{1,1} | $M_{1,1}$ \downarrow $Mds_{1,1}$ | $N_{1,1}$ \downarrow $Nds_{1,1}$ | $\begin{array}{l} \text{PValue}_{1,1} += \\ \text{Mds}_{1,0} * \text{Nds}_{0,1} + \\ \text{Mds}_{1,1} * \text{Nds}_{1,1} \end{array}$ | $\mathbf{M}_{1,3}$ \downarrow $\mathbf{M}ds_{1,1}$ | $N_{3,1}$ \downarrow $Nds_{1,1}$ | $PValue_{1,1} += \\ Mds_{1,0}*Nds_{0,1} + \\ Mds_{1,1}*Nds_{1,1}$ |

time

Execution Phases of Toy Example (cont.)

| | Phase 0 | | Phase 1 | | | |
|-----------------------|------------------------------------|---|---|---|------------------------------------|--|
| thread _{0,0} | $M_{0,0}$ \downarrow $Mds_{0,0}$ | $egin{array}{c} \mathbf{N_{0,0}} \\ \downarrow \\ \mathrm{Nds_{0,0}} \end{array}$ | $\begin{array}{c} PValue_{0,0} += \\ Mds_{0,0}*Nds_{0,0} + \\ Mds_{0,1}*Nds_{1,0} \end{array}$ | $\mathbf{M}_{0,2}$ \downarrow $\mathbf{Mds}_{0,0}$ | $N_{2,0}$ \downarrow $Nds_{0,0}$ | $\begin{array}{l} PValue_{0,0} += \\ Mds_{0,0}*Nds_{0,0} + \\ Mds_{0,1}*Nds_{1,0} \end{array}$ |
| thread _{0,1} | $M_{0,1}$ \downarrow $Mds_{0,1}$ | $N_{0,1}$ \downarrow $Nds_{1,0}$ | $PValue_{0,1} += Mds_{0,1} *Nds_{0,1} + Mds_{0,1} *Nds_{1,1}$ | $\mathbf{M}_{0,3}$ \downarrow $\mathbf{Mds}_{0,1}$ | $N_{2,1}$ \downarrow $Nds_{0,1}$ | $PValue_{0,1} += \\ Mds_{0,0}*Nds_{0,1} + \\ Mds_{0,1}*Nds_{1,1}$ |
| thread _{1,0} | $M_{1,0}$ \downarrow $Mds_{1,0}$ | $N_{1,0}$ \downarrow $Nds_{1,0}$ | $PValue_{1,0} += \\ Mds_{1,0}*Nds_{0,0} + \\ Mds_{1,1}*Nds_{1,0}$ | $M_{1,2}$ \downarrow $Mds_{1,0}$ | $N_{3,0}$ \downarrow $Nds_{1,0}$ | $PValue_{1,0} += \\ Mds_{1,0}*Nds_{0,0} + \\ Mds_{1,1}*Nds_{1,0}$ |
| $thread_{1,1}$ | $M_{1,1}$ \downarrow $Mds_{1,1}$ | $N_{1,1}$ \downarrow $Nds_{1,1}$ | $\begin{array}{l} \text{PValue}_{1,1} += \\ \text{Mds}_{1,0} * \text{Nds}_{0,1} + \\ \text{Mds}_{1,1} * \text{Nds}_{1,1} \end{array}$ | $\mathbf{M}_{1,3}$ \downarrow $\mathbf{M}d\mathbf{s}_{1,1}$ | $N_{3,1}$ \downarrow $Nds_{1,1}$ | $PValue_{1,1} += \\ Mds_{1,0}*Nds_{0,1} + \\ Mds_{1,1}*Nds_{1,1}$ |

time

Shared memory allows each value to be accessed by multiple threads

Barrier Synchronization

- Synchronize all threads in a block
 - __syncthreads()
- All threads in the same block must reach the __syncthreads() before any of the them can move on
- Best used to <u>coordinate(协调)</u> the phased execution tiled algorithms
 - To ensure that all elements of a tile are loaded at the beginning of a phase
 - To ensure that all elements of a tile are consumed(消费、使用) at the end of a phase



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Accelerated Computing



Module 4.4 - Memory and Data Locality

Tiled Matrix Multiplication Kernel

Objective

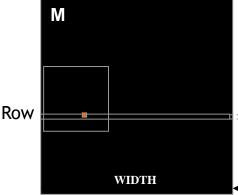
- To learn to write a tiled matrix-multiplication kernel
 - Loading and using tiles for matrix multiplication
 - Barrier synchronization, shared memory
 - Resource Considerations
 - Assume that Width is a multiple of tile size for simplicity(简化)

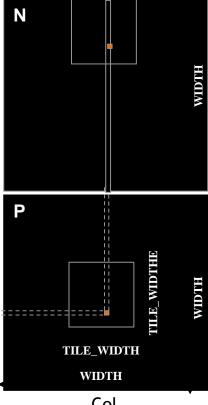
Loading Input Tile 0 of M (Phase 0)

 Have each thread load an M element and an N element at the same relative position (相对位置) as its P element.

```
int Row = by * blockDim.y + ty;
int Col = bx * blockDim.x + tx;
2D indexing for accessing Tile 0:
```

MIRowl[tx] N[ty][Col]

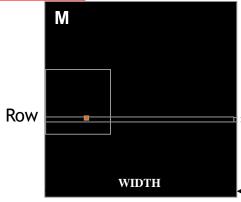


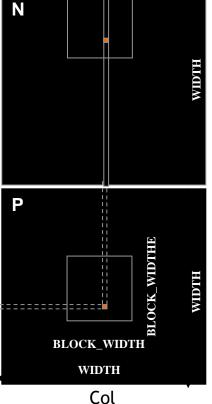


Loading Input Tile 0 of N (Phase 0)

 Have each thread load an M element and an N element at the same relative position as its P element.

N[ty][Col]

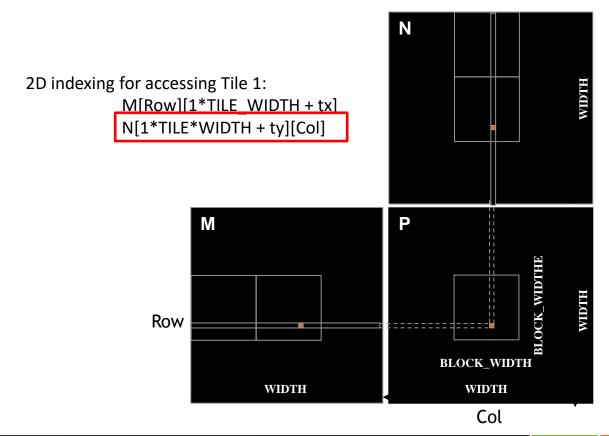




Loading Input Tile 1 of M (Phase 1)



Loading Input Tile 1 of N (Phase 1)



M and N are dynamically allocated (动态分配的)- use 1D indexing

- M[Row][p*TILE_WIDTH+tx]

 M[Row*Width + p*TILE_WIDTH + tx]
- N[p*TILE_WIDTH+ty][Col]
 N[(p*TILE_WIDTH+ty)*Width + Col]

where p is the sequence number of the current phase

Tiled Matrix Multiplication Kernel

__global__ void MatrixMulKernel(float* M, float* N, float* P, Int Width)

```
__shared__ float ds_M[TILE_WIDTH][TILE_WIDTH];
__shared__ float ds_N[TILE_WIDTH][TILE_WIDTH];

int bx = blockIdx.x; int by = blockIdx.y;
int tx = threadIdx.x; int ty = threadIdx.y;

int Row = by * blockDim.y + ty;
int Col = bx * blockDim.x + tx;
float Pvalue = 0;
```

```
// Loop over the M and N tiles required to compute the P element
for (int p = 0; p < n/TILE_WIDTH; ++p) {
    // Collaborative loading of M and N tiles into shared memory
    ds_M[ty][tx] = M[Row*Width + p*TILE_WIDTH+tx];
    ds_N[ty][tx] = N[(t*TILE_WIDTH+ty)*Width + Col];
    __syncthreads();

    for (int i = 0; i < TILE_WIDTH; ++i)Pvalue += ds_M[ty][i] * ds_N[i][tx];
    __synchthreads();
}
P[Row*Width+Col] = Pvalue;</pre>
```

Tiled Matrix Multiplication Kernel

```
global void MatrixMulKernel(float* M, float* N, float* P, Int Width)
 shared float ds M[TILE WIDTH][TILE WIDTH];
 shared float ds N[TILE WIDTH][TILE WIDTH];
int bx = blockIdx.x; int by = blockIdx.y;
int tx = threadIdx.x; int ty = threadIdx.y;
int Row = by * blockDim.y + ty;
int Col = bx * blockDim.x + tx;
float Pvalue = 0;
// Loop over the M and N tiles required to compute the P element
for (int p = 0; p < n/TILE WIDTH; ++p) {
  // Collaborative loading of M and N tiles into shared memory
  ds M[ty][tx] = M[Row*Width + p*TILE WIDTH+tx];
  ds N[ty][tx] = N[(t*TILE WIDTH+ty)*Width + Col];
    syncthreads();
   for (int i = 0; i < TILE WIDTH; ++i) Pvalue += ds M[ty][i] * ds N[i][tx];
  synchthreads();
P[Row*Width+Col] = Pvalue;
```

Tiled Matrix Multiplication Kernel

```
global void MatrixMulKernel(float* M, float* N, float* P, Int Width)
 shared float ds M[TILE WIDTH][TILE WIDTH];
 shared float ds N[TILE WIDTH][TILE WIDTH];
 int bx = blockIdx.x; int by = blockIdx.y;
 int tx = threadIdx.x; int ty = threadIdx.y;
 int Row = by * blockDim.y + ty;
 int Col = bx * blockDim.x + tx;
 float Pvalue = 0;
// Loop over the M and N tiles required to compute the P element
for (int p = 0; p < n/TILE WIDTH; ++p) {
   // Collaborative loading of M and N tiles into shared memory
   ds M[ty][tx] = M[Row*Width + p*TILE WIDTH+tx];
   ds N[ty][tx] = N[(t*TILE WIDTH+ty)*Width + Col];
   syncthreads();
   for (int i = 0; i < TILE WIDTH; ++i) Pvalue += ds M[ty][i] * ds N[i][tx];
    synchthreads();
 P[Row*Width+Col] = Pvalue;
```

Tile (Thread Block) Size Considerations

- Each thread block should have many threads
 - TILE_WIDTH of 16 gives 16*16 = 256 threads
 - TILE_WIDTH of 32 gives 32*32 = 1024 threads
- For 16, in each phase, each block performs 2*256 = 512 float loads from global memory for 256 * (2*16) = 8192 mul/add operations. (16 floating-point operations for each memory load)
- For 32, in each phase, each block performs 2*1024 = 2048 float loads from global memory for 1024 * (2*32) = 65536 mul/add operations. (32 floating-point operation for each memory load)

```
for (int i = 0; i < TILE_WIDTH; ++i) Pvalue += ds_M[ty][i] * ds_N[i][tx];
__synchthreads();</pre>
```

Shared Memory and Threading

- For an SM with 16KB shared memory
 - Shared memory size is implementation dependent!
 - For 16KB shared memory, one can potentially(潜在地) have up to 8 thread blocks executing
 - For <u>TILE_WIDTH = 16</u>, each thread block uses <u>2*256*4B = 2KB</u> of shared memory.
 - This allows up to **8*512 = 4,096** pending(待处理的) loads. (2 per thread, 256 threads per block)
 - The next <u>TILE_WIDTH 32</u> would lead to <u>2*32*32*4 Byte= 8K</u> Byte shared memory usage(用量) per thread block, allowing 2 thread blocks active at the same time
 - However, in a GPU where the thread count is limited to 1536 threads per SM, the number of blocks per SM is reduced to one!
- Each __syncthread() can reduce the number of active threads for a block
 - More thread blocks can be advantageous(有利的)



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Module 4.5 - Memory and Data Locality

Handling Arbitrary Matrix Sizes in Tiled Algorithms

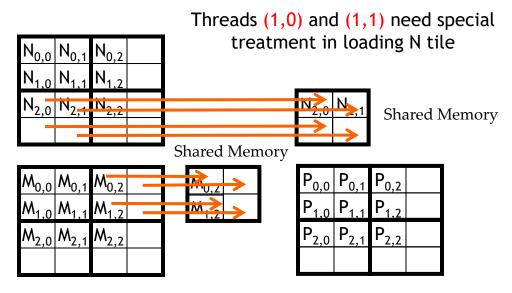
Objective

- To learn to handle arbitrary(任意的) matrix sizes in tiled matrix multiplication
 - Boundary(边界) condition checking
 - Regularizing(规范化) tile contents
 - Rectangular matrices

Handling Matrix of Arbitrary Size

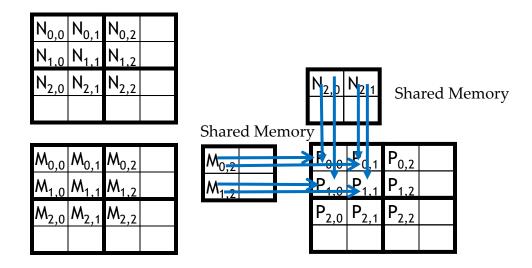
- The tiled matrix multiplication kernel we presented so far can handle only <u>square matrices(分降)</u> whose dimensions (Width) are multiples of the tile width (TILE_WIDTH)
 - However, real applications need to handle arbitrary sized matrices.
 - One could <u>pad(填充)</u> the rows and columns into multiples of the tile size, but would have significant space and data transfer time overhead(射闪开销).
- We will take a different approach.

Phase 1 Loads for Block (0,0) for a 3x3 Example



Threads (0,1) and (1,1) need special treatment in loading M tile

Phase 1 Use for Block (0,0) (iteration 0)



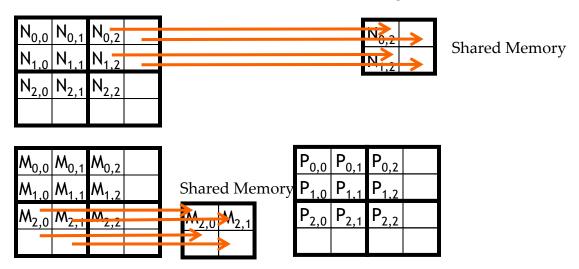
Phase 1 Use for Block (0,0) (iteration 1)

| $\begin{array}{c cccc} N_{0,0} & N_{0,1} & N_{0,2} \\ N_{1,0} & N_{1,1} & N_{1,2} \\ N_{2,0} & N_{2,1} & N_{2,2} \\ \end{array}$ | N _{2,0} N _{2,1} Shared Memory |
|--|---|
| $M_{0,0} M_{0,1} M_{0,2}$ $M_{1,0} M_{1,1} M_{1,2}$ $M_{2,0} M_{2,1} M_{2,2}$ | $M_{0,2}$ $M_{0,2}$ $P_{0,0}$ $P_{0,1}$ $P_{0,2}$ $P_{1,1}$ $P_{1,2}$ $P_{2,0}$ $P_{2,1}$ $P_{2,2}$ |

All Threads need special treatment.
None of them should introduce
invalidate(无效的) contributions to
their P elements.

Phase 0 Loads for Block (1,1) for a 3x3 Example

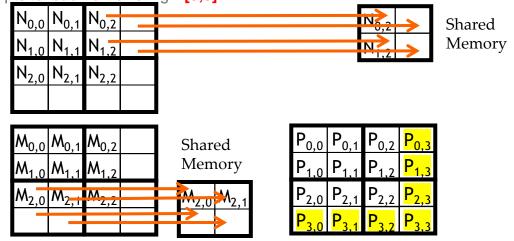
Threads (0,1) and (1,1) need special treatment in loading N tile



Threads (1,0) and (1,1) need special treatment in loading M tile

Major Cases in Toy Example

- Threads that do not calculate valid P elements but still need to participate in loading the input tiles
 - Phase 0 of Block(1,1), Thread(1,0), assigned to calculate non-existent P[3,2] but need to participate in loading tile element N[1,2]
- Threads that calculate valid P elements may attempt to <u>load</u> non-existing input elements when loading input tiles
 - Phase 0 of Block(0,0), Thread(1,0), assigned to calculate valid P[1,0] but attempts to load non-existing N[3,0]



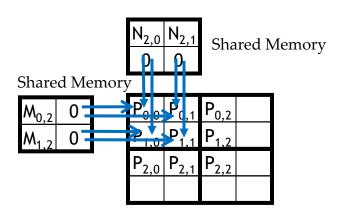
A "Simple" Solution

- When a thread is to load any input element, test if it is in the valid index range
 - If valid, proceed to load
 - Else, do not load, just write a 0
- Rationale(原因是): a 0 value will ensure that that the multiply-add step does not affect the final value of the output element
- The condition tested for loading input elements is different from the test for calculating output P element
 - A thread that does not calculate valid P element can still participate in loading input tile elements

Phase 1 Use for Block (0,0) (iteration 1)

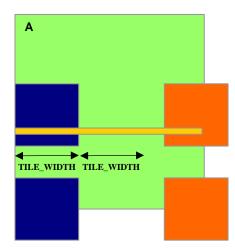
| $N_{0,0}$ | N _{0,1} | $N_{0,2}$ | |
|------------------|------------------|------------------|--|
| $N_{1,0}$ | N _{1,1} | $N_{1,2}$ | |
| N _{2,0} | N _{2,1} | N _{2,2} | |
| | | | |

| $M_{0,0}$ | $M_{0,1}$ | $M_{0,2}$ | |
|-----------|-----------|-----------|--|
| | | $M_{1,2}$ | |
| $M_{2,0}$ | $M_{2,1}$ | $M_{2,2}$ | |
| | | | |



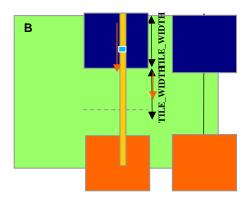
Boundary Condition for Input M Tile

- Each thread loads
 - M[Row][p*TILE_WIDTH+tx]
 - M[Row*Width + p*TILE_WIDTH+tx]
- Need to test
 - (Row < Width) && (p*TILE_WIDTH+tx < Width)</p>
 - If true, load M element
 - Else, load o



Boundary Condition for Input N Tile

- Each thread loads
 - N[p*TILE_WIDTH+ty][Col]
 - N[(p*TILE_WIDTH+ty)*Width+ Col]
- Need to test
 - (p*TILE_WIDTH+ty < Width) && (Col< Width)</p>
 - If true, load N element
 - Else, load 0



Loading Elements – with boundary check

```
for (int p = 0; p < (Width-1) / TILE_WIDTH + 1; ++p) {
  if(Row < Width && t * TILE_WIDTH+tx < Width) {
     ds M[ty][tx] = M[Row * Width + p * TILE WIDTH + tx];
  } else {
     ds_M[ty][tx] = 0.0;
 if (p*TILE_WIDTH+ty < Width && Col < Width) {
     ds_N[ty][tx] = N[(p*TILE_WIDTH + ty) * Width + Col];
 } else {
      ds N[ty][tx] = 0.0;
 __syncthreads();
```

Inner Product – Before and After

```
if(Row < Width && Col < Width) {
    for (int i = 0; i < TILE_WIDTH; ++i) {
        Pvalue += ds_M[ty][i] * ds_N[i][tx];
    }
    __syncthreads();
    } /* end of outer for loop */
    if (Row < Width && Col < Width)
        P[Row*Width + Col] = Pvalue;
    } /* end of kernel */</pre>
```

Some Important Points

- For each thread the conditions are different for
 - Loading M element
 - Loading N element
 - Calculating and storing output elements
- The effect of <u>control divergence(控制分歧)</u> should be small for large matrices

Handling General Rectangular Matrices

- In general, the matrix multiplication is defined in terms of rectangular matrices
 - A <u>j x k</u> M matrix multiplied with a <u>k x l</u> N matrix results in a j x l P matrix
- We have presented square matrix multiplication, a special case
- The kernel function needs to be generalized(泛化)
 to handle general rectangular matrices
 - The Width argument is replaced by three arguments: j, k, l
 - When Width is used to refer to the height of M or height of P, replace it with j
 - When Width is used to refer to the width of M or height of N, replace it with k
 - When Width is used to refer to the width of N or width of P, replace it with I



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