

CS 211 RECITATIONS

WEEK 6

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Content

- Number Systems
 - *decimal, binary, hexadecimal, octal*
 - *conversions*
- Representing Integers
 - *signed magnitude*
 - *one's complement*
 - *two's complement*
- Coding
 - *ASCII, Unicode and UTF-8*
- Floating Point Standard

Number Systems

■ Different Bases

- (10) Decimal (by default)
- (2) Binary (start with 0b, 0b10011, GCC extension)
- (16) Hexadecimal (start with 0x/0X, 0xABCD)
- (8) Octal (start with 0, 012 in octal is $1*8 + 2*1 = 10$ in decimal!)

■ Conversion Tips

- Hex to Binary: each digit in hex can be represented by FOUR bits
- Octal to Binary: each digit in octal can be represented by THREE bits
- Binary to Hex/Octal: treat each FOUR/THREE bits as a group
- Decimal to Binary
 - Memorize those bases, 1, 2, 4, 8, 16, 32, 64, etc.
 - Division

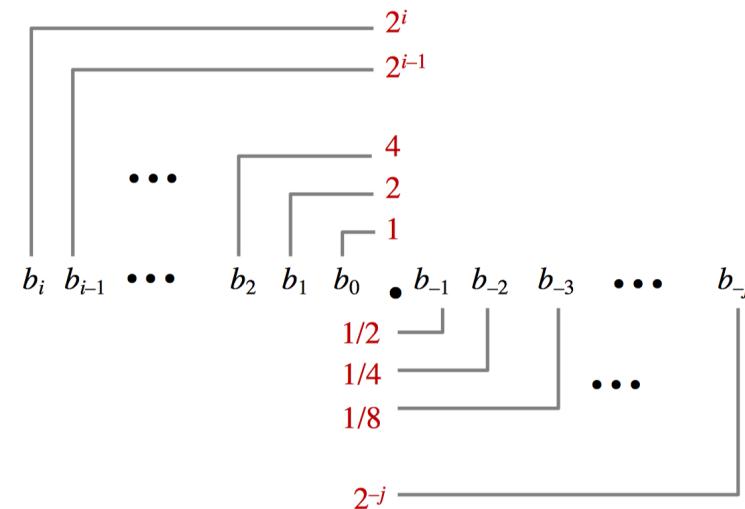
A handwritten diagram on lined paper showing the conversion of the decimal number 24 to binary. The process is shown as a series of divisions by 2:

- 2 | 24 0 ← MSB (Most Significant Bit)
- 2 | 12 0
- 2 | 6 0
- 2 | 3 1
- 2 | 1 1 ← MSB
- 0

The quotient 0b11000 is written to the right of the final step, indicating the binary representation of 24.

Number Systems

- Not just for integers
- $8.625 = 1000.101$
 - $0.625 = 1/2 + 1/8 = 2^{-1} + 2^{-3} = 0.101$
 - So 8.625 is exactly 1000.101 in binary



■ Representation

- Bits to right of “binary point” represent fractional powers of 2
- Represents rational number: $\sum_{k=-j}^i b_k \cdot 2^k$

Representing Integers

- Computers do not naturally know the concept of “positive/negative”, they store sequence of bits.
- Signed Magnitude
 - Use the first bit to represent positive(0)/negative(1)
- Problems
 - Two zeros?
 - Positive + Positive is okay
 - Negative + Negative is okay
 - Negative + Positive cannot be calculated directly

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

111	110	101	100	000	001	010	011
-3	-2	-1	-0	0	1	2	3

Representing Integers

- One's complement
 - *The first bit still represents positive (0)/negative(1)*
 - *Flip every bits and get the negative value*
- We still have two zeros!
- But $110 + 001$ now can be calculated directly!
 - $110 + 001 = 111$ (*negative zero*)
 - *But this is correct anyway*

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7

100	101	110	111	000	001	010	011
-3	-2	-1	-0	0	1	2	3

Representing Integers

- Two's complement
 - *One's complement plus one (in binary) in both directions*
 - *The first bit still represents positive(0)/negative(1)*
- One more negative number (only one zero!)
 - *Very convenient to use*
 - *Used in most computers today*

000	001	010	011	100	101	110	111
0	1	2	3	4	5	6	7
100	101	110	111	000	001	010	011
-4	-3	-2	-1	0	1	2	3

Coding

■ ASCII

- A character is stored as one byte according to the ASCII standard.
- Previously, 7 bits -> 128 characters, now 8 bits -> 256 characters.
- *char* in C programming language.

ASCII table

	0	1	2	3	4	5	6	7
0	NUL	DLE	space	0	@	P	'	p
1	SOH	DC1 XON	!	1	A	Q	a	q
2	STX	DC2	"	2	B	R	b	r
3	ETX	DC3 XOFF	#	3	C	S	c	s
4	EOT	DC4	\$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	'	7	G	W	g	w
8	BS	CAN	(8	H	X	h	x
9	HT	EM)	9	I	Y	i	y
A	LF	SUB	*	:	J	Z	j	z
B	VT	ESC	+	;	K	[k	{
C	FF	FS	,	<	L	\	l	
D	CR	GS	-	=	M]	m	}
E	SO	RS	.	>	N	^	n	~
F	SI	US	/	?	O	_	o	del

Coding

- Unicode is a standard that defines more than 100k characters in many languages.
- Unicode can be implemented by different character encodings.
- Most common: UTF-8
 - *Variable length: 1-4 Bytes for each character*
- Others:
 - *UFT-16*
 - *GBK*
 - *and many more*

Floating Point Standard

Numerical form:

$$V_{10} = (-1)^S * M * 2^E$$

- Sign bit **s** determines whether number is negative or positive
- Significand (mantissa) **M** normally a fractional value in range [1.0, 2.0)
- Exponent **E** weights value by a (possibly negative) power of two

Representation in memory:

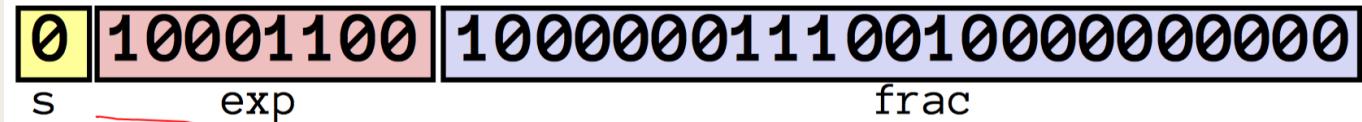
- MSB **s** is sign bit **s**
- exp field encodes **E** (but is *not equal* to E)
- frac field encodes **M** (but is *not equal* to M)



Floating Point Standard

- 12345 in decimal is 11000000111001 in binary
 - $1.1000000111001 * 2^{13}$ (scientific notations in binary)
- Mantissa
 - $M = 1.1000000111001$
 - $frac = 1000000111001$ followed by 0
- Exponent
 - $E = 13 = exp - Bias$
 - $Bias = 2^{k-1} - 1 = (127 \text{ in 32 bits} / 1023 \text{ in 64 bits})$
 - $exp = E + Bias = 140 \text{ in decimal, so } 10001100 \text{ in binary}$

$$V_{10} = (-1)^S * M * 2^E$$



Floating Point Standard

32 bits single precision (type float)

- 1 bit for sign, 8 bits for exponent, 23 bits for mantissa
 - Sign bit: 1 = negative numbers, 0 = positive numbers
 - Exponent is power of 2
- Have 2 zero's
- Range is approximately -10^{38} to 10^{38}

64 bits double precision (type double)

- 1 bit for sign, 11 bits for exponent, 52 bits for mantissa
- Majority of new bits for mantissa → higher precision
- Range is -10^{308} to $+10^{308}$

