RWR 4013

Digital Twins for Smart Cities

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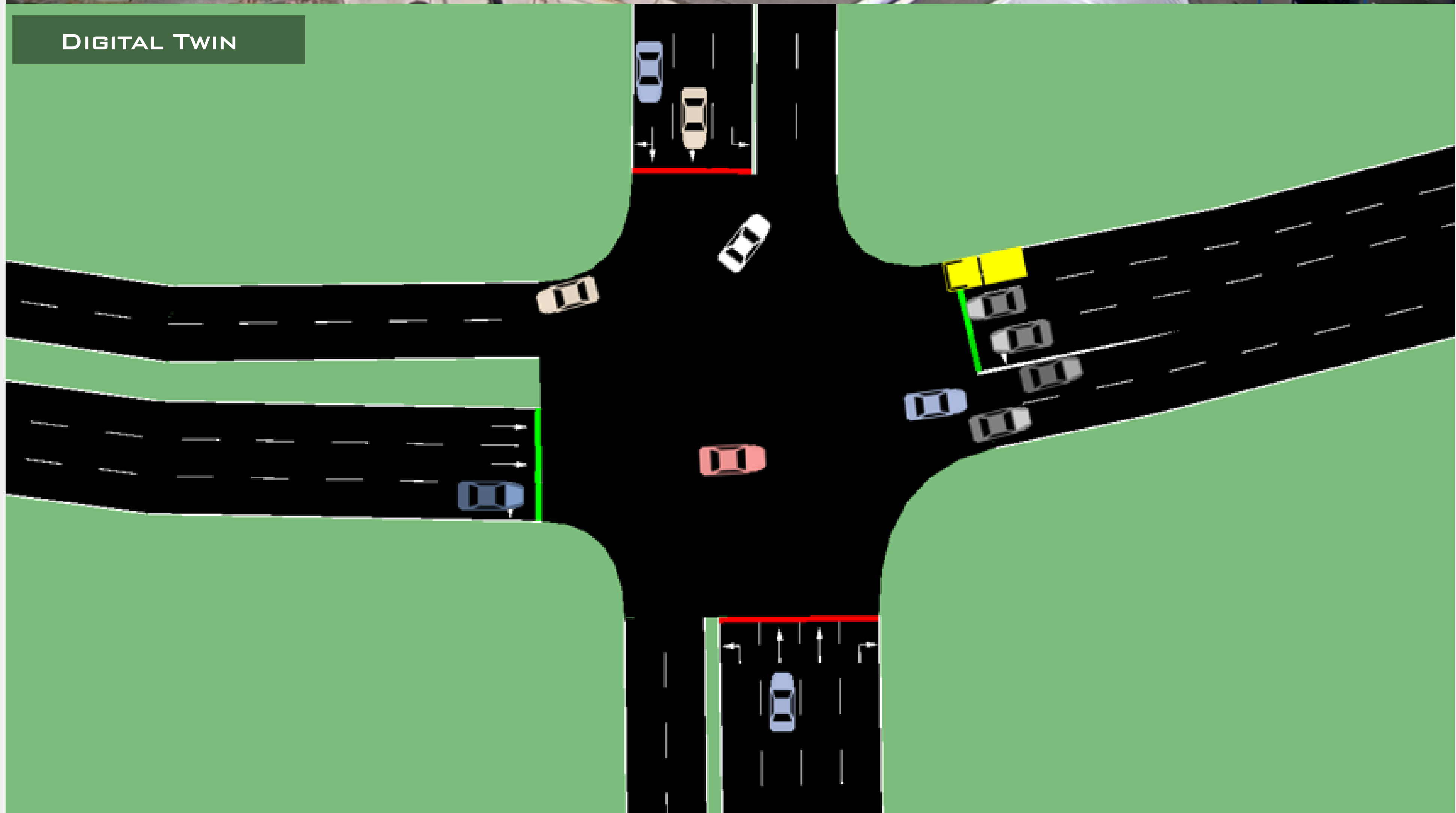
Week 9 | Session 2:
Optimization (Traffic Signal)

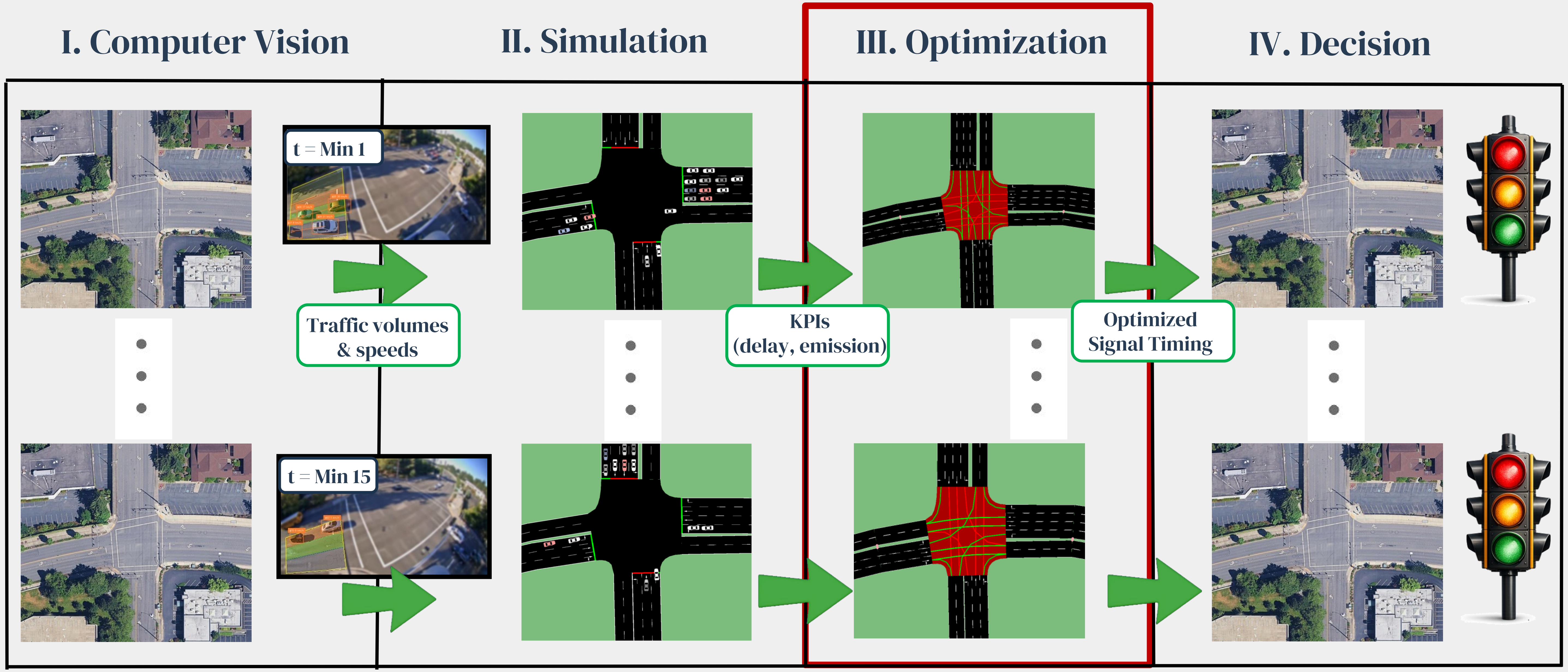
Fall 2026

RoadwayVR

roadwayvr.github.io/DigitalTwinsforSmartCities

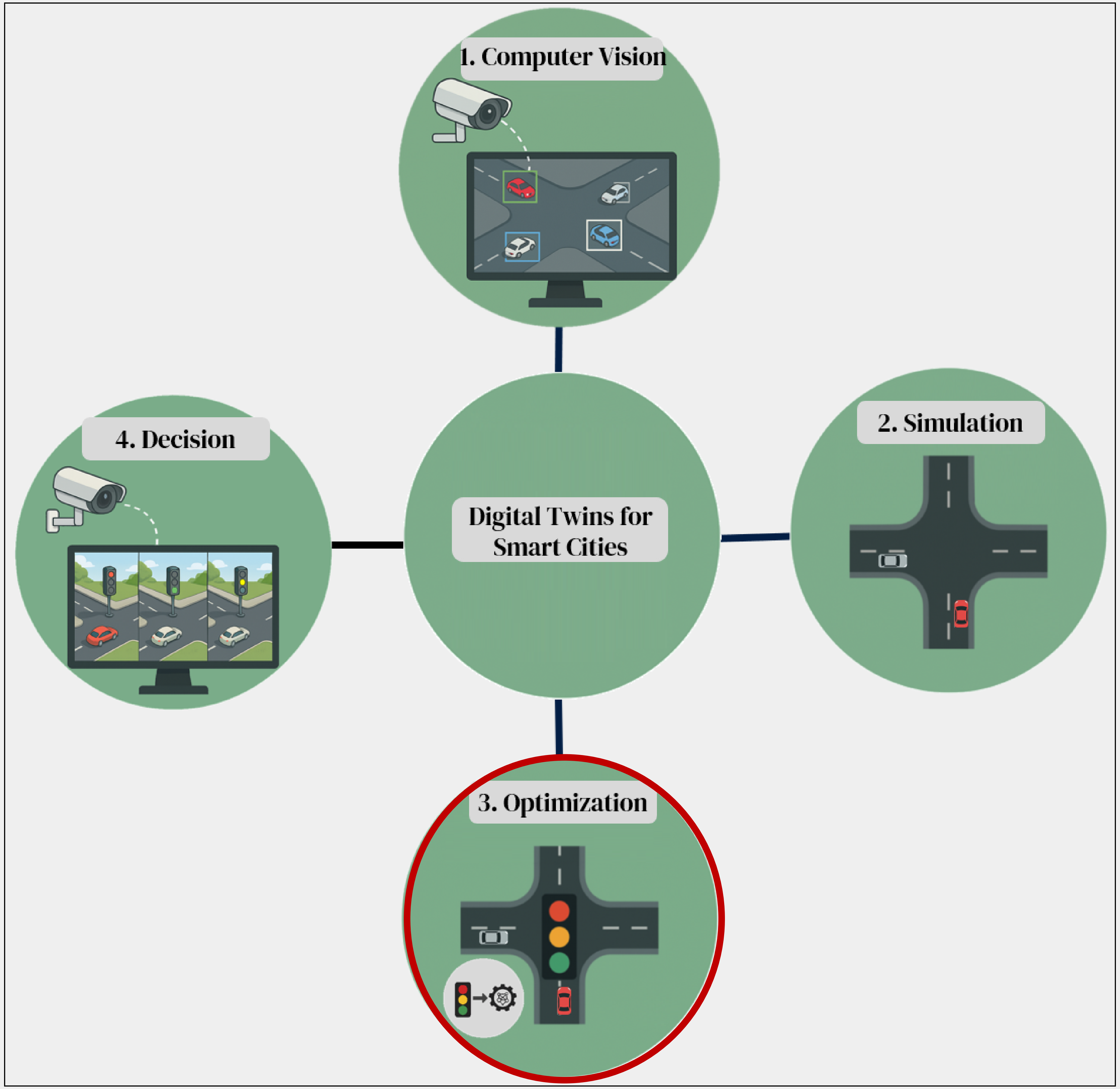


Overview of Course Syllabus in One Shot



Agenda

- ❑ Functional and Physical Areas of Intersection
- ❑ Key Performance Indicators (KPIs)
- ❑ From Delay to Level of Service (LOS)
- ❑ Optimization



Functional and Physical Areas of Intersection

Typical FHWA values (approach speed = 30 mph / 48 km/h):

- **Upstream functional area:** 395 ft (120 m)
- **Downstream functional area:** 200 ft (61 m)

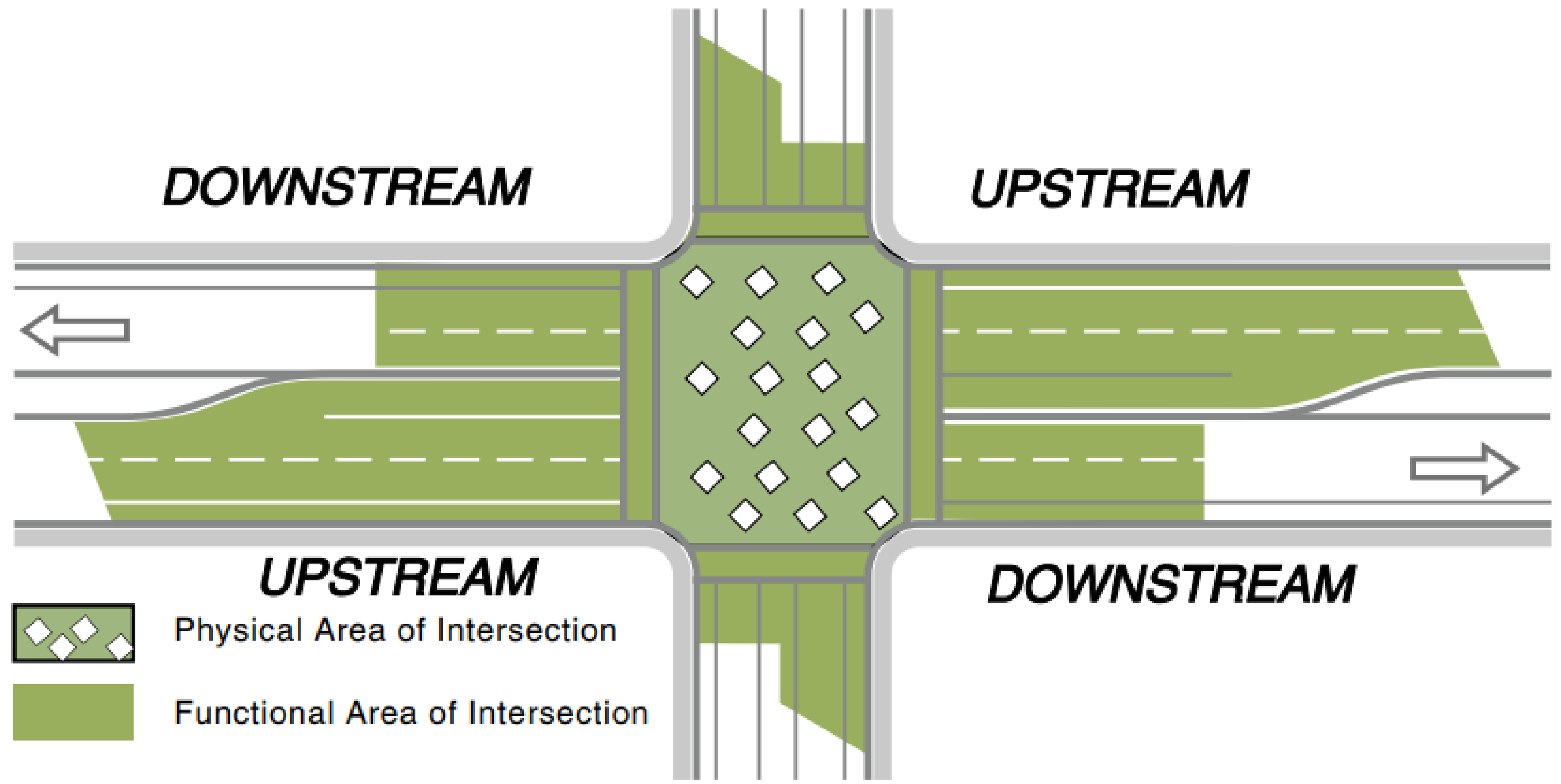


Figure 1: Functional and Physical Areas of an Intersection

Federal Highway Administration. (2010). *Access management manual* (FHWA-SA-10-002). U.S. Department of Transportation. https://highways.dot.gov/sites/fhwa.dot.gov/files/2022-06/fhwasa10002.pdf

Key Performance Indicators (KPIs)

What are KPIs?

- KPIs are measurable values that summarize intersection performance.
- They help compare before vs. after changes (e.g., signal timing)

Computed within the upstream functional area:

- **Average Delay (s/veh)** *(Lower is better)*
  - Extra travel time per vehicle compared to free-flow
  - Includes: slowdown + stopped + re-acceleration
- **Average Stopped Delay (s/veh)** *(Lower is better)*
  - Time spent stopped (near 0 speed) while waiting in a queue / at the stop line

Key Performance Indicators (KPIs)

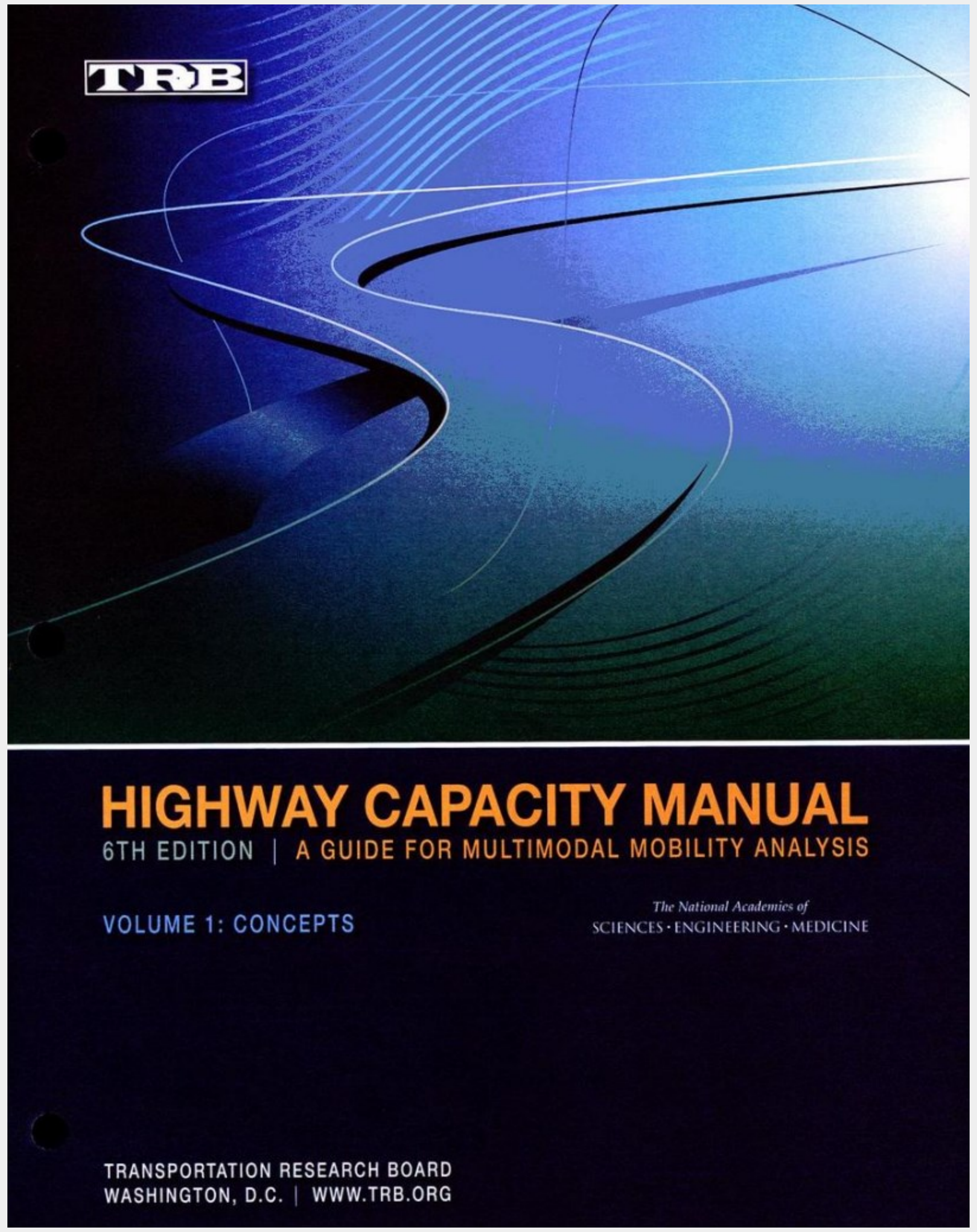
Computed within the upstream functional area:

- **CO_2 Emissions (mg)** *(Lower is better)*
  - Total CO_2 emitted during the analysis period
  - Includes emissions while **moving** and **idling/queuing**

- **Throughput (veh)** *(Higher is better)*
  - Number of vehicles that **successfully pass the stop line** during the analysis period

From Delay to Level of Service (LOS)

HCM (TRB): North America's standard for intersection performance evaluation



Level of Service (LOS):

- An **A–F grade** that describes how well a signalized intersection operates
- Based mainly on **Average Delay (s/veh)**

Quick interpretation:

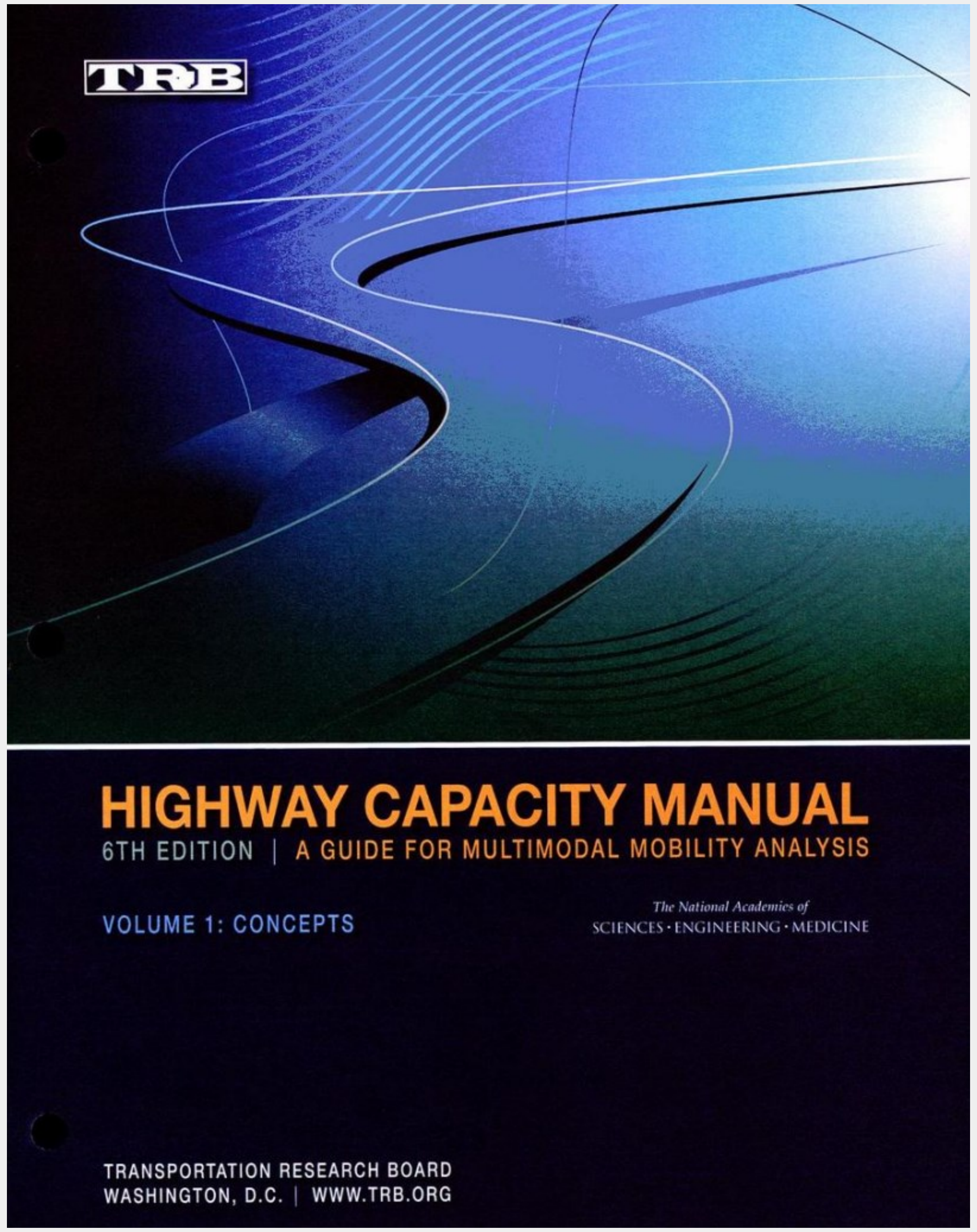
- **LOS A:** little delay, smooth flow
- **LOS F:** long delay, heavy congestion (often oversaturated)

A **Best**
B
C
D
E
F **Worst**

Next: We compute **average delay → map to LOS** using HCM thresholds.

HCM Level of Service (LOS)

- Compute average delay (s/veh) for the intersection.
- Compare it to the HCM thresholds below to assign LOS A–F.



HCM LOS Thresholds (Average Delay, s/veh)

LOS	Average Delay (s/veh)
A	10
B	> 10 and 20
C	> 20 and 35
D	> 35 and 55
E	> 55 and 80
F	> 80

Example: If delay = **42 s/veh**, then **LOS D** (35–55).

Optimization

Optimization:

- Choosing the best values of some decision variables to minimize or maximize a KPI
- To minimize average delay or emission or maximize throughput while respecting constraints

Signal Optimization:

- Choosing the best values of **signal timing parameters** (e.g., **green time per phase**)
- To minimize average delay while respecting constraints including, **minimum green**, safety **yellow/all-red**, etc.

Webster Singal Optimization:

- A widely used fixed-time signal-timing approach
- To minimize average delay by estimating an “optimal” green time for each phase based on upstream traffic volume.

Webster Signal Optimization

Step 1: Compute effective traffic volume

$E_{NS} = D_{NS} + \alpha Q_{NS}$

$E_{EW} = D_{EW} + \alpha Q_{EW}$

D_{NS} : Upstream traffic volumes in North and South (veh/min)
D_{EW}: Upstream traffic volumes in East and West (veh/min)
Q_{NS} : Queue lengths (veh) in North and South
Q_{EW}: Queue lengths (veh) in East and West
α : Queue-weighting parameter
α = 0 → Ignore queues;
α = 1 → Give equal weight to queues and demand

Step 2: Compute the proportional split (closed form)

$G = C - L$

$g_{NS} = \frac{E_{NS}}{E_{NS}+E_{EW}} \times G$

$g_{EW} = G - g_{NS}$

G = Available green time per cycle (s)
C : Signal Cycle length (s)
L : Total lost time per cycle (s)

Step 3: Enforce constraints by projection

$g_{min} \leq g_{NS} \leq G - g_{min}$

$g_{min} \leq g_{EW} \leq G - g_{min}$

g_{min} = Minimum green time (s)

Optimization (Exercise)

Exercise: Digital Twin Signal Timing Recommendation (15-Minute Data)

Problem context

A city is experiencing peak-period congestion at a two-phase signalized intersection. Drivers report long queues and delays, especially when demand shifts from East–West to North–South. The average delay is 80 on all directions for existing condition. The city wants to test a simple Digital Twin (DT) controller that updates green splits using minute by minute observed data.

The DT uses effective demand (demand + queue) to decide how to split the available green time G between:

- ❑ Phase NS: North–South movements
- ❑ Phase EW: East–West movements

You are the engineer. The city gives you minute by minute data and asks:

a) "How to split the available green time in each min?"

b) Assume you provided the avaibale green time and below is the average delay for each min for each direction, please calculate level of service for the existing condition and for Digital Twin Controller.

Optimization (Exercise)

Exercise: Digital Twin Signal Timing Recommendation (15-Minute Data)

Problem context

Compute D_{NS} and D_{EW}
Compute Q_{NS} and Q_{EW}
Compute G:
Compute g_{NS} and g_{EW}

Consider $g_{min} \leq g_{NS} \leq G - g_{min}$ (If g_{NS} is less than g_{min} in each min, you need to use $g_{NS} = g_{min}$ if g_{NS} is more than $G - g_{min}$ in each min, you need to use $g_{NS} = G - g_{min}$

Consider $g_{min} \leq g_{EW} \leq G - g_{min}$ (If g_{EW} is less than g_{min} in each min, you need to use $g_{EW} = g_{min}$ if g_{EW} is more than $G - g_{min}$ in each min, you need to use $g_{EW} = G - g_{min}$

Optimization (Exercise)

Download Week 9 Materials – SUMO Files, Data, Application

Run application →

In Base Scenario (Existing Signal Timing) → Assign SUMO Net, SUMO CFG, Traffic Deman Data (15-Min Calibrated Observed Data)

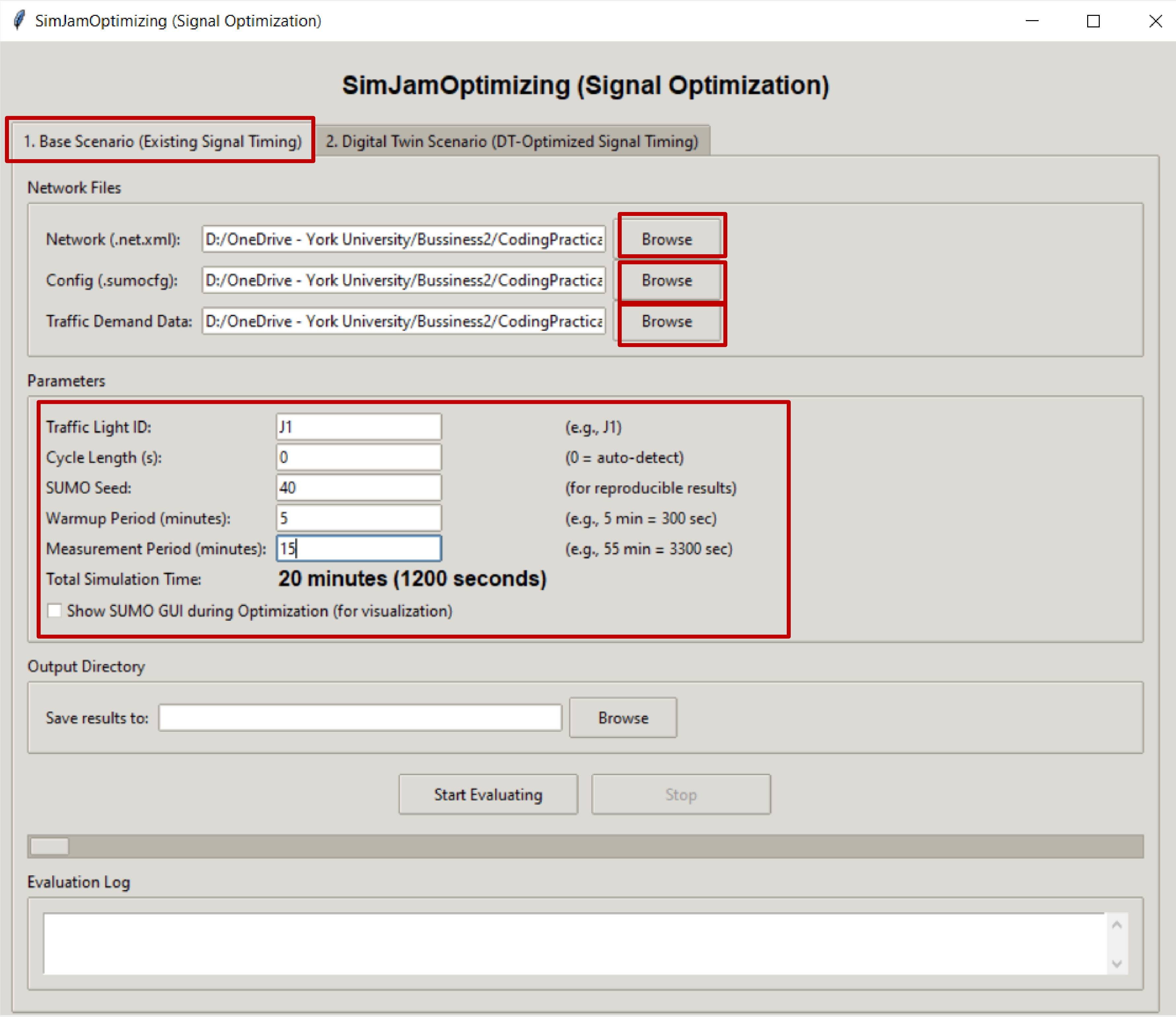
Parameters:

Assign Traffic Light ID: J1, Cycle Length (you can leave this as 0, it will automatically read this), SUMO Seed: 40, WarmUP period:5 Measurement Period: 15

Output Directory:

Create A folder "Result" and give path to the folder

Hit "Start Evaluation"



Optimization (Exercise)

Open Result Folder -> Open existing_lane_kpis.csv

All means 15 min | Lane id in NetEdit | Edge id in NetEdit | Approach | First KPI | Second KPI | Third KPI | Fourth KPI | Fifth KPI

	A	B	C	D	E	F	G	H	I
1	Minute	lane_id	edge_id	approach	avg_delay_s	vg_stopped_delay_	throughput	emission_co2_mg	los
2	all	E0_0	E0	EB	9.56	5.69	41	1780881.06	A
3	all	E0_1	E0	EB	5.95	1.28	80	5513512.48	A
4	all	E0_2	E0	EB	8.51	3.75	20	1172333.21	A
5	all	E2_0	E2	WB	3.81	2.19	25	266267.39	A
6	all	E2_1	E2	WB	3.07	1.84	114	1611076.94	A
7	all	E2_2	E2	WB	11.59	7.77	15	337890.35	B
8	all	E2_3	E2	WB	4.25	3	24	302868.69	A
9	all	E3_0	E3	NB	21.04	15.6	71	3775700.32	C
10	all	E3_1	E3	NB	16.45	9.67	91	10817603.93	B
11	all	E3_2	E3	NB	32.93	24.34	30	2940771.51	C
12	all	E7_0	E7	SB	11.3	9.21	24	522100.82	B
13	all	E7_1	E7	SB	12.53	10.01	83	3530998.04	B
14	all	E7_2	E7	SB	18.91	14.45	71	2287767.05	B
15	all	E7_3	E7	SB	15.84	13.42	22	566687.55	B
16	all	all	all	all	12.55	8.73	711	35426459.34	B

Aggregated Value for all min

Optimization (Exercise)

Open Result Folder -> Open existing_green_times.csv

All means 15 min

Green time East/West direction

Green time in Norht and South Direction

	A	B	C
1	Minute	Green_EW	Green_NS
2	all	39	15
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			

Optimization (Exercise)

Download Week 9 Materials – SUMO Files, Data, Application

Run application →

In Base Scenario (Existing Signal Timing) → Assign SUMO Net, SUMO CFG, Traffic Deman Data (Interval Calibrated Observed Data)

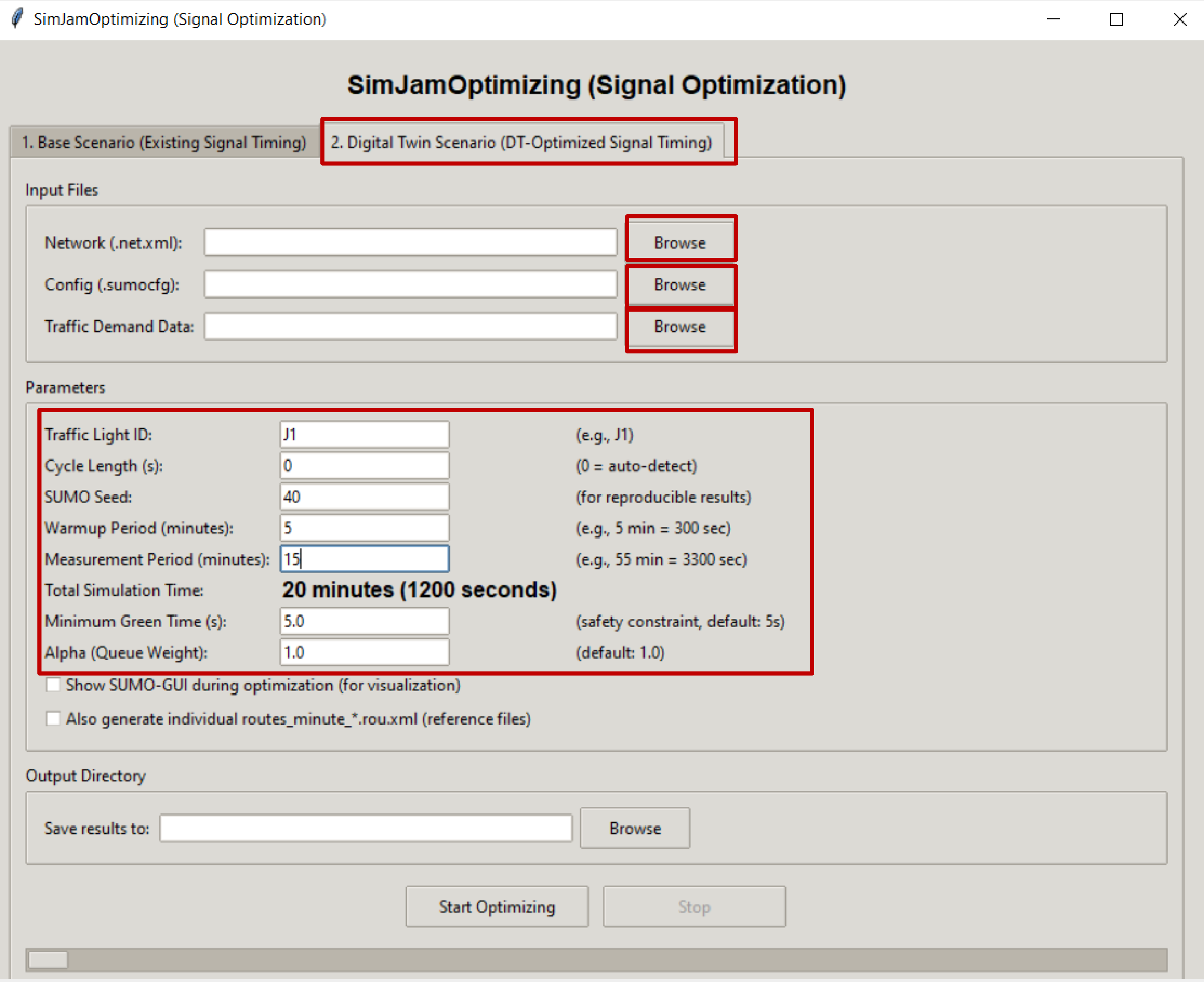
Patramters:

Assign Traffic Light ID: J1, Cycle Length (you can leave this as 0, it will automatically read this), SUMO Seed: 40, WarmUP period:5 Measurement Period: 15, Min green time: 5, Alpha: 1

Output Directory:

Create A folder "Result" and give path to the folder

Hit "Start Optimizing"



Optimization (Exercise)

Open Result Folder -> Open existing_lane_kpis.csv

All means 15 min	Lane id in NetEdit	Edge id in NetEdit	Approach	First KPI	Second KPI	Third KPI	Fourth KPI	Fifth KPI
A	B	C	D	E	F	G	H	I
Minute	lane_id	edge_id	approach	avg_delay_s	ıvg_stopped_delay_	throughput	emission_co2_mg	los
1	E0_0	E0	EB	11.39	7.5	1	448362.31	B
1	E0_1	E0	EB	16.58	14.33	0	75009.95	B
1	E0_2	E0	EB	1.08	0	1	38035.15	A
1	E2_0	E2	WB	1.37	0	1	6941.6	A
1	E2_1	E2	WB	8.74	7.67	1	111741.88	A
1	E2_2	E2	WB	17.02	13.5	0	42080.05	B
1	E2_3	E2	WB	0.31	0	1	10490.88	A
1	E3_0	E3	NB	3.17	0	10	856196.3	A
1	E3_1	E3	NB	0.97	0	2	40560.46	A
1	E3_2	E3	NB	3.66	1	2	84244.74	A
1	E7_0	E7	SB	4.07	2	2	14426.42	A
1	E7_1	E7	SB	1.29	0.56	6	91124.67	A
1	E7_2	E7	SB	3.09	1	2	12663.14	A
1	E7_3	E7	SB	11.68	9	1	19365.45	B
2	E0_0	E0	EB	6.48	3.7	4	347372.34	A
2	E0_1	E0	EB	4.91	2.83	4	63978.11	A
2	E0_2	E0	EB	1.38	0	1	39112.49	A
2	E2_0	E2	WB	8.81	7	1	33289.12	A
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
all	all	all	all	8.1	5.72	694	31824812.38	A

Aggregated Value for all min (row 212)

Optimization (Exercise)

Can we compare the result from Slide 12 to Slide 15?

Response: Yes, but, we need to aggregate for all min, see next slide

Optimization (Exercise)

Open Result Folder -> Open dt_lane_kpis_aggregated.csv

	All means 15 min	Lane id in NetEdit	Edge id in NetEdit	Approach	First KPI	Second KPI	Third KPI	Fourth KPI	Fifth KPI
	A	B	C	D	E	F	G	H	I
1	Minute	lane_id	edge_id	approach	avg_delay_s	avg_stopped_delay_s	throughput	emission_co2_mg	los
2	all	E0_0	E0	EB	6.67	3.37	63	7486293.46	A
3	all	E0_1	E0	EB	5.54	3.82	53	1013760.54	A
4	all	E0_2	E0	EB	3.81	2.1	20	897102.39	A
5	all	E2_0	E2	WB	4.66	3.03	25	384187.14	A
6	all	E2_1	E2	WB	4.75	3.54	92	2069526.37	A
7	all	E2_2	E2	WB	8.68	6.02	31	874084.03	A
8	all	E2_3	E2	WB	4.45	3.55	23	385166.02	A
9	all	E3_0	E3	NB	7.49	3.76	101	10565380.45	A
10	all	E3_1	E3	NB	7.81	5.04	58	1660796.91	A
11	all	E3_2	E3	NB	15.19	11.08	30	1863242.21	B
12	all	E7_0	E7	SB	8.54	6.67	24	391935.86	A
13	all	E7_1	E7	SB	6.23	4.61	99	2501243.34	A
14	all	E7_2	E7	SB	12.47	8.84	53	1150719.08	B
15	all	E7_3	E7	SB	17.08	14.68	22	581374.59	B
16	all	all	all	all	8.1	5.72	694	31824812.39	A

Aggregated Value for all min (row 16)

Optimization (Exercise)

Open Result Folder -> Open existing_green_times.csv

All means 15 min

Green time East/West direction

Green time in Norht and South Direction

	A	B	C
1	Minute	Green_EW	Green_NS
2	1	18.4	35.6
3	2	22.9	31.1
4	3	29.2	24.8
5	4	31	23
6	5	26	28
7	6	30.1	23.9
8	7	30.6	23.4
9	8	26.5	27.5
10	9	30.7	23.3
11	10	34.3	19.7
12	11	28.5	25.5
13	12	28.7	25.3
14	13	33.8	20.2
15	14	27.5	26.5
16	15	27	27
17	all	28.3	25.7

Aggregated Value for all min

Optimization (Exercise)

Optimization

4.2. Mathematical model

The Digital Twin signal control problem is formulated as a constrained optimization model, solved at each decision interval t. The objective is to optimally allocate green times to competing signal phases based on observed demand and queue conditions, subject to cycle length and safety constraints.

Decision variables

$$g_{NS}(t) \;:\; \text{green time allocated to the North–South phase (s)}$$

$$g_{EW}(t) \;:\; \text{green time allocated to the East–West phase (s)}$$

Parameters

$$d_N(t), d_S(t), d_E(t), d_W(t) \;:\; \text{approach demands (veh/min)}$$

$$q_N(t), q_S(t), q_E(t), q_W(t) \;:\; \text{queue lengths (veh)}$$

$$C \;:\; \text{signal cycle length (s)}$$

$$L \;:\; \text{total lost time per cycle (s)}$$

$$G = C - L \;:\; \text{available green time per cycle (s)}$$

$$\alpha \geq 0 \;:\; \text{queue-weighting parameter}$$

$$g_{\min} \;:\; \text{minimum green time (s)}$$

Effective demand formulation

The effective demand for each phase is defined as the sum of the observed demand and a weighted queue term:

$$E_{NS}(t) = [d_N(t) + d_S(t)] + \alpha\,[q_N(t) + q_S(t)] \tag{1}$$

$$E_{EW}(t) = [d_E(t) + d_W(t)] + \alpha\,[q_E(t) + q_W(t)] \tag{2}$$

Objective function

The objective minimizes the deviation from a proportional green allocation based on effective demand, which serves as a proxy for minimizing total intersection delay:

$$\min\ \text{Obj} = \left(g_{NS}(t) - \frac{E_{NS}(t)}{E_{NS}(t) + E_{EW}(t)}G\right)^2 + \left(g_{EW}(t) - \frac{E_{EW}(t)}{E_{NS}(t) + E_{EW}(t)}G\right)^2 \tag{3}$$

When $E_{NS}(t) + E_{EW}(t) = 0$, the green time is equally distributed between phases.

Constraints

Cycle length constraint

$$g_{NS}(t) + g_{EW}(t) = G \tag{4}$$

Minimum green time constraints

$$g_{NS}(t) \geq g_{\min} \tag{5}$$

$$g_{EW}(t) \geq g_{\min} \tag{6}$$

Implicit maximum green constraints

$$g_{NS}(t) \leq G - g_{\min} \tag{7}$$

$$g_{EW}(t) \leq G - g_{\min} \tag{8}$$

Non-negativity constraints

$$g_{NS}(t) \geq 0, \qquad g_{EW}(t) \geq 0 \tag{9}$$

Change this problem to that three steps

Step 0 — Define the constants in Image 1

From Image 1, define the "target" green splits:

$$a = \frac{E_{NS}(t)}{E_{NS}(t) + E_{EW}(t)} G$$

$$b = \frac{E_{EW}(t)}{E_{NS}(t) + E_{EW}(t)} G$$

These are just numbers once E_{NS}, E_{EW}, G are known at minute t.

Step 1 — Rewrite Image 1 using a, b

Image 1 objective becomes:

$$\min \ \Big(g_{NS}(t) - a\Big)^2 + \Big(g_{EW}(t) - b\Big)^2$$

That's simply "choose (g_{NS}, g_{EW}) as close as possible to (a, b)."

Step 2 — Add the cycle constraint (what your signal must satisfy)

Your signal must satisfy:

$$g_{NS}(t) + g_{EW}(t) = G$$

Also notice something important:

$$a + b = \frac{E_{NS}}{E_{NS} + E_{EW}} G + \frac{E_{EW}}{E_{NS} + E_{EW}} G = G$$

So the target (a, b) already lies on the feasible line $g_{NS} + g_{EW} = G$.

Step 3 — Show why the minimum happens at $g_{NS} = a, \ g_{EW} = b$

Because squares are always nonnegative:

$$(g_{NS} - a)^2 \geq 0, \qquad (g_{EW} - b)^2 \geq 0$$

So their sum is:

$$(g_{NS} - a)^2 + (g_{EW} - b)^2 \geq 0$$

The **smallest possible value** is **0**, and the only way to get 0 is:

$$g_{NS} = a \quad \text{and} \quad g_{EW} = b$$

And since (a, b) is feasible (Step 2), that is the optimal solution.

Step 4 — Substitute back a, b to get Image 2

So the optimizer is:

$$g_{NS}(t) = \frac{E_{NS}(t)}{E_{NS}(t) + E_{EW}(t)} G$$

and then automatically

$$g_{EW}(t) = G - g_{NS}(t)$$

✅ That is exactly **Image 2**.

closed-form, analytically solvable (convex) optimization

A **closed-form solution** means you can compute the answer using a **direct formula** (algebra), like:

So you don't need:

iterative search,

numerical optimization,

gradient methods,

or a solver.