

Data

Merge Sort					
n = 10					
k	1	n/4	n/2	3n/4	n
	2700	3700	5100	6000	7100
	1400	4800	4700	4400	5200
	1100	2500	6900	4600	6100
	900	2400	3700	9100	4300
	1000	1900	2700	4000	5200
	1000	2300	2500	4000	6800
	900	2600	2700	3800	5800
	1000	3700	3200	4500	6800
	900	2500	4300	7500	6800
	900	2600	3000	4700	6300
	900	2900	4000	5400	4000
	900	4600	5900	4700	6700
	900	1700	2800	3500	4500
	900	2600	6800	5000	11400
	900	2600	3100	4500	7100
	1000	4100	3700	5300	6800
	900	4600	4400	3800	6800
	900	2800	4400	18600	16900
	2000	2000	3700	5700	6700
	1100	2600	4000	22300	6700
average	1110	2975	4080	6570	6900

Merge Sort					
n = 50					
k	1	n/4	n/2	3n/4	n
	2700	8300	13900	19300	23300
	4900	5800	14400	17400	19700
	1300	12300	18800	21300	27300
	1000	5300	12400	23500	21300
	1000	5200	14100	18500	19700
	900	8300	22300	29200	32300
	1000	6600	15100	28800	18300
	2000	5600	24000	26100	17300
	1100	9900	9600	20100	17100

	1000	5300	10600	10100	18900
	900	16200	20200	5800	15700
	1000	11300	10500	7000	6800
	2500	19700	10800	2800	3600
	1400	6400	5200	4600	2400
	1000	4700	3100	4900	4700
	1000	4600	1400	5100	3500
	1000	5900	4000	4700	2300
	2100	4800	4700	4500	3400
	2100	5500	1700	3600	3400
	1900	6900	3400	2100	2200
average	1590	7930	11010	12970	13160

Merge Sort					
n = 100					
k	1	n/4	n/2	3n/4	n
	2700	13700	28400	39200	44100
	1400	14300	25000	46200	56100
	2000	12400	32400	48200	51800
	1100	12200	34700	44400	37800
	1000	27400	32100	32200	18800
	1700	21900	21900	34800	10100
	1000	15300	20400	9900	11100
	1100	25600	9500	6900	11600
	1000	10300	4100	12200	7800
	900	10900	4000	7800	9800
	900	20700	4000	9500	6100
	1000	13800	3000	5600	11300
	1000	12100	4300	5500	7900
	1000	5800	5700	7100	14700
	1000	3700	4700	7600	7600
	1000	1400	2800	7600	9900
	900	2400	5900	9200	38500
	900	4800	4500	5700	9500
	900	1500	6000	7100	7900
	1200	2700	3000	22600	11900
average	1185	11645	12820	18465	19215

Merge Sort					
n = 500					
k	1	n/4	n/2	3n/4	n
	2800	63000	163100	228100	264800
	1800	77400	33400	54600	64300
	1700	63900	23500	37400	110600
	1100	15900	18800	52000	56500
	900	11400	30400	43400	86000
	900	19700	20100	36200	63100
	900	13500	27600	38600	89100
	1000	10300	19500	60800	50400
	900	11900	26600	37300	55700
	900	22900	24800	35000	76900
	1000	13200	33700	32700	60400
	900	14300	19500	51800	60200
	1000	12800	21100	52000	51600
	900	12800	18800	34000	59800
	900	11900	28900	33000	83900
	1900	12500	22500	35600	86700
	1100	12000	33100	33800	84200
	1100	14200	52600	36700	89600
	1100	12000	33700	38300	86900
	1300	11700	23100	41400	72500
average	1205	21865	33740	50635	82660

Merge Sort					
n = 1000					
k	1	n/4	n/2	3n/4	n
	2900	133900	277400	413600	661800
	1700	66700	49400	98500	215000
	2000	25100	59500	95100	116400
	1300	19500	46000	86300	105800
	1000	51400	45100	65900	119100
	1000	23000	68700	90700	58400
	900	22900	45100	71200	42600
	900	22500	46300	70600	45700
	900	23000	63400	70700	50700
	900	22800	41100	86200	39300

	900	30300	43400	64400	38800
	900	21900	44400	69900	41000
	900	22700	45200	69000	49500
	900	22400	42600	77000	42100
	1000	21500	44600	72700	213800
	1000	26400	40800	68900	38000
	1000	21500	52900	71200	36300
	900	32100	54600	66700	35600
	1300	20100	45300	71200	45100
	1900	21600	79400	79200	41700
average	1210	32565	61760	92950	101835

mmRule					
n = 10					
k	1	n/4	n/2	3n/4	n
	263200	227000	270500	277900	228300
	6600	5800	13300	8400	6500
	7500	5100	8400	9900	5800
	5800	4700	6700	10000	5800
	5300	4600	6300	8200	5700
	5300	4800	6200	8700	5800
	5300	4600	6300	16700	5700
	5300	6800	8800	9500	5700
	5200	6600	8400	8600	7200
	5300	16100	6700	8100	7200
	5200	7100	6400	11300	7100
	5300	5400	10800	8800	13600
	5700	7500	8300	10900	6800
	5800	6600	6500	8500	8200
	5400	5200	9700	7200	7800
	5300	6800	8200	9000	5900
	5200	6700	6500	11200	7900
	7400	5500	6200	8600	8900
	5900	6500	10800	18500	8100
	5300	5400	9300	8500	6200
average	18565	17440	21215	23425	18210

mmRule					
n = 50					
k	1	n/4	n/2	3n/4	n

	251800	256400	261600	233200	255200
	19500	19000	12500	20600	25100
	17100	16800	9900	17300	27600
	18400	16700	16300	16800	22500
	16000	16600	9200	21000	27100
	22900	20100	9100	21700	19400
	16600	16400	10800	18900	22900
	25900	26100	8700	17900	35000
	16800	17400	11200	17400	33900
	16300	26200	11100	31000	35400
	50100	59000	11200	54400	38000
	19700	14600	10600	31400	13700
	24600	21400	10700	13500	18200
	13100	13700	10500	11900	21000
	18800	11100	10900	34500	16600
	8900	9200	11500	13300	12800
	9600	22600	8900	7900	11300
	12200	11900	9900	7600	21600
	11300	10800	10400	9600	14200
	7300	10900	10300	7500	10000
average	29845	30845	23265	30370	34075

mmRule					
n = 100					
k	1	n/4	n/2	3n/4	n
	266200	290500	301200	257900	517700
	36800	44600	44700	50700	55600
	34600	43000	40000	41600	42600
	30000	32100	33200	32100	47100
	35600	35300	32800	49000	54900
	48800	80900	60000	45200	45700
	32400	59600	36300	33400	27900
	45600	44900	42100	25500	26000
	32300	26800	30600	21200	27400
	25500	28000	25100	15600	83100
	14400	18400	33600	36600	18700
	20400	29000	16500	18300	26900
	16500	15500	19200	17000	29300
	18200	20200	23400	13600	15100
	26900	16200	14600	15700	8500
	15300	25700	15400	15100	12300

	12200	15700	15200	15400	8700
	18900	6100	22500	22000	12800
	17000	4000	31900	3700	14300
	14700	3800	14100	3500	6000
average	38115	42015	42620	36655	54030

mmRule					
n = 500					
k	1	n/4	n/2	3n/4	n
	444200	439900	612800	441700	434500
	183900	127900	158100	145900	177300
	105600	134900	121600	144000	125400
	97300	55800	86500	131800	89000
	69500	55500	63600	92700	59300
	57100	54200	61200	56300	61500
	61900	50700	60100	74700	57600
	54400	45500	48900	47600	53200
	53000	41900	56500	20800	34600
	49400	40200	49100	30400	61800
	124300	32500	59100	31000	19500
	17100	22500	20800	28700	24900
	16900	17700	18800	26900	17500
	17500	20300	18500	21800	20500
	23700	16300	19200	30700	23100
	17900	18800	21000	24300	34900
	23800	30300	33200	48800	16200
	20100	16700	19000	24700	24000
	21700	17100	21400	19400	21400
	19300	16800	18200	16700	26200
average	73930	62775	78380	72945	69120

mmRule					
n = 1000					
k	1	n/4	n/2	3n/4	n
	592600	596300	560000	687000	606800
	255800	193300	242000	226700	225800
	118500	101400	106700	132400	112100
	110400	103700	104200	114300	100900
	106100	127500	95200	93400	99200

	102000	71600	54700	65200	79000
	38600	59200	38500	61500	42500
	37300	36000	49200	58600	37600
	34000	34600	40700	69900	35300
	49700	49700	56300	54800	42600
	43500	44200	37100	54900	40200
	37900	31700	44500	45400	46200
	43200	39200	37700	49400	37600
	36200	37300	34000	55200	38600
	161100	37200	54200	243900	155100
	35300	169800	169600	48900	36100
	35200	34700	36500	53700	35400
	37800	35200	35100	42900	32800
	47100	41300	34000	48100	37100
	37100	35900	33900	52200	34400
average	97970	93990	93205	112920	93765

qSort recursive					
n = 10					
k	1	n/4	n/2	3n/4	n
	2100	1800	1700	1600	1700
	900	1000	1100	900	2000
	800	1100	1100	800	800
	600	2900	800	600	1400
	1100	800	700	700	1100
	700	700	600	600	1200
	700	1100	3300	700	1100
	1000	700	700	600	1100
	1000	700	800	1100	800
	700	1100	600	600	1100
	700	1000	1200	600	1100
	700	1100	1100	700	1100
	600	800	700	900	1000
	600	800	700	800	1100
	600	900	1100	700	1200
	700	700	800	600	1300
	700	600	700	700	1100
	700	700	1100	900	700
	900	1100	1100	1100	1100
	700	1100	1100	1100	1100

average	825	1035	1050	815	1155
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qSort recursive					
n = 50					
k	1	n/4	n/2	3n/4	n
	2200	2400	2400	2600	2300
	1800	2000	2100	2000	2100
	1600	1600	2100	2200	3100
	1700	1600	1700	1700	1800
	1800	1800	1800	1700	1800
	1800	1700	2200	1700	2000
	1900	1700	1700	2100	1800
	1700	1800	2100	1800	1800
	1900	1600	1700	1700	2100
	1700	2100	2000	1700	2200
	1700	1800	1800	1700	1900
	1800	4700	1700	1800	2100
	1800	1900	1700	1700	2000
	1700	1700	1700	1700	1900
	1800	1900	1700	1700	2200
	1900	1800	1700	1800	2000
	1800	1700	1700	1800	2100
	1800	2200	1700	1700	2200
	2000	2000	1900	5400	2200
	1700	2100	2100	2300	1800
average	1805	2005	1875	2040	2070

qSort recursive					
n = 100					
k	1	n/4	n/2	3n/4	n
	3100	3600	3900	4000	4000
	3400	3200	2900	3800	3000
	3000	3100	3000	3400	6000
	2900	3500	2900	3700	3500
	3000	3300	3300	3200	3200
	2900	4300	3400	3500	3000
	3200	3800	5300	3400	3200
	3300	3300	4000	3300	3100

	3100	3400	3300	3300	4000
	3100	3300	3200	3200	4000
	4500	6200	8600	6700	6100
	3300	3500	3300	3200	4300
	3300	3500	3400	3400	3200
	3400	3300	3400	3400	3100
	3100	3400	3300	3700	3300
	3000	3300	3400	3400	3000
	2900	3200	3300	3300	3500
	3100	3300	3500	3300	3100
	3400	3000	3500	3400	3500
	3400	3300	3400	3400	3300
average	3220	3540	3715	3600	3670

qSort recursive					
n = 500					
k	1	n/4	n/2	3n/4	n
	13400	14500	11000	14600	14800
	11600	14800	11300	13200	12600
	14500	16100	13200	15700	14800
	13800	13400	13900	12800	12900
	14700	16200	18800	14500	16700
	13500	13200	19100	16300	12900
	14400	15300	16200	19600	15700
	13800	12700	13400	13500	12700
	14700	15800	17400	16300	15700
	14200	13200	13200	13900	13400
	14900	15900	15500	16000	15600
	13900	13200	13000	13000	13400
	14600	15800	15300	14100	14800
	13800	13300	13300	13200	12900
	14700	14200	15700	14000	16100
	13500	13200	13500	13100	12900
	14300	14400	14500	15100	15600
	13400	13300	13600	15700	12800
	14400	15200	15800	15800	15600
	13200	13300	13300	13100	12900
average	13965	14350	14550	14675	14240

qSort recursive					
n = 1000					
k	1	n/4	n/2	3n/4	n
	19900	22800	27600	25500	17200
	28400	29000	28800	27900	20300
	29100	24200	35200	27300	26800
	27400	28600	29200	26800	26200
	28600	28900	27100	27400	28500
	29000	28500	27700	28800	29100
	28800	27800	30200	27600	27600
	29500	28100	28300	28500	28500
	28300	31100	28400	28100	28000
	27400	30900	27400	36200	28600
	26900	37300	27500	35800	28600
	28600	36200	26700	37000	28500
	27200	37700	26000	35100	27800
	28700	34200	27300	33300	27600
	27100	40700	28700	32000	26800
	26800	39100	28600	40600	28900
	28000	35200	29800	28900	33100
	28900	34200	27500	39400	28400
	27900	50200	31200	38200	28700
	27500	28300	28400	34600	27300
average	27700	32650	28580	31950	27325

qSort Partition					
n = 10					
k	1	n/4	n/2	3n/4	n
	1600	1600	1700	1600	1500
	900	900	1100	1100	1700
	1000	1000	1000	1200	1100
	700	700	1000	1000	700
	1100	700	3000	1100	600
	600	1100	800	1100	1100
	600	4500	1100	2100	1400
	600	1000	700	1300	1100
	600	1100	700	1000	900
	1100	900	600	1200	1100
	600	1200	600	1200	1000

	700	700	600	1200	1000
	500	700	700	1000	1100
	600	1100	1100	900	1100
	600	900	1000	1000	700
	600	800	1000	1900	700
	500	1100	900	1100	700
	2000	1100	700	1000	1100
	800	1000	700	1300	1100
	1200	1000	1100	1100	700
average	845	1155	1005	1220	1020

qSort Partition					
n = 50					
k	1	n/4	n/2	3n/4	n
	2400	2700	2500	2700	2800
	1800	4500	1700	2900	1800
	1600	2100	1600	1800	1500
	1600	1900	1600	1700	1900
	1500	1600	1600	1500	1800
	1500	2000	1600	1600	4300
	1600	2000	1900	1500	2600
	1600	1800	2500	2100	1700
	1600	2100	2000	1900	1500
	1700	2000	1800	1600	2700
	1500	1800	1600	1900	2000
	1600	1900	1800	2000	2000
	1500	1900	1600	2100	1900
	1500	2000	1500	1800	2000
	1800	1900	1600	2000	2000
	1700	1900	1800	1600	1700
	1700	2000	2000	2000	1900
	1900	1800	1900	1600	2000
	1900	2100	1600	1500	1800
	2000	1900	1600	2000	2000
average	1700	2095	1790	1890	2095

qSort Partition					
n = 100					

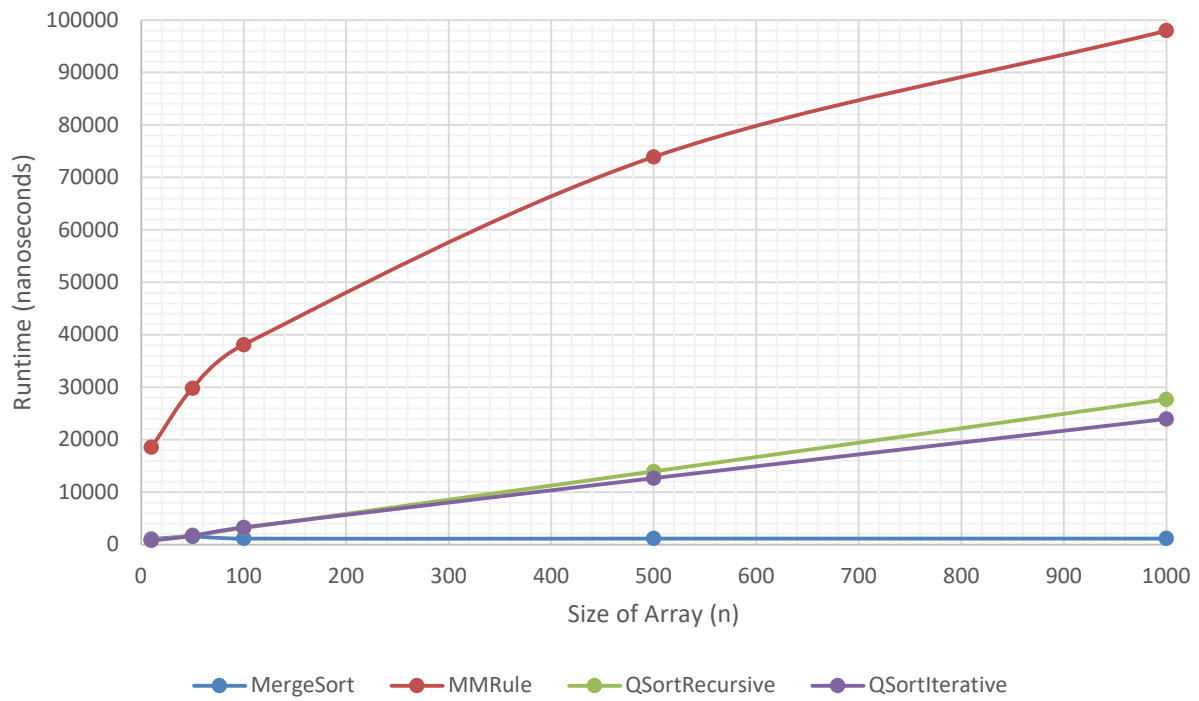
k	1	n/4	n/2	3n/4	n
	3000	2400	3800	3600	3700
	2000	3700	3000	2800	2900
	3100	4200	3000	2900	2800
	2800	3000	2800	2900	2900
	3000	3500	2700	2700	2700
	3000	6900	5200	2800	4500
	2800	3100	3200	3000	3100
	3000	5500	3200	2900	3100
	3200	3300	3100	5300	3100
	6100	3100	3000	3300	2800
	7000	5900	5200	8600	5800
	3000	3100	2900	3000	3100
	2600	3000	3000	3200	3100
	3100	3100	3000	3000	2800
	3000	3200	2900	3000	3000
	3000	4000	3100	3000	2900
	3100	3100	3100	3100	2800
	3000	3000	3400	4200	2900
	3000	2700	3100	3100	2700
	3100	3000	3100	3400	2900
average	3295	3640	3290	3490	3180

qSort Partition					
n = 500					
k	1	n/4	n/2	3n/4	n
	13400	10600	12600	13100	13000
	11400	8700	11200	11600	11600
	13100	13000	14900	13900	13600
	12000	11800	11900	11800	13200
	13100	13100	13800	14200	13100
	12200	12300	12900	12000	11600
	12700	14500	12600	13400	14100
	12200	12000	12100	12500	12400
	13100	12600	14100	13200	13400
	14400	12900	11900	11700	11900
	13900	13400	12600	13500	18200
	11600	12000	12200	11700	11900
	12900	13200	12800	13400	14000
	12300	12200	12200	12600	11400

	12700	12700	12900	13200	14100
	12200	12200	12100	14700	12400
	13100	13000	13000	13100	14000
	12200	12100	12100	11900	11900
	13200	13900	12700	14400	16900
	12200	11900	12100	11500	15500
average	12695	12405	12635	12870	13410

qSort Partition					
n = 1000					
k	1	n/4	n/2	3n/4	n
	23400	22200	22500	34200	23700
	19100	20700	23800	26500	24600
	23600	24500	22500	30200	25300
	24400	24600	28200	30600	24900
	23800	24700	23900	33900	25200
	24400	25000	30800	31400	25000
	24900	25100	25600	29500	24900
	23000	24900	25200	31000	24500
	23900	25700	27800	49400	28900
	24700	25600	25100	33100	27400
	23700	25900	25100	37100	25900
	24400	26200	26800	32000	26000
	24400	25100	25100	32100	25800
	23600	24200	30100	32300	25300
	24300	25400	25100	33800	23800
	24400	25000	25000	62500	24000
	24300	26000	25100	30800	24000
	24400	24100	24800	29900	25300
	25600	25600	24600	36500	23600
	25400	25300	25900	38500	23900
average	23985	24790	25650	34765	25100

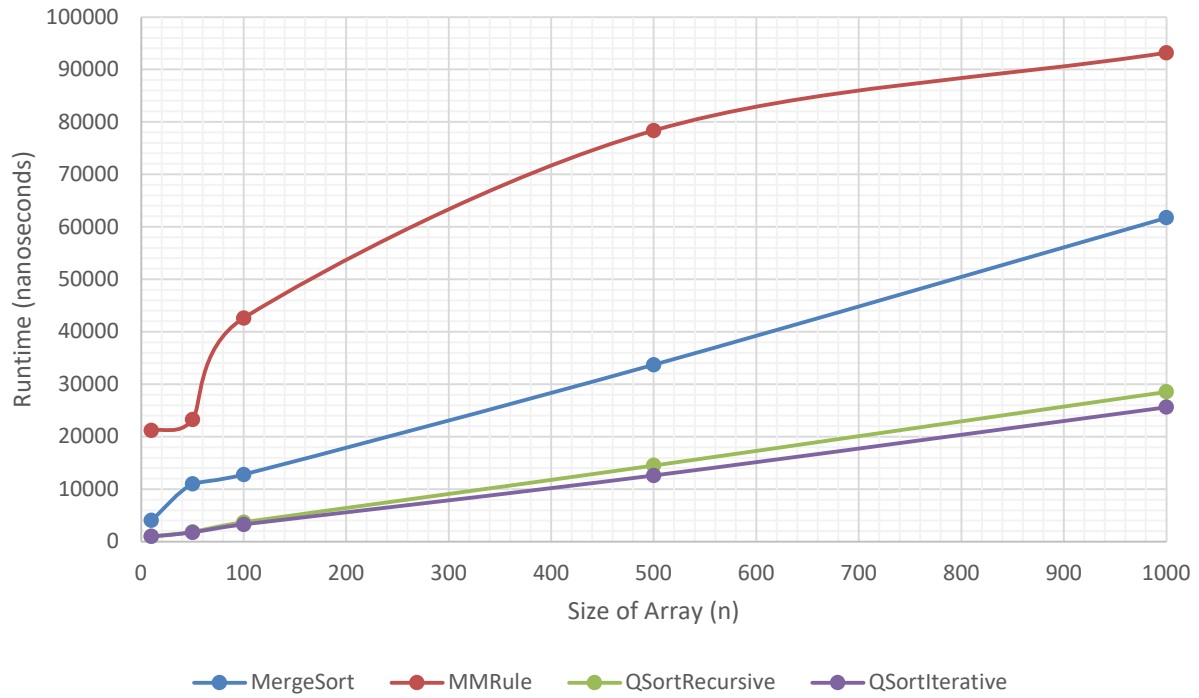
Runtime vs. Size of array at Kth smallest element ($k = 1$)



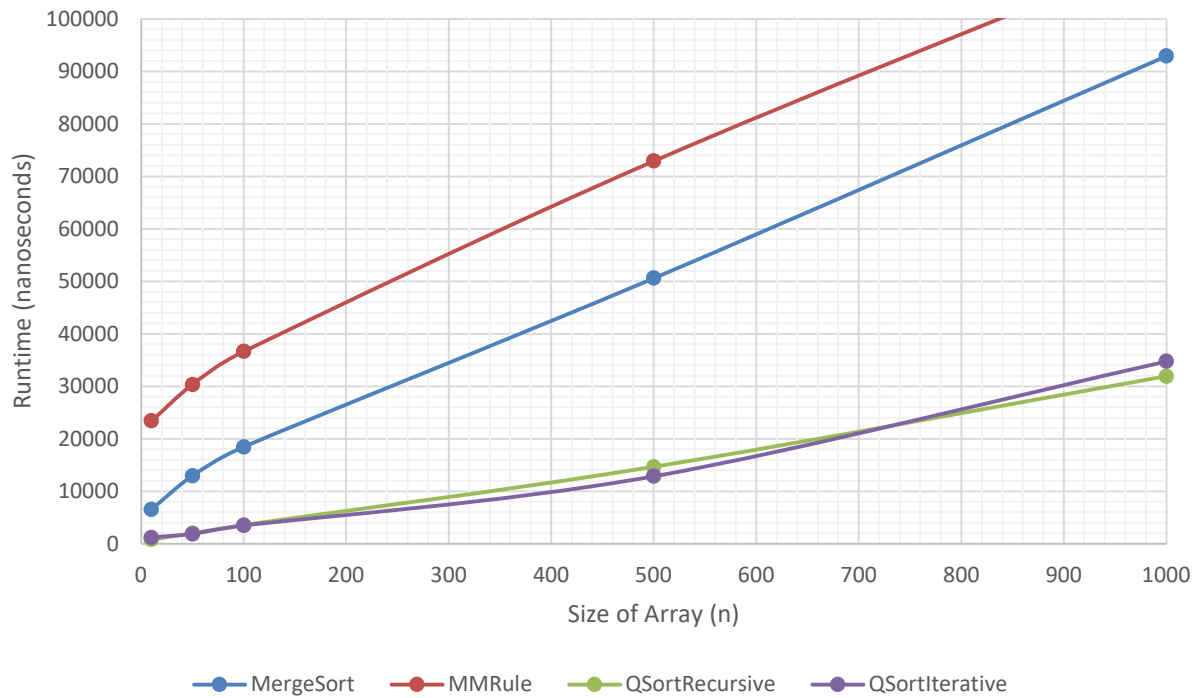
Runtime vs. Size of array at Kth smallest element ($k = n/4$)



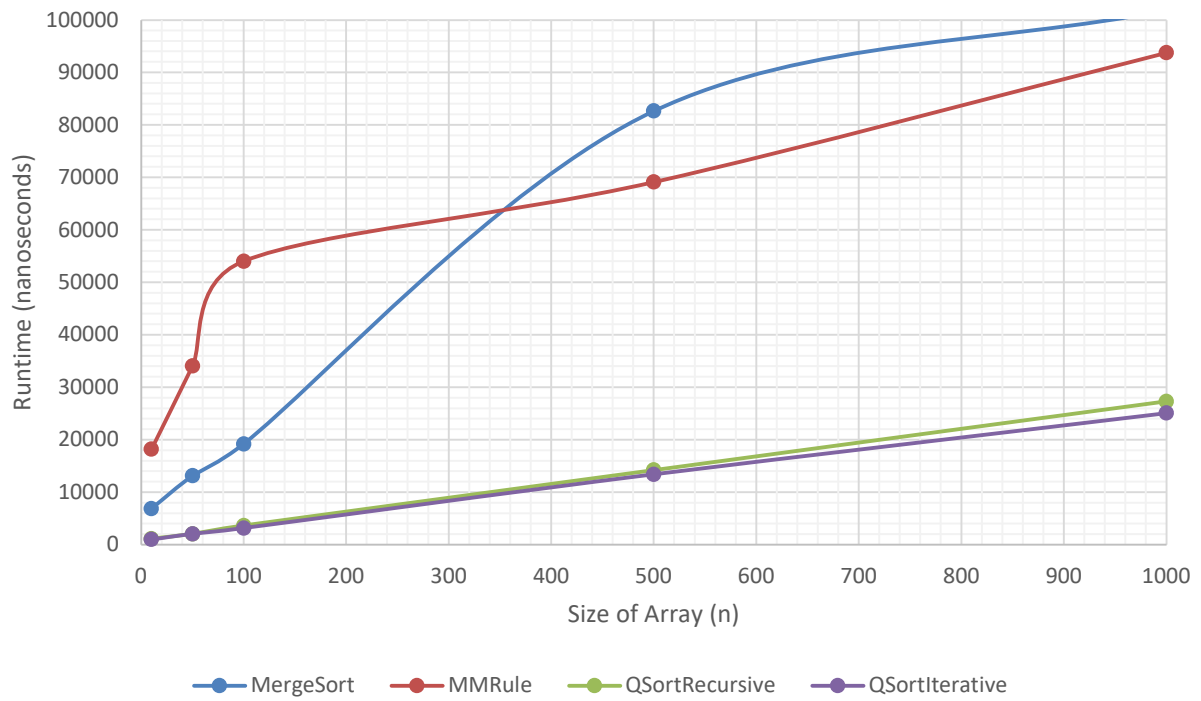
Runtime vs. Size of array at Kth smallest element ($k = n/2$)



Runtime vs. Size of array at Kth smallest element ($k = 3n/4$)



Runtime vs. Size of array at Kth smallest element ($k = n$)



Test Strategies

The first algorithm uses Merge Sort to find the k^{th} smallest element. The algorithm first separates the array into two smaller subarrays. It will then recursively call itself to repeat this process on both halves of the subarray and continuously create halves of the array. The process continues until the array has been sorted with the smallest element on the left and the largest element on the right.

The Quick Sort algorithm has a recursive and partition method for the second algorithm. For the partition method of Quick Sort, the last element of the array was used as a pivot. Using the first element of the array will provide the same complexity. At the point of pivot on the array, starting from the beginning of the array at index zero, the algorithm will then move the values that are less than the pivot to the left and swap with those that are greater. Essentially, values that are less than the pivot will be on the left and values that are greater than the pivot will be on the right. The recursive method of Quick Sort continuously executes this method until it is order from least to greatest.

Finally for the third algorithm, the median of medians rule was used. Similar to the previous algorithms, we would divide the array into smaller subarrays and find the median of all the subarrays. The partition method will then be called to determine the position of the element. Multiple methods were created to support the median of medians rule.

To test the algorithms, each algorithm was executed with five different sized arrays (10, 50, 100, 500 and 1000) and with each size, a different k^{th} element was being searched for (1, $n/2$, $n/2$, $3n/4$, and n). A method called "fillArray" was created in order to randomly create an array of size n . Each group was executed twenty times. This means that there is a total of 500 data points collected.

Results

The data collected from the tables and graphs shows us that the Quick Sort algorithms, on average, had a faster runtime compared to the other algorithms. As expected, with the small sample size the test was conducted, these two algorithms performed faster than the others. Quick sort partition method has a time complexity of $O(n)$ for its best and average case but a time complexity of $O(n^2)$ for its worst case. The case is similar for Quick sort recursive. If we increased the arrays size to an unfathomable size, then perhaps we can find the Merge Sort method to be the fastest. Merge sort has a tie complexity of $O(n \log n)$ for its best, worst and average cases. In this test, our sample size was fairly small, so we do not see the potential Algorithm 1 has. Similarly, for the median of medians rule, the time complexity of its worst case is $O(n)$.

The value of the k^{th} element increases from 1 to the size of the array n . At $k = 1$, all the algorithms have the lowest runtime it can possibly be. The Quick sort algorithms have the fastest times with an average of around 800 nanoseconds and the mm Rule had the slowest time of an average of 1800 nanoseconds. The Merge Sort method fell in between with an average of 1100 nanoseconds. However, as the size of the array increased, the Quick Sort methods quickly fell behind to the Merge Sort method. Ideally this is not the case and when we compare the data to the other k values, we find the Quick Sort is the fastest with all k values. This could be an error in the code since it was also being modified during the attempts of running the tests (found errors while running). Ideally, for $k = 1$, we do not expect Merge Sort to be the fastest however this is a special case. As the value of k increases, we see a trend of Quick Sort methods being the fastest, Merge Sort in the middle and mmRule the slowest. Another instance of a differing trend is at $k = n$. The mmRule is the slowest for the smaller sized arrays, however as the size of the array increases, the Merge Sort method quickly overtakes it and obtains a slower runtime. This is not expected but there is also no guarantee of protection from interferences. This could be from a program running the background during the runtime of the test. However, if we continued to test the larger sizes, we can see from the graph that the lines of Merge Sort and mm Rule

will possibly meet and potentially cross again. Further testing is needed with minimal interferences.

Theoretically, the size of the array is not large enough to compare the potential of Merge Sort. In the case that I do obtain the resources to use an extremely large array size, we can expect the Quick Sort methods to increase in time exponentially, overtaking the other algorithms. However we cannot see it here.

Theoretical Complexity and Comparisons

Merge Sort is a recursive algorithm with a time complexity of $O(n \log n)$ for its best, worst and average case. As mentioned before, it takes the array and divide it into two subarrays. This process continues recursively and to merge all the elements of all arrays together takes linear time (n). This gives us a total time complexity of $O(n \log n)$. Due to the algorithm needing to copy the elements of an array into a temporary array, the space complexity is $O(n)$. This algorithm, for smaller sized arrays, will tend to perform worse than Quick sort because Merge Sort still needs perform its entire process even after the array has been completely sorted.

The Quick sort had a best and average time complexity of $O(n \log n)$ and a worst time complexity of $O(n^2)$. The best case of this algorithm is when the pivot is chosen as a mean. Then a sort of binary tree will form with a height of $\log n$. Each level of the tree, with the best case, will be traversed n times. There so the best case is $O(n \log n)$. The worst case occurs when either ends of the array is used instead of the mean. The height of the tree formed will be n and each top node will perform n operations continuously $n - 1$ until only 1 is reached. This creates the worst time complexity of $O(n^2)$. Considering the best-case scenario, it may be possible to make it even better. If we know the half of the array the k^{th} index resides in, we can simply perform the algorithm on that side of tree only. Therefore, the better time complexity will be $O(n)$.

The partition algorithm with the median of medians rule has a time complexity of $O(n)$. similar to the Quick sort algorithm, we take the median of the subarrays that will be created, hence the name median of medians. The time it takes to find the median is only $O(1)$ because it is a simple formula. The remainder of the code that creates subarrays is only $O(n)$. The worst case is only the largest number to the median of medians or the smallest number to the median of medians.

Strengths and Constraints

The strength in the experiment followed my expectations well. The time complexity and runtime was expected for the most part. There are few outliers such as when $k = 1$ as mentioned as above. An easy solution of improvement for that is to perform testing first before utilizing the code for data. I could have made sure that the entirety of the codes worked before starting the tests. Even with this slight error, the main image of the algorithms has been portrayed properly. Another major improvement for this code is the size of the array. It could have been increased more to better visualize the runtime of the algorithms. However, that also falls into my constraints. I would need a computer capable of dealing with that much data. My PC also had many other programs running in the background. Such as videos, games and even video games which can take a good portion of the RAM. This could have fixed the outlier data as well. Otherwise, most of the data collected was within reasonable ranges considering the capabilities of my available resources.