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Control of a 3DoF Hybrid SPM gimbal mechanism.

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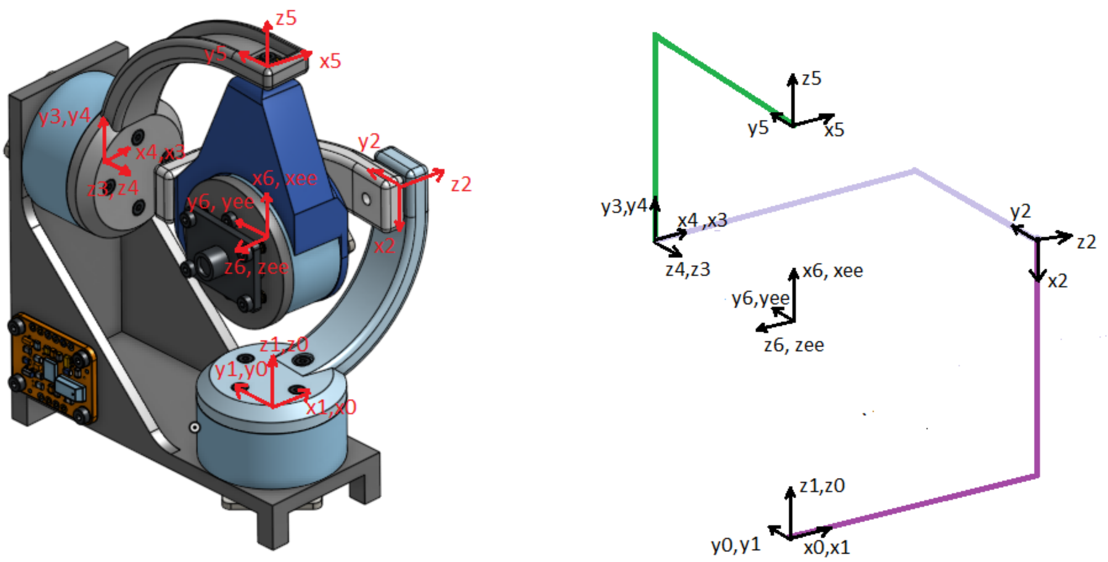
1 | Inverse Kinematics

The inverse kinematics are derived for manipulator shown in figure 1.1a. This mechanism consists of two kinematic chains sharing a common end effector. The first chain is made from the joints q1, q2, q3, and the second chain from q4, q5, q6. The inverse kinematics are for the 2-DOF manipulator, since the third axis is trivial. The motors are at the joints q1, q4 and q6.



(a) *CAD manipulator model.* **(b)** *Kinematic chains and coordinate frames.*

Figure 1.1: *Manipulator used for the inverse kinematics derivation.*

First Kinematic Chain

The rotation matrix of the first chain is given by:

$$R_{03} = R_z(q_1)\,R_y(\frac{\pi}{2}\,R_z(q_2)\,R_x(\frac{\pi}{2}). \tag{1.1}$$

Explicitly,

$$R_{03} = \begin{bmatrix} -\sin q_1 \sin q_2 & \cos q_1 & \cos q_2 \sin q_1 \\ \cos q_1 \sin q_2 & \sin q_1 & -\cos q_1 \cos q_2 \\ -\cos q_2 & 0 & -\sin q_2 \end{bmatrix}. \tag{1.2}$$

Second Kinematic Chain

The rotation matrix of the second chain is:

$$R_{06} = R_x(\frac{\pi}{2})\,R_z(q_4)\,R_x(-\frac{\pi}{2})\,R_z(q_5)\,R_y(\frac{\pi}{2}). \tag{1.3}$$

In expanded form:

$$R_{06} = \begin{bmatrix} \sin q_4 & -\cos q_4 \sin q_5 & \cos q_4 \cos q_5 \\ 0 & \cos q_5 & \sin q_5 \\ -\cos q_4 & -\sin q_4 \sin q_5 & \cos q_5 \sin q_4 \end{bmatrix}. \tag{1.4}$$

End-Effector Rotation

The end-effector rotation matrix is expressed as:

$$R_{ee} = R_y(\phi_2)\,R_z(\phi_3), \tag{1.5}$$

which can be written as:

$$R_{ee} = \begin{bmatrix} \cos\phi_2\cos\phi_3 & -\cos\phi_2\sin\phi_3 & \sin\phi_2 \\ \sin\phi_3 & \cos\phi_3 & 0 \\ -\cos\phi_3\sin\phi_2 & \sin\phi_2\sin\phi_3 & \cos\phi_2 \end{bmatrix}. \tag{1.6}$$

The second column of R_{06} is the negative of the third column of R_{03}. Therefore:

$$-\cos q_4 \sin q_5 = \cos q_2 \sin q_1, \tag{1.7}$$
$$\cos q_5 = -\cos q_1 \cos q_2, \tag{1.8}$$
$$-\sin q_4 \sin q_5 = -\sin q_2. \tag{1.9}$$

Dividing equation 1.7 by 1.8 gives:

$$-\frac{\cos q_4 \sin q_5}{\cos q_5} = -\frac{\sin q_1}{\cos q_1}. \tag{1.10}$$

Hence,

$$\tan q_1 = \tan q_5 \cos q_4, \tag{1.11}$$

which leads to:

$$q_1 = \arctan\big(\tan q_5 \cos q_4\big). \tag{1.12}$$

Joint Angles from End-Effector Orientation

To relate q_5 and q_4 to the end-effector orientation, we compare the second column of R_{06} and R_{ee}, since $y_{ee} = y_{R06}$:

$$-\cos q_4 \sin q_5 = -\cos\phi_2 \sin\phi_3, \tag{1.13}$$
$$\cos q_5 = \cos\phi_3, \tag{1.14}$$
$$-\sin q_4 \sin q_5 = \sin\phi_2 \sin\phi_3. \tag{1.15}$$

From the second equation, it follows that:

$$q_5 = \phi_3. \tag{1.16}$$

By dividing the equation 1.15 by 1.13, we obtain:

$$\frac{\sin q_4}{\cos q_4} = \frac{\sin\phi_2}{\cos\phi_2}. \tag{1.17}$$

Since the tangent function is odd, this implies:

$$q_4 = -\phi_2. \tag{1.18}$$

Final Expressions

Finally, the joint angles are:

$$q_1 = \arctan\big(\cos\phi_2\,\tan\phi_3\big), \tag{1.19}$$
$$q_4 = -\phi_2, \tag{1.20}$$
$$q_5 = \phi_3. \tag{1.21}$$