Assignment 2

ELG3106 – Fall 2025

Section: A00

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Introduction:

This assignment investigates reflection and transmission of a uniform plane wave (UPW) at oblique incidence on the planar boundary $z = 0$ separating two non-magnetic, lossless dielectrics: medium 1 (air) for $z \leq 0$ and medium 2 with $\varepsilon_r = 8$ for $z \geq 0$. The goals of the experiments are to (i) plot reflectivity and transmissivity versus incident angle for both polarizations (TE and TM), (ii) confirm Brewster's angle using the data collected from part (i), and (iii) using the specified parameters, solve for the incident, reflected, and transmitted phasors**.** The analysis follows standard boundary-condition methods for plane waves and uses the Fresnel coefficients and Snell's law.

Theory and Formulas:

For a plane wave in a lossless medium, the ratio of electric to magnetic field magnitudes equals the intrinsic impedance $\eta = \sqrt{\mu/\varepsilon}$. At a planar interface, tangential E and H are continuous, leading to the Fresnel reflection and transmission coefficients below for TE (perpendicular polarization)**,**

$$\Gamma_{\mathrm{TE}} = \frac{\eta_2 \cos\ \theta_t - \eta_1 \cos\ \theta_i}{\eta_2 \cos\ \theta_t + \eta_1 \cos\ \theta_i}, \tau_{\mathrm{TE}} = \frac{2\eta_2 \cos\ \theta_i}{\eta_2 \cos\ \theta_i + \eta_1 \cos\ \theta_t}. \quad \text{Eq. 1.0}$$

For TM (parallel polarization)**,**

$$\Gamma_{\mathrm{TM}} = \frac{\eta_2 \cos\ \theta_i - \eta_1 \cos\ \theta_t}{\eta_2 \cos\ \theta_i + \eta_1 \cos\ \theta_t}, \tau_{\mathrm{TM}} = \frac{2\eta_2 \cos\ \theta_i}{\eta_2 \cos\ \theta_i + \eta_1 \cos\ \theta_t}. \quad \text{Eq. 1.1}$$

Angles are influenced by Snell's law,

$$k_1 \sin\theta_i = k_2 \sin\theta_t\,, \qquad \mathrm{k} = \omega\sqrt{\mu\varepsilon}. \quad Eq. 1.2$$

For non-magnetic media ($\mu_1 = \mu_2 = \mu_0$), Brewster's angl**e** exists only for TM polarization and represents the angle at which the maximum transmission occurs.

$$\theta_B = \arctan\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) \qquad Eq\ 1.3$$

These are the equations to be implemented to compute power reflectivity $R = |\Gamma|^2$ and power transmissivity $T = |\tau|^2 \frac{(\eta_1 \cos\theta_t)}{(\eta_2 \cos\theta_i)}$ as functions of θ_i.

We can decompose the wave into its incident, reflected and transmitted components in the phasor representations (fields varying as $e^{j\omega t}$):

- **Incident wave**

$$\tilde{\mathbf{E}}_i = \hat{\mathbf{a}}_{e,i} E_{i0}\, e^{-j\mathbf{k}_i \cdot \mathbf{R}}, \tilde{\mathbf{H}}_i = \hat{\mathbf{a}}_{h,i} H_{i0}\, e^{-j\mathbf{k}_i \cdot \mathbf{R}}. \quad \text{Eq. } 1.4$$

- **Reflected wave**

$$\tilde{\mathbf{E}}_r = \hat{\mathbf{a}}_{e,r} E_{r0}\, e^{-j\mathbf{k}_r \cdot \mathbf{R}}, \tilde{\mathbf{H}}_r = \hat{\mathbf{a}}_{h,r} H_{r0}\, e^{-j\mathbf{k}_r \cdot \mathbf{R}}. \quad Eq. 1.5$$

- **Transmitted wave**

$$\tilde{\mathbf{E}}_t = \hat{\mathbf{a}}_{e,t} E_{t0}\, e^{-j\mathbf{k}_t \cdot \mathbf{R}}, \tilde{\mathbf{H}}_t = \hat{\mathbf{a}}_{h,t} H_{t0}\, e^{-j\mathbf{k}_t \cdot \mathbf{R}}. \quad Eq. 1.6$$

Here $\hat{a}_{e,\cdot}$ and $\hat{a}_{h,\cdot}$ are unit polarization vectors set by the TE/TM geometry; amplitudes are connected within each medium by $E_0 = \eta H_0$. These forms, together with the boundary conditions at $z = 0$, produce the Fresnel relations above.

Computational Tasks:

1. Angle sweep and plots. For $0 \le \theta_i \le 90$ (at 1° intervals), compute $\Gamma_{\text{TE/TM}}$ and $\tau_{\text{TE/TM}}$ using the formulas above, convert to reflectivity (R) and transmissivity (T), and plot all four curves $R_{\text{TE}}, T_{\text{TE}}, R_{\text{TM}}, T_{\text{TM}}$ on the same figure, clearly labeled.

2. Confirm Brewster's angle**.** Using $\varepsilon_{r_1} = 1$ and $\varepsilon_{r_2} = 8$, evaluate θ_B using *Eq. 1.3* and verify numerically that $R_{\text{TM}}(\theta_B) \approx 0$ in the plot (TE has no Brewster angle for non-magnetic media).

3. Phasor solutions for a specified case. For TM polarization, $\theta_i = 60°$, $f = 1$ GHz, and $H_{i0} = 1$ A/m, compute k_1, k_2; find θ_t via Snell's law and compute $\Gamma_{\text{TM}}, \tau_{\text{TM}}$; then obtain

$$H_{r0} = \Gamma_{\text{TM}} H_{i0}, \qquad H_{t0} = \tau_{\text{TM}} \frac{\eta_1}{\eta_2} H_{i0}, \qquad Eq. 1.7$$

and the corresponding electric-field amplitudes from $E_0 = \eta H_0$. Assemble the incident, reflected, and transmitted phasors using the forms above with propagation vectors $\mathrm{k}_i, \mathrm{k}_r, \mathrm{k}_t$ consistent with the geometry in *Eq. 1.2*. The results for these phasors, wave-vector components and magnitudes are to be tabulated in the simplest numeric form, as required.

Flow Diagram:

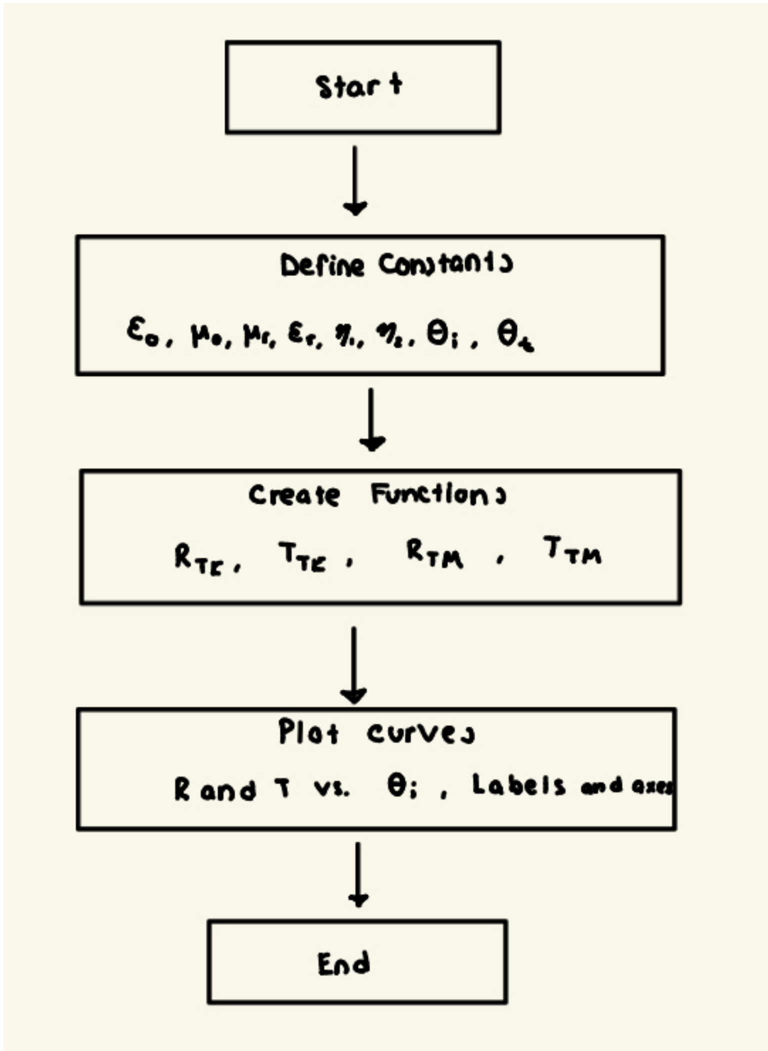

Figure 1: Flow diagram of written code.

Code:

```matlab
clear all;
%Define constants
epsilon_0 = 8.854*10^-12;
mu_0      = 4*pi*10^-7;
epsilon_r1= 1; %in medium 1
epsilon_r2= 8; %in medium 2
mu_r      = 1; %magnetic material

%Define variables
medium_1 = epsilon_r1*epsilon_0; %air
medium_2 = epsilon_r2*epsilon_0; %8*epsilon_0
global eta_1 eta_2 n1 n2;
eta_1    = sqrt((mu_r*mu_0)/medium_1);
eta_2    = sqrt((mu_r*mu_0)/medium_2);
n1       = sqrt(mu_r*epsilon_r1);
n2       = sqrt(mu_r*epsilon_r2);

incident_angle_deg    = 0:90; %independant variable
reflected_angle_deg   = incident_angle_deg; %incident=reflected
incident_angle_rad    = deg2rad(incident_angle_deg);
transmitted_angle_rad = asin(sin(incident_angle_rad).*(n1/n2)); %snells law

%Calculating reflectivity (R) for perpendicular polarization
function calc_reflectivity_TE = reflectivity_TE(incident_angle_deg,transmitted_angle_rad)
    incident_angle_rad = deg2rad(incident_angle_deg);
    global eta_1 eta_2;
    reflection_coeffecient_TE = (eta_2.*cos(incident_angle_rad)-eta_1.*cos(transmitted_angle_rad))./(eta_2.*cos(incident_angle_rad)+eta_1.*cos(transmitted_angle_rad));
    calc_reflectivity_TE = (abs(reflection_coeffecient_TE)).^2;
end

```

Figure 2: Code for simulation (1 of 4).

```matlab
%Calculating transmittivity (T) for perpendicular polarization
function calc_transmittivity_TE = transmittivity_TE(incident_angle_deg,transmitted_angle_rad)
    incident_angle_rad = deg2rad(incident_angle_deg);
    global eta_1 eta_2;
    transmittivity_coeffecient_TE = (2.*eta_2.*cos(incident_angle_rad))./(eta_2.*cos(incident_angle_rad)+eta_1.*cos(transmitted_angle_rad));
    calc_transmittivity_TE = ((abs(transmittivity_coeffecient_TE)).^2).*(eta_1.*cos(transmitted_angle_rad))./(eta_2.*cos(incident_angle_rad));
end

%Calculating reflectivity (R) for parallel polarization
function calc_reflectivity_TM = reflectivity_TM(incident_angle_deg,transmitted_angle_rad)
    incident_angle_rad = deg2rad(incident_angle_deg);
    global eta_1 eta_2;
    reflection_coeffecient_TM = (eta_2.*cos(transmitted_angle_rad)-eta_1.*cos(incident_angle_rad))./(eta_2*cos(transmitted_angle_rad)+eta_1.*cos(incident_angle_rad));
    calc_reflectivity_TM = (abs(reflection_coeffecient_TM)).^2;
end

%Calculating transmittivity (T) for parallel polarization
function calc_transmittivity_TM = transmittivity_TM(incident_angle_deg,transmitted_angle_rad)
    incident_angle_rad = deg2rad(incident_angle_deg);
    global eta_1 eta_2;
    transmittivity_coeffecient_TM = (2.*eta_2.*cos(incident_angle_rad))./(eta_2.*cos(transmitted_angle_rad)+eta_1.*cos(incident_angle_rad));
    calc_transmittivity_TM = ((abs(transmittivity_coeffecient_TM)).^2).*(eta_1.*cos(transmitted_angle_rad))./(eta_2.*cos(incident_angle_rad));
end


% Evaluate curves once
Rte = reflectivity_TE(incident_angle_deg, transmitted_angle_rad);
Tte = transmittivity_TE(incident_angle_deg, transmitted_angle_rad);
Rtm = reflectivity_TM(incident_angle_deg, transmitted_angle_rad);
Ttm = transmittivity_TM(incident_angle_deg, transmitted_angle_rad);

```

Figure 3: Code for simulation (2 of 4).

```
% Plot
figure; hold on; grid on
xlabel('Incident angle (degrees)');
ylabel('Transmittivity / Reflectivity');
title('Transmittivity and Reflectivity vs. Incident Angle');

h1 = plot(incident_angle_deg, Rte, 'r', 'LineWidth', 1.6);
h2 = plot(incident_angle_deg, Tte, 'g', 'LineWidth', 1.6);
h3 = plot(incident_angle_deg, Rtm, 'b', 'LineWidth', 1.6);
h4 = plot(incident_angle_deg, Ttm, 'c', 'LineWidth', 1.6);

% leave some room to the right for inline labels
xlim([min(incident_angle_deg) max(incident_angle_deg)+5]);

% Inline labels (auto-places at last finite point; color matches line)
placeLabel(incident_angle_deg, Rte, 'R_{TE}', h1, 1, 0);
placeLabel(incident_angle_deg, Tte, 'T_{TE}', h2, 1, 0);
placeLabel(incident_angle_deg, Rtm, 'R_{TM}', h3, 1, 0);
placeLabel(incident_angle_deg, Ttm, 'T_{TM}', h4, 1, 0);

legend('R_{TE}','T_{TE}','R_{TM}','T_{TM}','Location','best'); % optional

% ==== Helper (can live at end of the same script) ====
function placeLabel(x, y, txt, h, dx, dy)
    % place a label at the last finite point of curve (x,y)
    if nargin < 5, dx = 1; end
    if nargin < 6, dy = 0; end
    x = x(:); y = y(:);                          % ensure same shape
    k = find(isfinite(x) & isfinite(y), 1, 'last');
    if isempty(k), return; end
    text(x(k)+dx, y(k)+dy, txt, ...
        'Color', get(h,'Color'), ...
        'HorizontalAlignment','left', ...
```

Figure 4: Code for simulation (3 of 4).

```
        'HorizontalAlignment','left', ...
        'VerticalAlignment','middle', ...
        'Interpreter','tex', ... % use 'latex' if you prefer
        'FontSize', 11, 'FontWeight','bold');
end
```

Figure 5: Code for simulation (4 of 4).

Virtual Simulations:

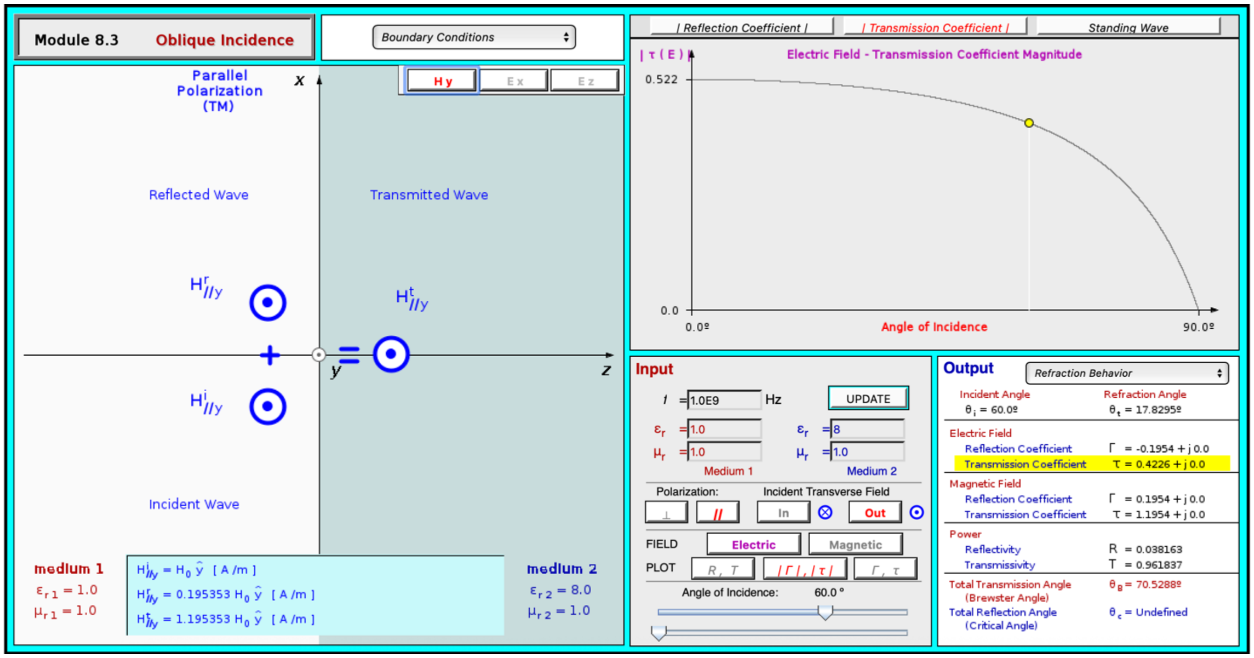

Figure 6: Simulated results for magnetic field components.

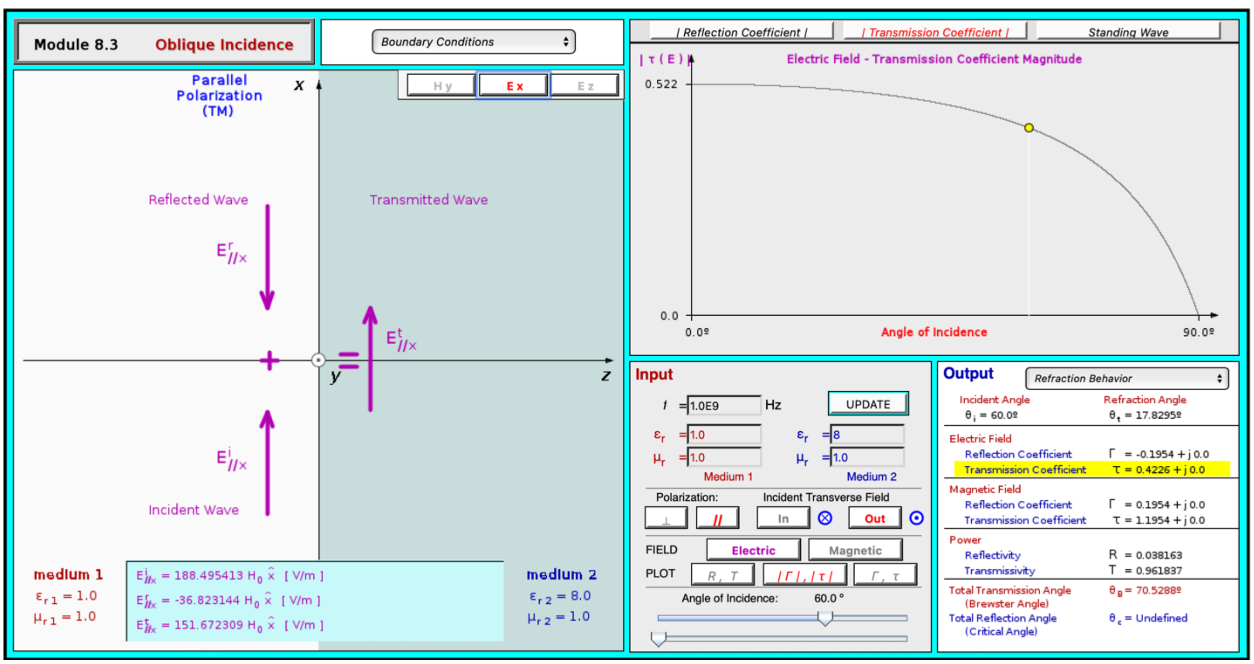

Figure 7: Simulated results for electric field in the $x-direction$.

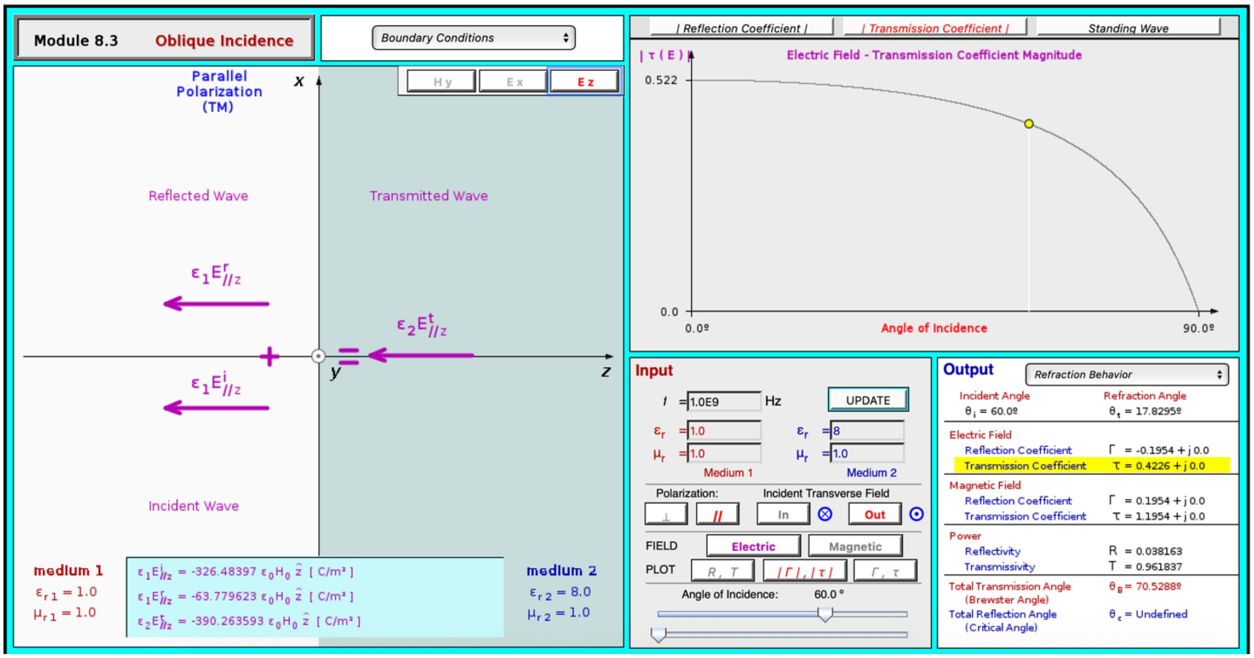

Figure 8: Simulated results for electric field in the $y-$ direction.

Results:

Part 1 – Plot of $R_{TE/TM}$ and $T_{TE/TM}$ $vs. incident\ angle$:

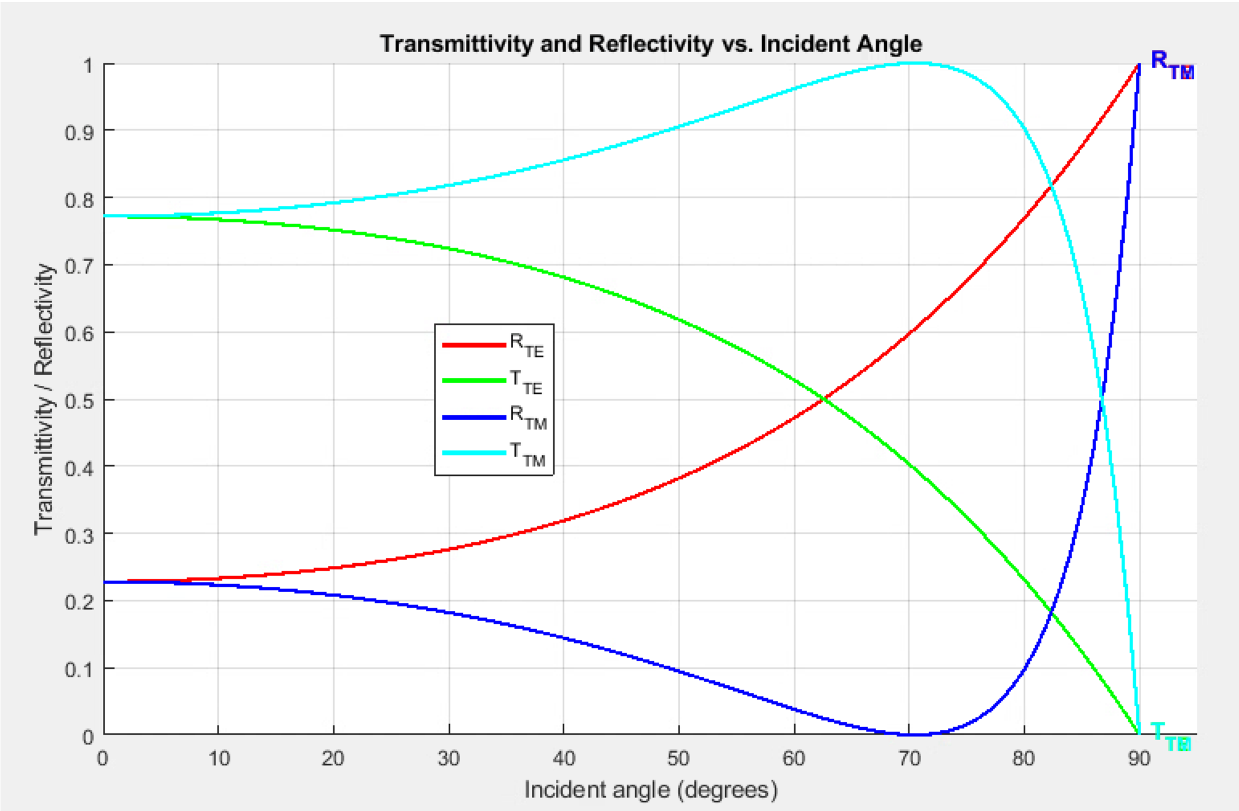

Figure 9: Tabulated results of all 25 combinations.

Part 2 – Calculated Brewster's Angle:

$$\theta_B = \arctan\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

$$\theta_B = \arctan(\sqrt{8}) \approx 70.53°$$

Part 3 – Tabulated Results:

Table 1: Wave Vector Components

Wave Component	$\hat{x}$	y	$\hat{z}$
$\tilde{E}_i$	$0.5e^{-j(18.15x+10.48z)}$	0	$-0.866e^{-j(18.15x+10.48z)}$
$\tilde{H}_i$	0	$e^{-j(18.15x+10.48z)}$	0
$\tilde{E}_r$	$0.5e^{-j(18.15x-10.48z)}$	0	$0.866e^{-j(18.15x-10.48z)}$
$\tilde{H}_r$	0	$-e^{-j(18.15x-10.48z)}$	0
$\tilde{E}_{tr}$	$0.952e^{-j(18.15x+56.43z)}$	0	$-0.306e^{-j(18.15x+56.43z)}$
$\tilde{H}_{tr}$	0	$e^{-j(18.15x+56.43z)}$	0

Table 2: Wave Component Magnitudes.

Wave Component	Magnitudes
E_{i_0}	376.991
H_{i_0}	1
E_{r_0}	-73.513
H_{r_0}	-0.195
E_{tr_0}	159.282
H_{tr_0}	1.195

Discussion:

Part 1 – Plot of $R_{TE/TM}$ *and* $T_{TE/TM}$ *vs. incident angle*:

- Normal incidence agreement (TE = TM): At $\theta_i = 0°$, the TE and TM curves are equivalent, because the Fresnel forms reduce to the same normal-incidence coefficients. accordingly, a single reflectivity value appears at $\theta_i = 0°$ in *Figure 9*. This is consistent with the theory section used to generate the curves.

- We can observe that there is no Brewster angle in the TE graph. The TE reflectivity rises consistently with the incident angle (no Brewster angle for non-magnetic media), whereas the TM reflectivity exhibits a clear minimum at Brewster's angle. That dip to (near) zero in the R_{TM} curve, with a corresponding peak in T_{TM}, is the most prominent feature on the plot. The absence/presence of Brewster's angle for TE/TM matches the assignment's theory. Brewster's angle exists where no reflection occurs meaning that this all transfers into transmission which is why we exhibit a maximum transmission at $\theta_i = 70°$.

- Energy balance (R + T ≈ 1): Using power definitions $R = \mid \Gamma \mid^2$ and $T = \mid \tau \mid^2$ with the appropriate impedance/angle factors, the curves collectively obey power conservation across the interface; numerically, the sum $R_{\mathrm{TE/TM}} + T_{\mathrm{TE/TM}}$ stays near 1 for each polarization.

- Incidence limit: As $\theta_i \to 90°$, both polarizations approach R → 1 and T → 0, consistent with the Fresnel forms and the mismatch seen at large incidence angles.

Part 2 – Calculated Brewster's Angle:

From the TM-only expression,

$$\theta_B = \arctan\sqrt{\frac{\varepsilon_2}{\varepsilon_1}},$$

your calculation gives $\theta_B \approx 70.53°$ for $\varepsilon_1 = 1$ and $\varepsilon_2 = 8$. On *Figure 9*, this is the angle where the TM reflectivity curve crosses zero (or its minimum if finite sampling prevents it from reaching exactly zero) and where TM transmissivity is maximized. A practical graphical procedure is: (i) read the θ −coordinate of the minimum of $R_{\mathrm{TM}}(\theta_i)$; (ii) confirm it coincides with the maximum of $T_{\mathrm{TM}}(\theta_i)$; (iii) compare against the analytic θ_B. As we observed, these angles match, confirming our plots accuracy.

The Brewster's angle, TM-polarized reflections "disappear" (in an ideal setting) at a dielectric interface. This is exploited in glare-reduction (e.g., polarized eyewear relative to road/water reflections), and optical coatings which will be important when designing our design study project.

Part 3 – Tabulated Results:

The Part 3 tabulated phasors follows the stated procedure: use Snell's law to obtain θ_t, compute Γ_{TM} and τ_{TM}, then assemble E, H for the incident, reflected, and transmitted waves in phasor form with consistent propagation vectors $\mathrm{k}_i, \mathrm{k}_r, \mathrm{k}_t$. We defined $H_{i0} = 1$ A/m and utilized the transformation between fields ($E = \mu H$).

When we multiply the magnitude phasor components by the corresponding wave-vector component directions, the virtual-simulation values agree with the calculated values listed in *Table 1* (wave-vector components) and *Table 2* (component magnitudes) including the sign conventions for the reflected TM fields and the expected scaling between E and H via the intrinsic impedances in each medium. This validates our numerical calculations.

Conclusion

The simulation campaign and analytic calculations collectively demonstrate the Fresnel behavior of a plane wave incident from air onto a dielectric with $\varepsilon_r = 8$: TE shows no Brewster angle and increases in reflectivity with angle; TM exhibits a Brewster angle near 70.53° with a corresponding transmission peak; and power balance is respected across our value for the incidence angle. The TM phasor solutions at $f = 1$ GHz and $\theta_i = 60°$ are consistent with Snell's law and the Fresnel coefficients**,** and they numerically match the virtual-simulation readouts when interpreted with the tabulated wave-vector components and magnitudes. These consistencies across theory, code, plots (*Figure 9*), and tables *(Tables 1–2*) satisfy the assignments objectives and provide us with the basics of wave propagation through different boundaries.

References

University of Ottawa. (2025). *ELG3106: Module 04 – Reflection and transmission of plane waves* [Lecture notes]. Department of Electrical Engineering, University of Ottawa.

University of Ottawa. (2025). *ELG3106 2025 Assignment 2* [Course assignment]. Department of Electrical Engineering, University of Ottawa.

Appendix:

Incident wave equation:

$$\tilde{E}_i = (\hat{x}\cos\ \theta_i - \hat{z}\sin\ \theta_i)E_{i0}e^{-jk_1(x\sin\ \theta_i + z\cos\ \theta_i)}$$
$$\widetilde{H}_i = \hat{y}\ \frac{E_{i0}}{\eta_1}e^{-jk_1(x\sin\ \theta_i + z\cos\ \theta_i)}$$

Reflected wave equation:

$$\tilde{E}_r = (\hat{x}\cos\ \theta_r + \hat{z}\sin\ \theta_r)E_{r0}e^{-jk_1(x\sin\ \theta_r - z\cos\ \theta_r)}$$
$$\widetilde{H}_r = -\hat{y}\ \frac{E_{r0}}{\eta_1}e^{-jk_1(x\sin\ \theta_r - z\cos\ \theta_r)}$$

Transmitted wave equation:

$$\tilde{E}_{tr} = (\hat{x}\cos\ \theta_t - \hat{z}\sin\ \theta_t)E_{tr0}e^{-jk_2(x\sin\ \theta_t + z\cos\ \theta_t)}$$
$$\widetilde{H}_{tr} = \hat{y}\ \frac{E_{tr0}}{\eta_2}e^{-jk_2(x\sin\ \theta_t + z\cos\ \theta_t)}$$

Medium 1:

$$k_1 = \omega\sqrt{\mu_r\mu_0\ \varepsilon_r\varepsilon_0} = 2\pi f\sqrt{\mu_0\varepsilon_0} = 20.958\,\frac{rad}{m}$$
$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = 120\pi$$

Medium 2:

$$k_2 = \omega\sqrt{\mu_r\mu_0\ \varepsilon_r\varepsilon_0} = 2\pi f\sqrt{\mu_0(8\varepsilon_0)} = 59.279\,\frac{rad}{m}$$
$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = 133.29$$

Snell's Law:

$$\theta_i = \theta_r = 60°$$
$$\theta_t = \arcsin\left(\sin\theta_i \cdot \frac{k_1}{k_2}\right) = 17.829°$$

Electric Magnitude Relation:

$$\tilde{E} = -\eta(\hat{\mathrm{k}} \times \tilde{H})$$

$$\tilde{H} = \frac{1}{\eta}(\hat{\mathrm{k}} \times \tilde{E})$$

(applies to incident, reflected, and transmitted waves)

TM Polarization Relations:

$$\Gamma_{TM} = \frac{E_{r0}}{E_{i0}} = \frac{\eta_1 H_{r0}}{\eta_1 H_{i0}} \Longrightarrow H_{r0} = H_{i0}\,\Gamma_{TM}$$

$$\tau_{TM} = \frac{E_{t0}}{E_{i0}} = \frac{\eta_2 H_{t0}}{\eta_1 H_{i0}} \Longrightarrow H_{t0} = \frac{\eta_1}{\eta_2} H_{i0}\,\tau_{TM}$$