Assignment 4
ELG3106 – Fall 2025

Section: A00
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Date Assigned: Sunday November 3rd, 2025
Due Date: Saturday November 17th, 2025

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Introduction:

In many RF and microwave systems we must deliver power from a source and feedline to a load without standing waves or excess loss. The practical problem is to match a complex load Z_L to a lossless feedline of characteristic impedance Z_{01} by inserting an in-series quarter-wave ($\lambda/4$) transformer. Doing so eliminates reflections at the feedline, maximizes power transfer, and produces a flat VSWR at the match point. Because a complex load does not lie on the feedline's resistance circle, we first transform the load along its own (identical) lossless line until the seen impedance is purely real; then a $\lambda/4$ section with an appropriate characteristic impedance provides the exact conjugate match to the feedline.

Theory:

1. Lossless transmission line input impedance (general):

$$Z_{\text{in}} = Z_0 \, \frac{Z_L + jZ_0 \tan(\beta\, l)}{Z_0 + jZ_L \tan(\beta\, l)},$$

With $\beta = 2\pi/\lambda$ and line length l. This relationship lets us rotate the load around the Smith chart and compute the impedance seen at any distance from the load.

2. Quarter-wave property for a real load:
   When $l = \lambda/4$ on a *lossless* line,

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} \text{ (for real } Z_L\text{).}$$

This is the basis of $\lambda/4$ impedance transformers: a quarter-wave section inverts a real load about Z_0.

3. Matching condition for the in-series $\lambda/4$ transformer:
   Let $Z(d)$ be the impedance of the load line at the transformer's input plane, a distance d from the load. At voltage maxima or minima on the load line, $Z(d)$ is purely real, giving two unique placements $d_{\max}$ and $d_{\min}$. The $\lambda/4$ transformer of characteristic impedance Z_{02} then satisfies

$$Z_{01}\, Z(d) = Z_{02}^2,$$

so that the feedline sees a perfect match $Z_{\text{in}} = Z_{01}$.

These relations justify the two-step design: (i) move from Z_L to the nearest real-axis point(s) on the Smith chart (at $d_{\max}$ or $d_{\min}$) on the load line; (ii) choose Z_{02} from the product condition above so that the $\lambda/4$ section transforms that real value to Z_{01}. The assignment specifies $Z_{01} = 50$ and $Z_L = 100 - j\,200$, assumes air lines at $f = 1.0$ GHz, and uses high-resolution Smith charts plus the Module 2.7 Tutorial/Design applets to construct and verify both solutions (zero feedline reflection).

Goal of the assignment

Design in an explicit, algorithmic fashion, both valid $\lambda/4$ transformer solutions that match the given complex load to the 50 feedline: determine $d_{\max}$, $d_{\min}$, the corresponding real impedances $Z(d_{\max})$ and $Z(d_{\min})$, compute the required transformer impedances Z_{02} from $Z_{01}Z(d) = Z_{02}^2$, and confirm, via the Module 2.7 Design app, that the feedline reflection coefficient is zero for each case. Document the steps, tabulate the final values, and include annotated Smith-chart constructions and verification screenshots.

Detailed Algorithm:

1. The Setup

We are given:

- Load: $Z_L = 100 - j200$
- Characteristic impedance of the main line: $Z_0 = 50\Omega$.

We want to design a quarter-wave transformer so that the overall input impedance is exactly 50Ω to create a matched-line(no reflections).

Because the load is complex, we cannot directly match it with a single real quarter-wave section. Therefore we must,

1. Use a section of 50-Ω line (some distance from the load) to move along the constant-$|\Gamma|$ circle until the impedance becomes purely real.
2. At that point insert a quarter-wave transformer with some characteristic impedance Z_{02} that converts that real resistance to 50Ω.

The first half of the algorithm is about finding the locations along the line where the impedance is purely real.

2. Normalize the load impedance

On a Smith chart it is convenient to work with normalized impedance

$$z = \frac{Z}{Z_0}$$

For the given load:

$$z_L = \frac{Z_L}{Z_0} = \frac{100 - j200}{50} = 2 - j4$$

This is the starting point on the Smith chart.

3. Compute the load reflection coefficient

The reflection coefficient at the load is

$$\Gamma_L = \frac{z_L - 1}{z_L + 1}$$

Substitute $z_L = 2 - j4$:

$$\Gamma_L = \frac{(2 - j4) - 1}{(2 - j4) + 1} = \frac{1 - j4}{3 - j4}$$

$$\Gamma_L = \frac{(1 - j4)(3 + j4)}{(3 - j4)(3 + j4)} = \frac{3 + j4 - j12 + 16}{9 + 16} = \frac{19 - j8}{25} \approx 0.76 - j0.32$$

Now find magnitude and phase:

$$|\Gamma_L| = \sqrt{0.76^2 + (-0.32)^2} \approx 0.8246$$
$$\angle\Gamma_L = \tan^{-1}\left(\frac{-0.32}{0.76}\right) \approx -22.6°$$

This tells us that the point corresponding to the load lies on a circle of radius $|\Gamma_L|$ and is rotated about $-22.6°$ from the positive real axis on the Smith chart.

4. Determining Maximum and Minimum Voltage Distance

We already found

$$\Gamma_L \approx 0.76 - j0.32, \qquad |\Gamma_L| \approx 0.8246, \angle\Gamma_L \approx -22.62°$$

On the Smith chart:

- The radius of the circle is $|\Gamma_L|$.
- The starting angle at the load is $-22.62°$.

To move from the load toward the generator, we must rotate clockwise (more negative phase). We are looking for where this constant $|\Gamma|$ circle crosses the real axis (purely resistive impedance).

From $-22.62°$, rotating clockwise we hit:

1. The negative real axis at $-180°$.
   Clockwise rotation needed:

$$\Delta\theta_{\mathrm{CW},1} = \mid -180^\circ - (-22.62^\circ) \mid = 157.38^\circ$$

2. The positive real axis at $-360^\circ \equiv 0^\circ$.
   Clockwise rotation needed:

$$\Delta\theta_{\mathrm{CW},2} = \mid -360^\circ - (-22.62^\circ) \mid = 337.38^\circ$$

So along the 50-Ω line, going toward the generator, we will encounter:

- First: a negative real Γ (voltage minimum) after a 157.38° CW rotation.
- Second: a positive real Γ (voltage maximum) after a 337.38° CW rotation.

5. Converting Maximum and Minimum Voltage Distance

For a lossless line, a phase change of 720° in Γ corresponds to one wavelength of travel. Therefore

$$\frac{d}{\lambda} = \frac{\Delta\theta_{\mathrm{CW}}}{720^\circ}$$

Using the magnitudes of the clockwise rotations:

1. Voltage minimum (negative real Γ)

$$d_{\min} = \frac{157.38^\circ}{720^\circ}\,\lambda \approx 0.2186\,\lambda$$

2. Voltage maximum (positive real Γ**)**

$$d_{\max} = \frac{337.38^\circ}{720^\circ}\,\lambda \approx 0.4686\,\lambda$$

As you move clockwise (toward the generator) from the load:

- After 0.2186λ the impedance is real and corresponds to a voltage minimum.
- After 0.4686λ the impedance is real and corresponds to a voltage maximum.

6. Convert to physical distances (for 1 GHz in air)

Assuming propagation in air at $f = 1$ GHz:

$$\lambda = \frac{c}{f} \approx \frac{3 \times 10^8}{10^9} = 0.3 \text{ m} = 300 \text{ mm}$$

Then

- $d_{\text{min}} = 0.2186\,\lambda \approx 0.2186 \times 300 \approx 65.6$ mm
- $d_{\text{max}} = 0.4686\,\lambda \approx 0.4686 \times 300 \approx 140.6$ mm

These are the physical distances from the load toward the generator along the 50-Ω line to reach purely real impedances.

6. Find the real impedances at those locations

At both locations the magnitude of Γ is still $|\Gamma_L| = 0.8246$; only the sign changes:

- At d_{min}: $\Gamma(d_{\text{min}}) = -|\Gamma_L| = -0.8246$
- At d_{max}: $\Gamma(d_{\text{max}}) = +|\Gamma_L| = +0.8246$

We convert Γ back to normalized impedance using

$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

1. At d_{min} (negative real Γ)

$$z(d_{\text{min}}) = \frac{1 - 0.8246}{1 + 0.8246} \approx 0.0961$$

De-normalize:

$$Z(d_{\text{min}}) = z(d_{\text{min}})Z_0 \approx 0.0961 \times 50 \approx 4.81\ \Omega$$

2. At d_{max} **(**positive real Γ**)**

$$z(d_{\text{max}}) = \frac{1 + 0.8246}{1 - 0.8246} \approx 10.40$$

De-normalize:

$$Z(d_{\max}) = z(d_{\max})Z_0 \approx 10.40 \times 50 \approx 520\ \Omega$$

So, when we move clockwise:

- At 0.2186λ from the load: $Z \approx 4.81\Omega$ (real, small, voltage minimum).
- At 0.4686λ from the load: $Z \approx 520\Omega$ (real, large, voltage maximum).

7. Quarter-wave transformer impedances

A quarter-wave transformer with characteristic impedance Z_{02} transforms a real load R to

$$Z_{\text{in}} = \frac{Z_{02}^2}{R}$$

For perfect matching to the 50-Ω line we require $Z_{\text{in}} = Z_0 = 50\Omega$, so

$$Z_{02} = \sqrt{Z_0\, R}$$

We can choose either real resistance (at $d_{\min}$ or $d_{\max}$) as the "load" of the quarter-wave transformer.

1. Transformer placed at $d_{\max}$ (matching a large resistance ≈ 520 Ω)

$$Z_{02,\max} = \sqrt{50 \times 520} \approx \sqrt{26\,000} \approx 1.61 \times 10^2\Omega \approx 161\Omega$$

2. Transformer placed at $d_{\min}$ (matching a small resistance ≈ 4.81 Ω)

$$Z_{02,\min} = \sqrt{50 \times 4.81} \approx \sqrt{240} \approx 15.5\ \Omega$$

Both designs are mathematically valid but in practice you would pick the one whose Z_{02} is feasible to realize with your transmission-line technology.

Virtual Simulations:

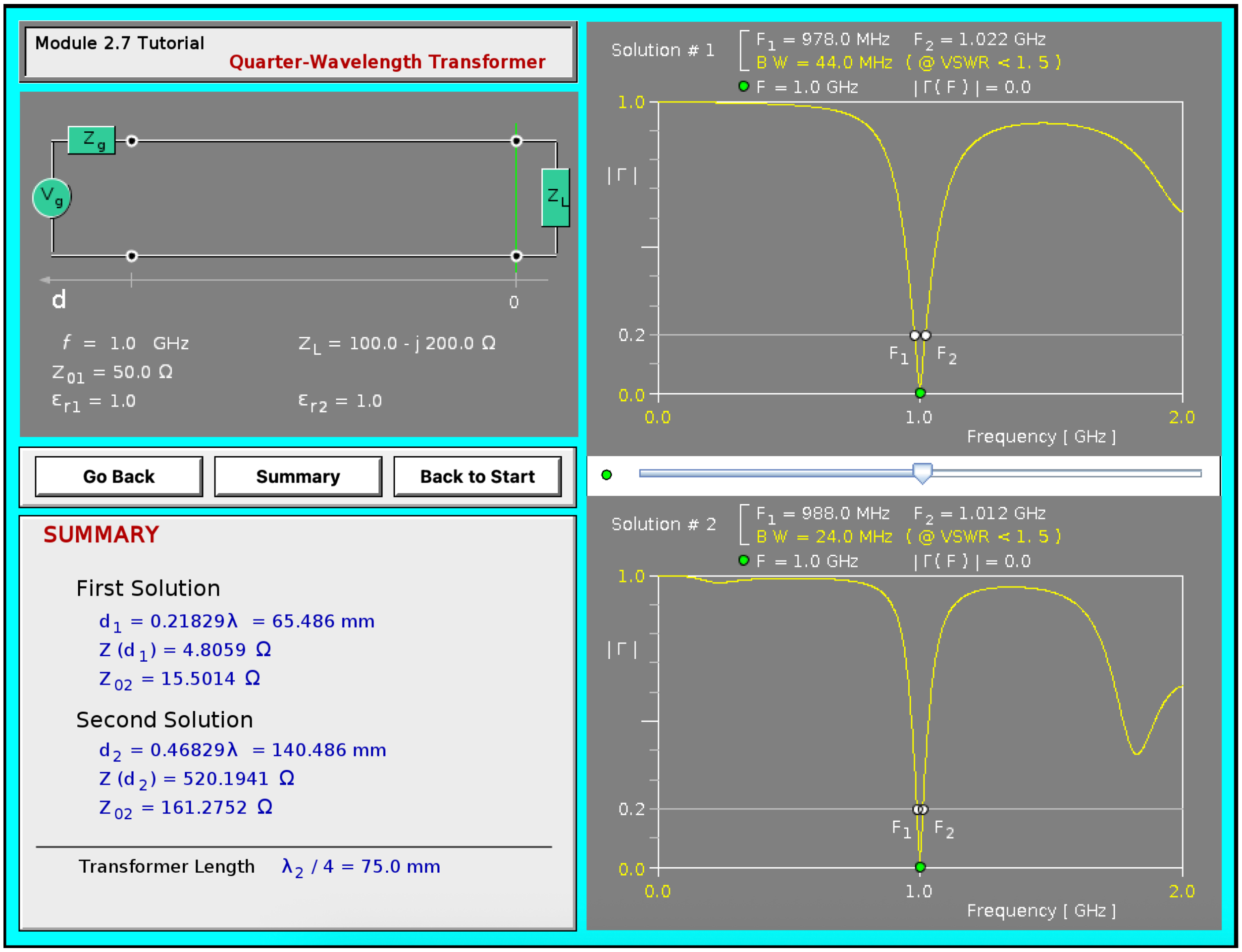


Figure 1: Tutorial design simulation for quarter-wave length transformer.

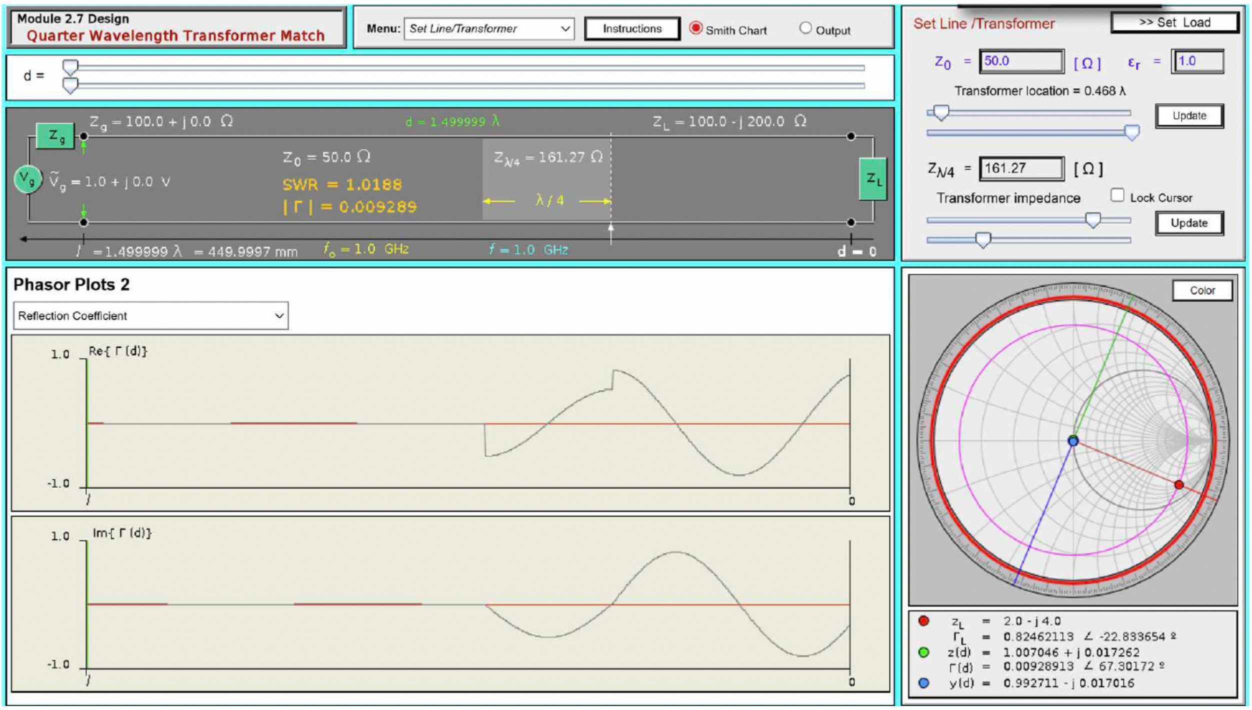


Figure 2: Simulation for d_{max} results

Output Data

Cursor	d	$= 1.499999\,\lambda$
		$= 449.9997$ mm
Impedance [Ω]	$Z(d)$	$= 50.352314 + j\,0.863082$
		$= 50.359711 \angle 0.0171$ rad
Admittance [S]	$Y(d)$	$= 0.019854 - j\,340.32 \times 10^{-6}$
		$= 0.019857 \angle -0.0171$ rad
Reflection Coefficient	$\Gamma(d)$	$= 0.00358447 + j\,0.00856968$
		$= 0.00928913 \angle 1.174637$ rad
		$= 0.00928913 \angle 67.30172$ º
Voltage [V]	$\tilde{V}(d)$	$= 0.334917 + j\,0.003818$
		$= 0.334939 \angle 0.0114$ rad
Current [A]	$\tilde{I}(d)$	$= 0.006651 - j\,38.18 \times 10^{-6}$
		$= 0.006651 \angle -0.0057$ rad

SWR = 1.0188 (cursor)

Length and Location of Transformer Line

$L_{\lambda/4} = 75.0$ mm $\quad d_{\lambda/4} = 140.4$ mm

Figure 3: Output data for d_{max} results

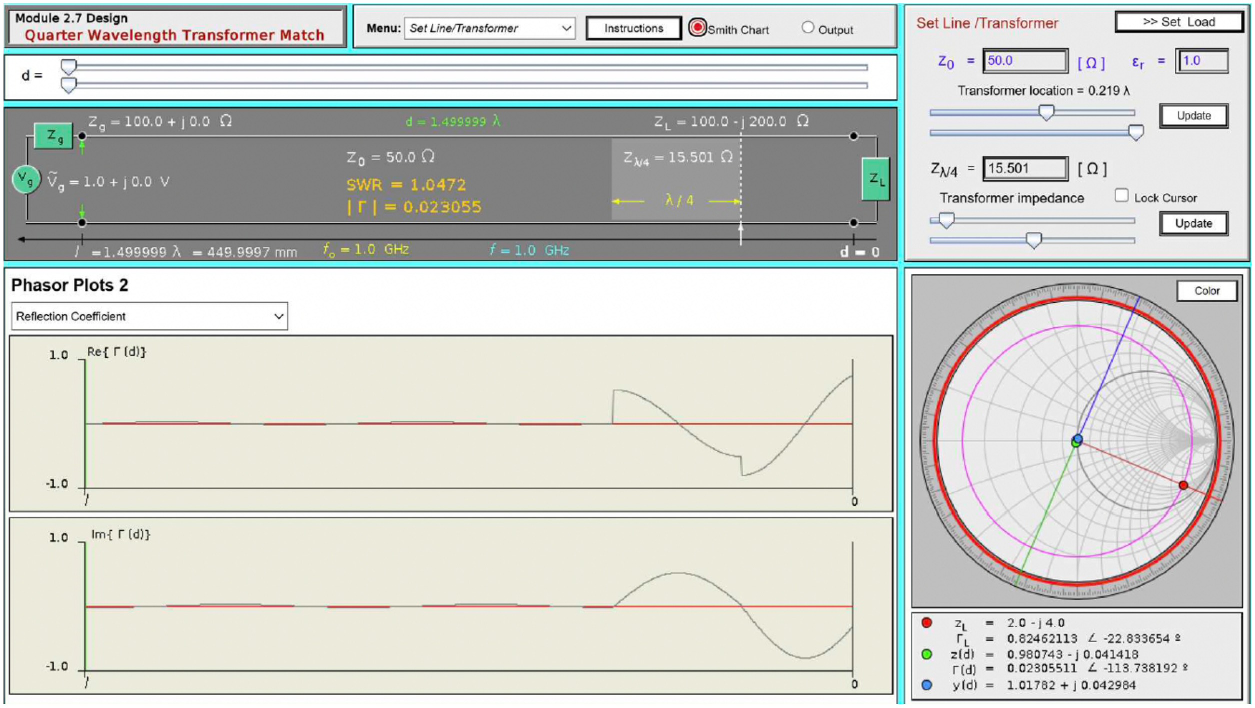


Figure 4: Simulation for d_{min} results

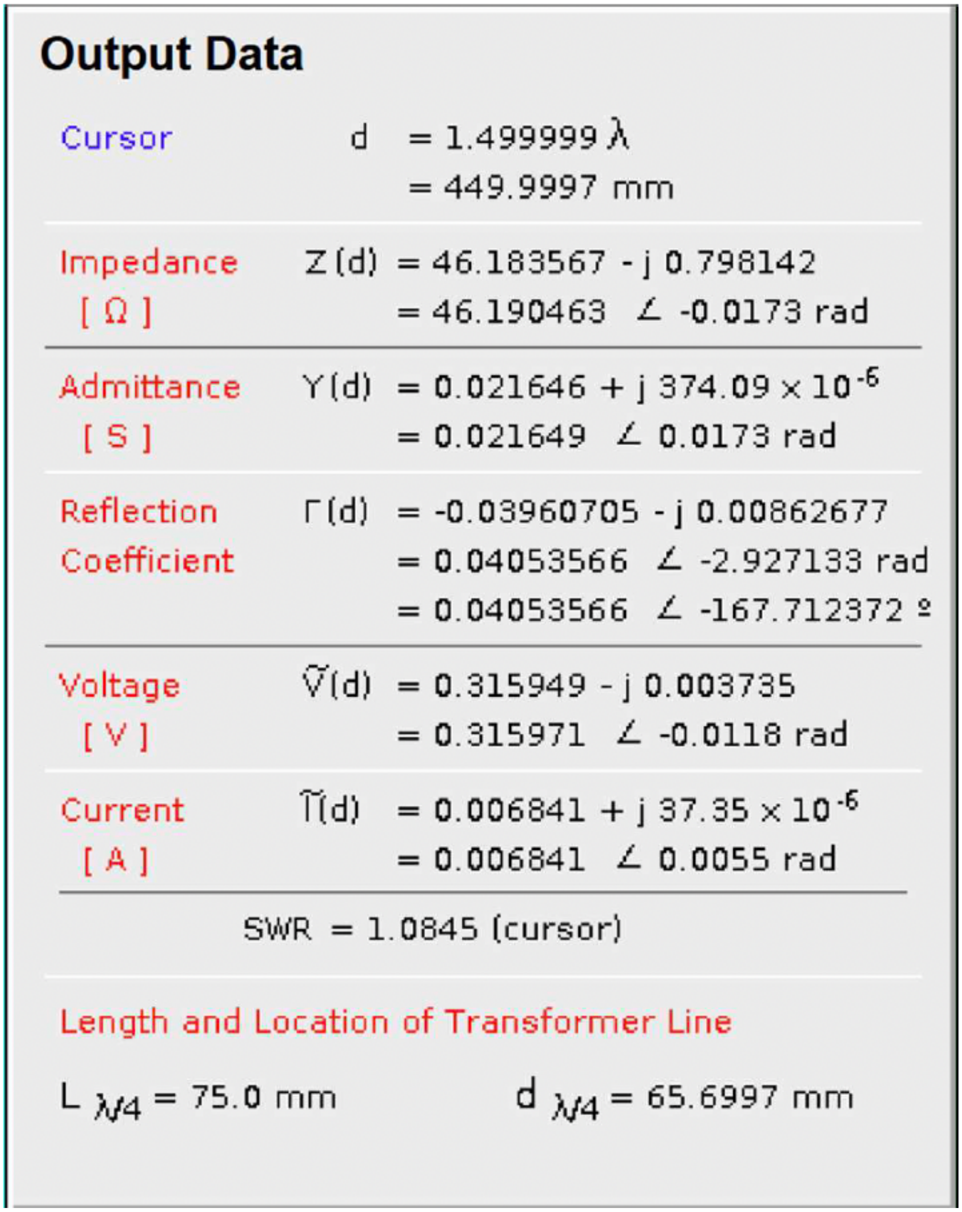


Figure 5: Output data for d_{max} $results$

Results:

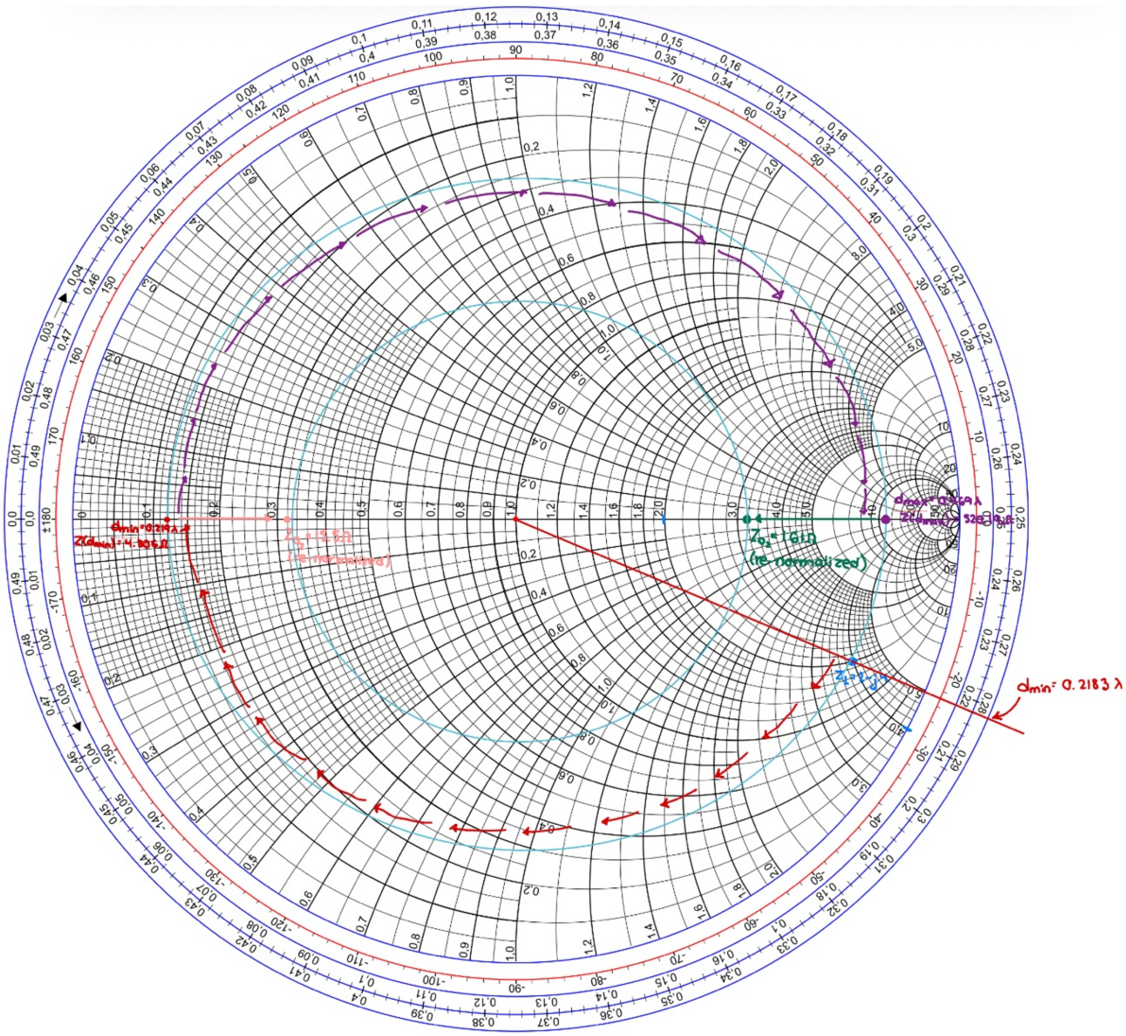


Figure 6: Annotated Smith-Chart results

Table 1: Calculated Results for Distances

Parameter	Simulated Value	Simulated Distance (mm)	Calculated Value	Calculated Distance (mm)
λ	λ	300	λ	300
d_{min}	0.2183λ	65.486	0.2186λ	65.58
d_{max}	0.4683λ	140.486	$0.4686\,\lambda$	140.58

Table 2: Calculated Results for Impedances

Parameter	Value	Simulated Value (Ω)	Calculated Value Ω)
$Z(d_{min})$	0.0961	4.8059	4.81
$Z(d_{max})$	10.40	520.1941	520
$Z_{02}(d_{min})$	-	15.5014	15.5
$Z_{02}(d_{max})$	-	161.2752	161

Discussion:

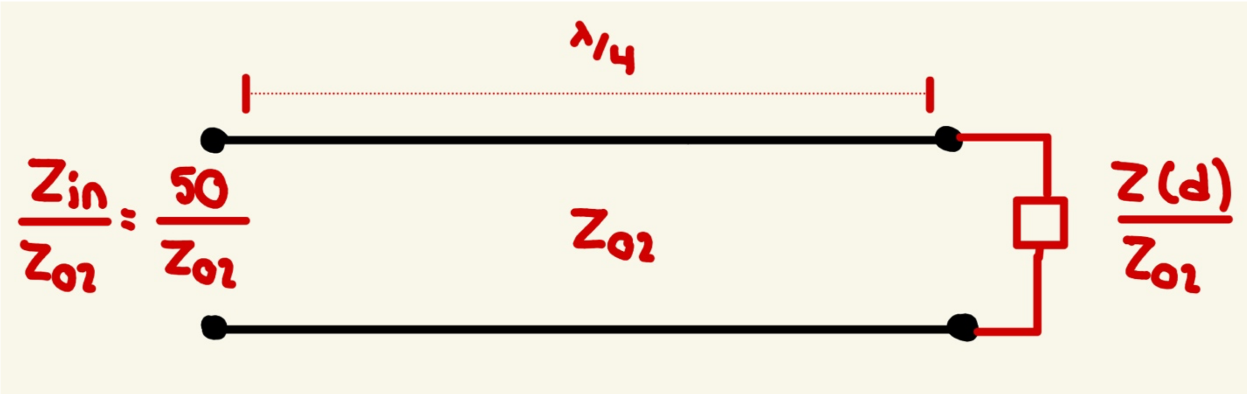
Step 1 – Part 1:

To design this matching network, it helps to think of the problem in two parts. Even though we normally solve it starting from the load and moving toward the generator, it is actually easier to explain the logic by going backwards. The idea is to understand what the quarter-wavelength transformer must accomplish, and then work our way toward the load to see what impedance the line segment before it must create. By breaking the task into these two problems, the entire process becomes much clearer and easier to follow.

Step 2 – Part 1:

The second part of the problem is determining the characteristic impedance Z02 of the quarter-wavelength line. The key requirement is that, when looking into the matching network from the generator, the input impedance must be a real 50 ohms. On a Smith chart, a purely real impedance always lies on the horizontal axis, since that is where all reactance's are zero.

A quarter-wave line has a very specific effect: it rotates the impedance by exactly 180 degrees on the Smith chart. This means that whatever impedance appears at the far end of the quarter-wave line must also be purely real, since rotating a point on the horizontal axis by 180 degrees keeps it on that same axis. Because Z02 is real, the impedance seen at one end of the quarter-wave section must also be real after normalization. Once we know that one coordinate of Z(d) must lie on the horizontal axis, we can search for the values of Z(d) that satisfy this requirement and then determine the correct value of Z02 that links the two real impedances together.



Step 1 – Part 2:
Now that we know the impedance Z(d) must lie on the horizontal axis at the point where it connects to the quarter-wave transformer, we move one step backward toward the load. We start at the known load impedance zL. By adding a short length of transmission line before the

quarter-wave section, we can rotate zL along a constant-gamma circle. This rotation does not change the magnitude of the reflection coefficient; it only changes the phase. The goal is to choose the length of this line so that the rotated impedance becomes purely real. In other words, we remove the reactive part of zL simply by rotating it to a point on the horizontal axis.

At first, it seems like there is a problem: a quarter-wave line only rotates impedances by 180 degrees. How can the quarter-wave section also adjust the magnitude of the impedance if all it does is rotate? The key insight is that the magnitudes of the characteristic impedances are different. Because Z02 and Z0 are not the same, this mismatch in characteristic impedances causes the impedance on the horizontal axis to shift to a new real value when it is renormalized. This gives the quarter-wave section the ability to change the magnitude of the impedance, even though it only rotates points by 180 degrees. By applying the quarter-wave transformation equation and using the two normalized real impedances, we can then solve for Z02.

Smith Chart Solutions:

Because we are searching for points where Z(d) crosses the horizontal axis as it rotates along a constant-gamma circle, we naturally find two possible distances along the line. Each crossing corresponds to a different purely real impedance, and each one leads to a different value of Z02. Both solutions are valid, and they lie on opposite sides of the Smith chart. Depending on which solution is easier to fabricate or more practical for the design, either one may be chosen.

Simulation Solutions:

The results from the Java Smith chart application and the manual calculations agree well with each other. When entering the values into the app, the reflection coefficient ended up very close to zero but not exactly zero. This slight discrepancy comes from rounding the values before entering them into the program. If the exact values are used, the reflection coefficient becomes extremely small, confirming that the matching network is behaving as expected.

Conclusion:

In conclusion, by working with both the Smith chart and the Java applications, we were able to determine the two valid solutions for the matching network. These solutions arise because the impedance travels along a constant-gamma circle and intersects the real axis at two different positions. Each intersection leads to a different real impedance, and therefore a different characteristic impedance for the quarter-wavelength line.

A key part of this design is understanding the purpose of the λ/4 transformer. Its role is to match the load to the 50-ohm system so that reflections are minimized or eliminated. It accomplishes this in two steps. First, the λ/4 line rotates the impedance by 180 degrees on the Smith chart, effectively flipping the impedance across the center. Second, because the characteristic impedance of the λ/4 transformer (Z02) is different from the main line, this rotation is accompanied by a scaling of the impedance magnitude. This combination of rotation and scaling allows a real impedance at one end of the transformer to appear as a different real impedance at the input, making it possible to transform the intermediate value Z(d) into exactly 50 ohms.

The two impedance crossings found on the Smith chart reflect two possible choices for Z02, both of which achieve the same matching goal. The mismatch between the characteristic impedances of the two lines is what allows the impedance magnitude to change, while the quarter-wavelength section itself is what determines the direction of this transformation through its 180-degree rotation.