

# Overview document

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## Libraries Used

```
library(gmodels)
library(PSYC201)
library(ggplot2)
library(dplyr)
library(magrittr)
library(moments)
library(combinat)
library(gtools)
```

## Permutation Rules

Number of permutations with replacement, any possible order

$n^r$  :  $n$  is number of items to choose from,  $r$  is number chosen

Total Number of permutations without replacement,

$$\frac{n!}{(n-r)!}$$

Number of permutations without replacement, but not requiring a specific order

$$\frac{n!}{r!(n-r)!}$$

## Univariate descriptive Statistics

### Definition of moments.

Point of clarification: Moments refers just to the set of numbers that describe a set of points, it makes sense in a physical system or to probability. They are defined according to the following function.

$$\mu'_i = \int_a^b (x-c)^i f(x) dx$$

For probability  $f(x) = p(x)$  so :

$$\mu'_i = \int_a^b (x-c)^i p(x) dx$$

As  $i$  increases, you move up moments. The total number of moments as  $i$  goes to  $\infty$  completely describes a set of points. *Seems like you need might only need  $n$  moments, but what do I know.* In general, each moment is referred to as  $\mu_i$ , my guess is that using  $\mu$  to represent mean is a convention to drop the 1.

Zeroeth Moment- the probability mass function

$$M_0 = \mu_0 = \Omega = 1$$

First moment, the mean

$$M_1 = \mu_1 = \mu_x = \int_a^b (x-c)^1 f(x) dx = \int_a^b x f(x) dx = E[X]$$

Second Moment, variance. Note that  $\sigma$  is the standard deviation.

$$M_2 = \mu_2 = \sigma_x^2 = \int_a^b (x-c)^2 f(x) dx = E[(X - E[X])^2] = E[X^2] - E[X]^2$$

Confirming with R

```
# need to adjust for sample calculation by subtracting 0, but not identitcal
x <- rnorm(200, 0, 4)
var(x)
```

```
## [1] 16.54169
```

```
sum((x^2))/(length(x) - 1) - mean(x)^2
```

```
## [1] 16.54298
```

This continues for the definition of moment, but using these in a non-normalized fashion doesn't appear normal

Third moment

$$M_3 = \mu_3 = \int_a^b (x - c)^3 f(x) dx$$

Fourth Moment

$$M_4 = \mu_4 = \int_a^b (x - c)^4 f(x) dx$$

At this point, there now are what called “normal moments”, where they are now “normalized” by dividing by the standard deviation. Now the  $i_{th}$  moment is given by

$$\frac{\mu_i}{\sigma^i}$$

The third normalized or standardized moment is called Pearson moment coefficient of skewness.

$$M_3 = \gamma = \frac{\mu_3}{\sigma^3} = E\left[\left(\frac{x - \mu}{\sigma}\right)^3\right] = E\left[\left(\frac{E[(X - \mu)^3]}{E[(X - \mu)^2]^{\frac{3}{2}}}\right)\right]$$

Confirming with R

```
x <- rgamma(200, shape = 10)
skewness(x)
```

```
## [1] 0.5260652
```

```
mean(mean((x - mean(x))^3)/(mean((x - mean(x))^2)^(3/2)))
```

```
## [1] 0.5260652
```

Fourth Moment, kurtosis.

$$M_4 = \beta_2 = \frac{\mu_4}{\sigma^4} = E\left[\left(\frac{E[(X - \mu)^4]}{E[(X - \mu)^2]^2}\right)\right]$$

Confirming with R

```
x <- rnorm(200)
kurtosis(x)
```

```
## [1] 2.761481
```

```
mean(mean((x - mean(x))^4)/(mean((x - mean(x))^2)^(2)))
```

```
## [1] 2.761481
```

However, because the kurtosis is equal to three in a normal distribution, it is often subtracted from the measure of kurtosis so that the sign of the value indicates whether a distribution is platykurtic (less than 0, low kurtosis) or leptokurtic (greater than 0, high kurtosis). It is simultaneously a measure of three values. It generally increases with peakedness, how much variance is right next to the mean and tails and the tail thickness, but decreases with increased mass held in the shoulders, or more modest deviations. However, it generally is seen as mostly a measure of tail thickness.

$$M_4 = \beta_2 = \frac{\mu_4}{\sigma^4} = E\left[\frac{(X - \mu)^4}{E[(X - \mu)^2]^2}\right] - 3$$

## Central Limit theorem.

Proof by assertion. It's an interesting idea. What happens when you start to sum  $n$ , iid variables. This is interesting as it assumes its observation is itself drawn from an independent distribution (which I guess is restating its assumption). If you know the nature of the underlying distribution, you can calculate your expectation about the mean, variance, skew and kurtosis of the sum of the variables.

Rules for adding or summing variables.

$$Mean(X + Y) = Mean(X) + Mean(Y)$$

$$Mean(aX) = a * Mean(X)$$

$$Var(X + Y) = Var(X) + Var(Y)$$

$$Var(aX) = a^2 * Var(X)$$

Question:

$$Var(X+Y) = Var(X) + Var(Y)$$

if  $Y = X$

why does  $Var(2*X) \neq Var(X + X)$ ? Because they are different transformations, doing something very different. This is a dumb question rob.

This means that if one starts to sum iid variables,  $n$  increases linearly. However, by assertion the kurtosis and skew start to fall towards zero, meaning that distribution of the sum of iid variables starts to look like a normal distribution. Apparently they increase like this, regardless of the initial distribution.

$$Mean[\sum^n X] = n * Mean[X]$$

$$Variance[\sum^n X] = n * Variance[X]$$

$$skew(\sum^n X) = \frac{1}{\sqrt{n}} * skew(x)$$

$$kurtosis(\sum^n X) = \frac{1}{n} * kurtosis(x)$$

This more or less underlies most statistics. There is the assumption that as you increase  $n$ , the expectation about the sum of the added random variables starts to resemble a normal variable (though normally you divide this value by  $n$ ). This is cool.

## Normal variable

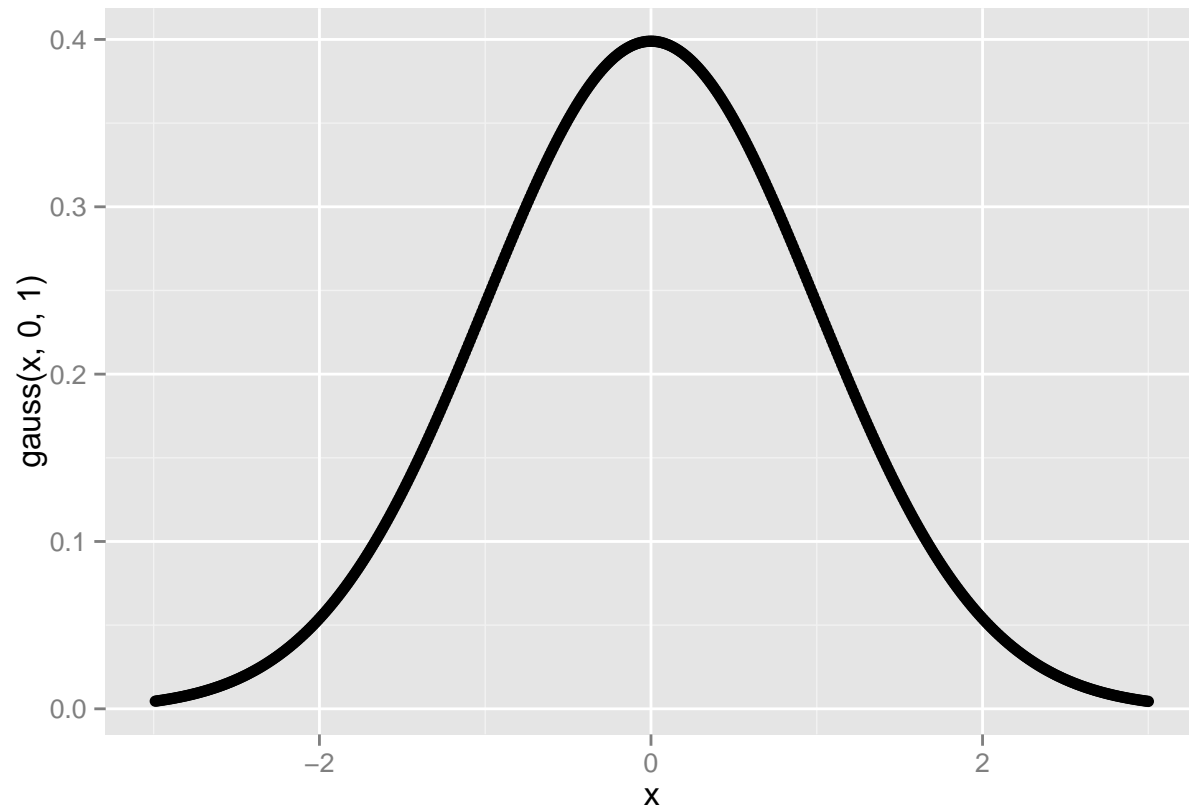
Here  $X$  is distributed according to the following function.

$$P(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left[-\frac{x-\mu}{2\sigma^2}\right]}$$

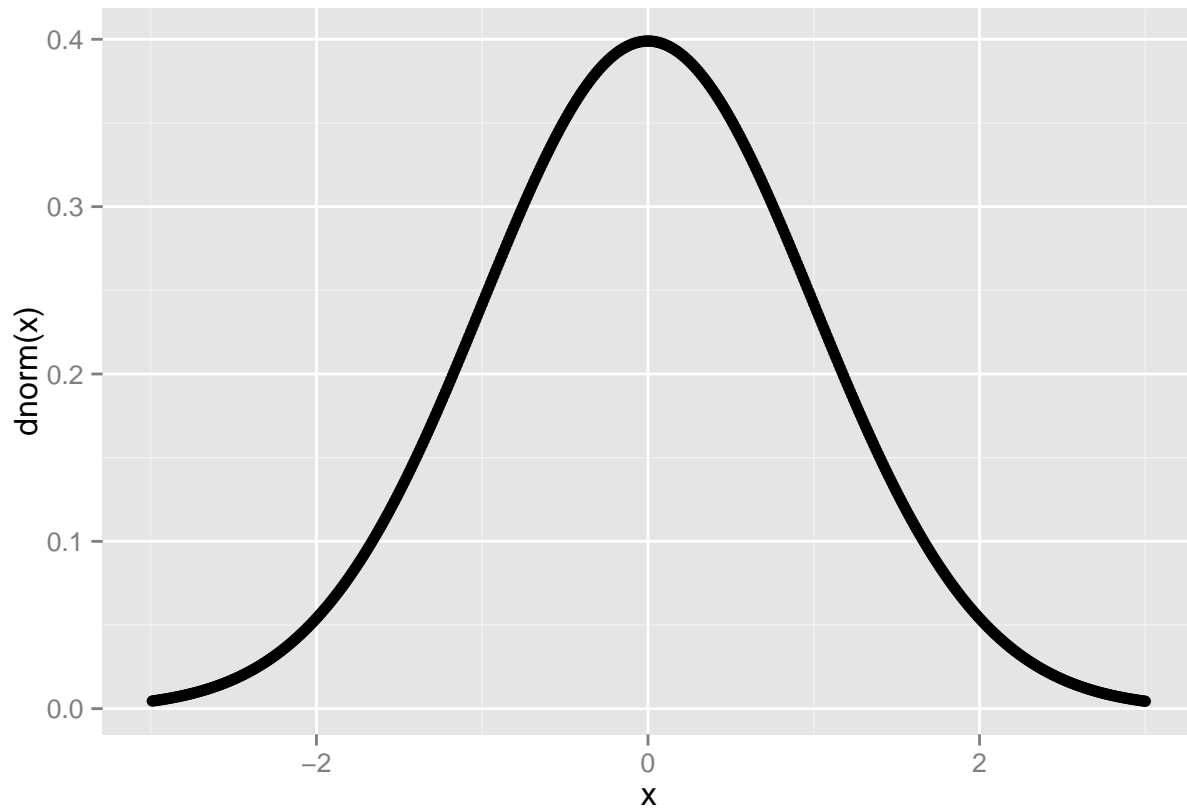
Check in R

```
gauss <- function(x, mu, sigma){  
  const <- 1 / sqrt( 2 * pi * sigma^2)  
  probability <- const * exp( - (x-mu)^2 / (2 * sigma^2) )  
  return(probability)  
}
```

```
}  
x<-1:600/100 - 3  
qplot(x,gauss(x,0,1))
```



```
qplot(x,dnorm(x))
```



## Powerball Example

Expected Value of Powerball lottery. - Five white balls drawn without replacement from 59, order irrelevant - red ball drawn from bin of 35 - \$2 to play - assume no other players

What is the expected value

```
#probability of getting 5 white balls
#five factorial 5 chances for first value, then 4 chances for second value, then..
#out of 59 chances for first value, 58 chances for second value.
white <- factorial(5) / prod((59-5):59)
red <- 1/ 35
Prob <- white*red
```

```
#does it work for simplier example
#deck of 5 cards, draw three. need to get the three target values 1,2,3
```

```
#first draw could be any of the three
d1 <- 3/5
#second draw the same.
d2 <- 2/4
#third draw the same
d3 <- 1/3
p <- d1*d2*d3
p
```

```
## [1] 0.1
```

*#this gives us correct answer- check with permutations function.*

```
permutations(n=5,r=3,repeats.allowed=F) %>%  
  data.frame %>%  
  summarise(win = mean(X1+X2+X3==6))
```

```
##    win  
## 1 0.1
```

## Law of large numbers

If you have now taken the sum of this set of random variables, you can now get the expected value of the mean and variance by dividing by  $n$ . I think this means that there is a careful set

## Multivariate extension

## Calculating Sample Variance

## Combining variances.

Shape - skewness kurtosis

-How do you calculate these, can you recognize them in a graph.

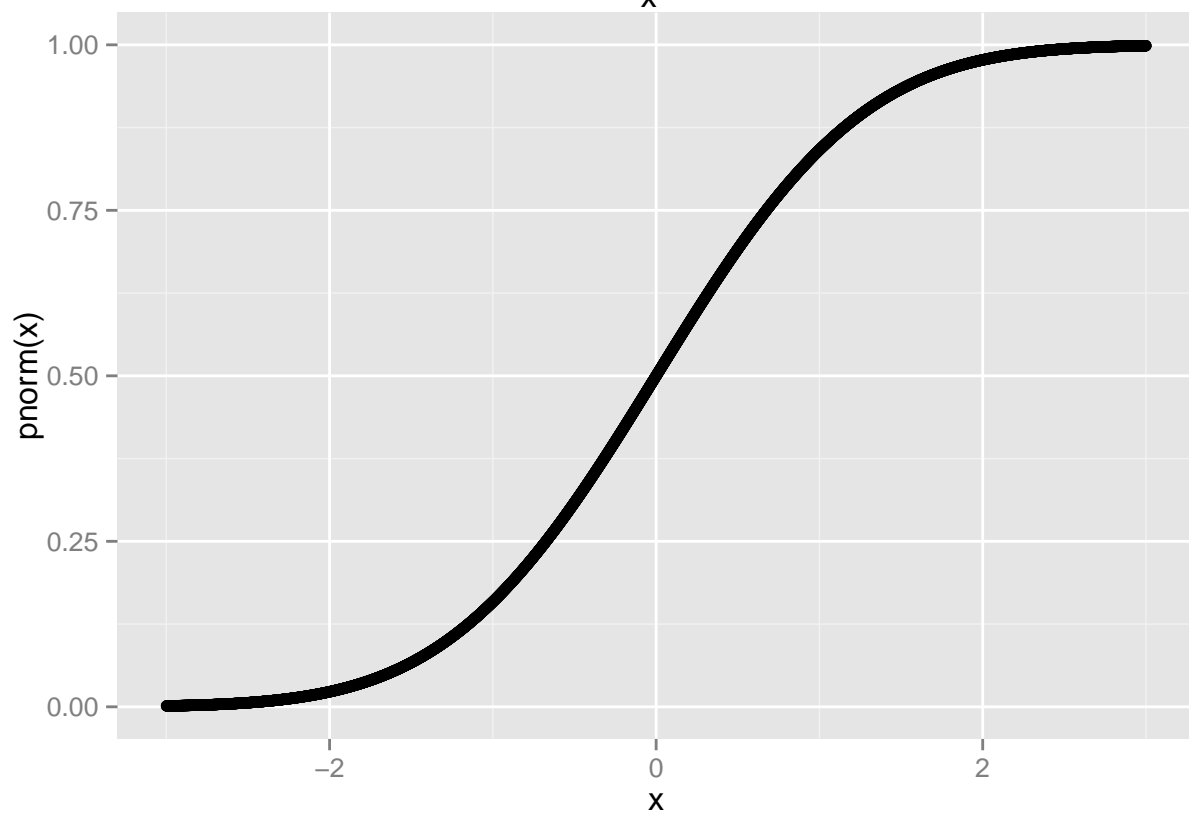
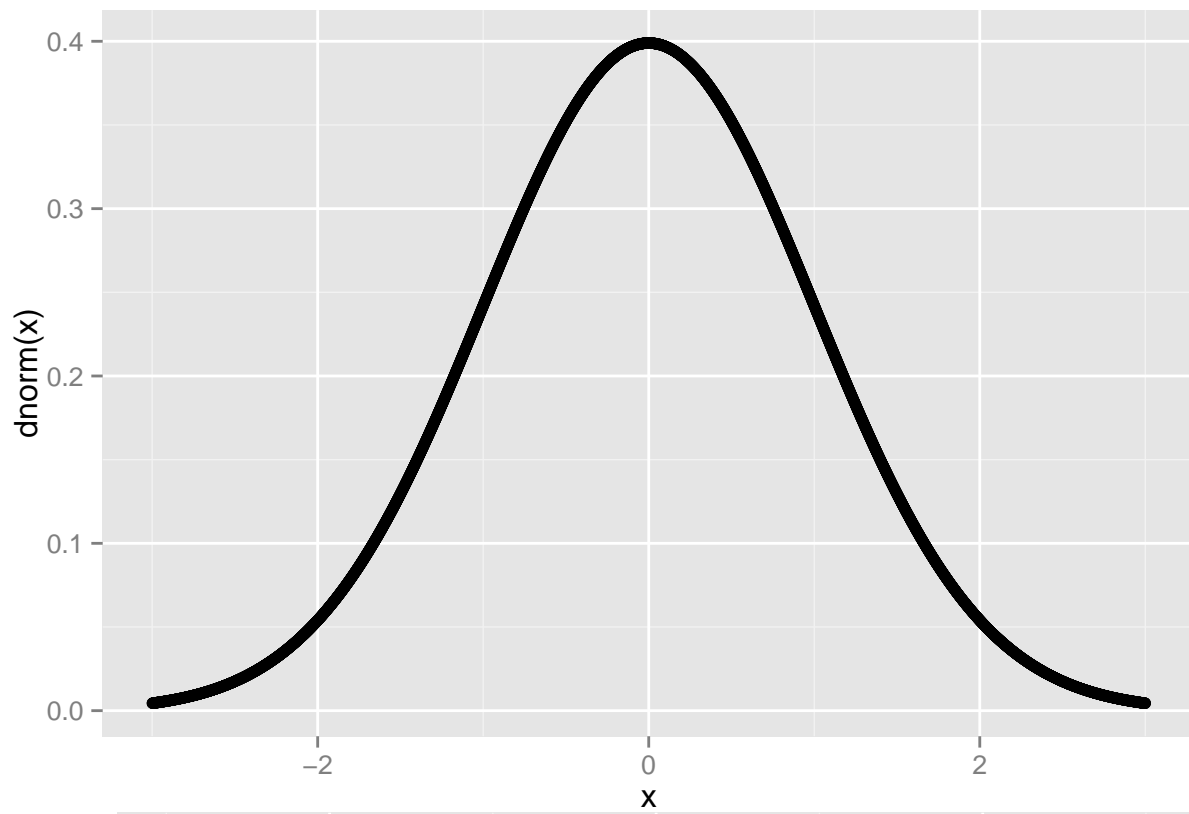
## Reading and estimating from graphs

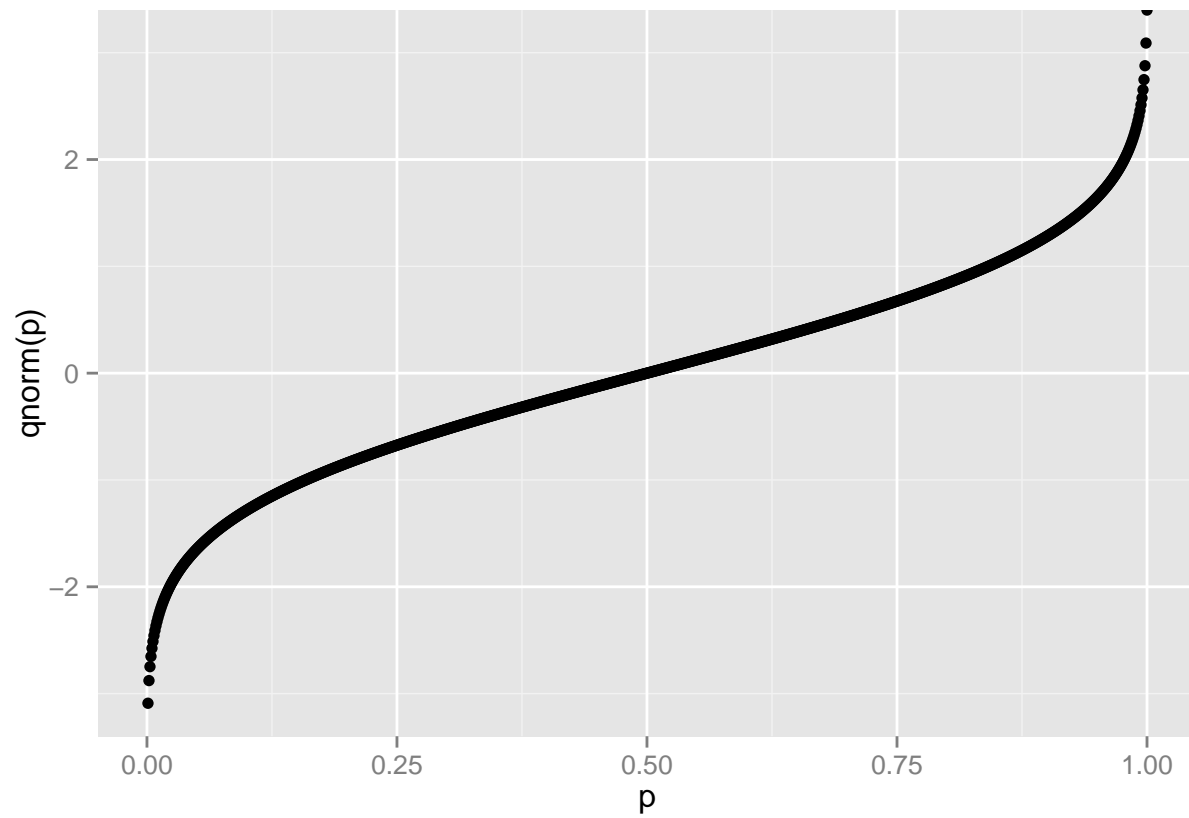
## Probability Distributions.

## pdf,cdf, icdf

Here are the R commands, same for continuous and discrete probability density functions.







-Not refer to law of total probability in the answers, particularly as numerator in bayes equation.

## Random Variables and Expectation

### Null Hypothesis Sampling Testing.