LITERATURE REVIEW

[1] has suggested a methodology to choose between the first and the best improvement strategy for a local search algorithm: "selecting the best improvement at each iteration gives worse results on average than selecting the first improvement, if the initial solution is chosen at random. However, starting with 'greedy' or 'nearest neighbor' constructive heuristics, the best improvement is better and faster on average". In this litterature review we want to determine what would be the number of papers impacted if this conclusion is not true. Different cases need to be separated

Among the 119 papers that cite [1] on Google Scholar, 45 mentions this part of their conclusion. However, the impact is different depending of the importance of the conclusion for each paper. We will split papers into three categories:

1. Papers that use [1] to choose an improvement algorithm as a part of their contribution.
2. Papers that made their own experiments to show which improvement strategy is better.
3. Papers that only mention the conclusion without using it for the core of the contribution

1 Papers using the conclusion directly

22 papers use directly the conclusion of [1]. These papers tackles problems similar to [1], the TSP, or other NP-complex problems. However, as said in [2], we cannot be sure that the property will be still valid for other problems or for other algorithms.

7 papers tackle routing problems and use directly the conclusion of Hansen et al. They all use the 2-opt heuristic except [3] which uses a genetic algorithm. 15 papers tackle 8 different problems (clustering, knapsack, jobshop, (r—p)-centroid problem, ...). 2 papers are not problem specific and are analysing algorithms performances.

2 Papers testing both strategies

13 papers test the first and the best improvement strategy for their problem. We can distinguish papers by problem types as in the previous section. 4 papers tackle a problem near the TSP whereas 9 tackle a different problem (NK landscapes, problems on graphs, facility allocation problems...).

Among all of the papers that test both strategies, the authors or the results provide sufficient information to verify or contradict the property of [1]. 10 papers get results agreeing with [1] and 3 papers get different results.

2.1 Papers finding similar results as [1]

Among the paper tackling problem similar to the TSP, 3 of them find the same results: [2] tests both a first and best improvement strategy in a Sequential VND algorithm. The paper tests

both algorithms for a random and a greedy initialization and finds similar results as [1]. To obtain this result, averages final costs of each possibility (initialization, improvement method combinations especially) are compared. The initial points are generated in a 100x100 square as in [1]. [4] finds a similar conclusion for a VND algorithm with granular search for the vehicle capacited problem. On the other hand [5] tests the two improvement strategies on TSPLIB instances (different instances from those of [1] except berlin52). The two methods are compared with averages.

For problems different from the TSP with similar result as [1], averages are also the main way of comparison. [6] takes a combination of parameters as example and makes a scatter plot of 500 random instances with as x axis the "distance between local optimal and best known solution" and as y axis the "% dev between local optimum and best known solution". A wide variety of problems have been tackled in these papers: the MaxMinSum problem ([5]), bi-objective quadratic assignment problems ([7]), heaviest k-subgraph problems ([6]), obnoxious p-median problem ([8]), multi-objective coordination graphs ([9]), simultaneous optimization in development project ([10]), multi objective optimization for energy supply systems ([11]).

2 papers can be added to this category. These papers do not mention the conclusion of [1] but compare both strategies and get similar results ([12] and [13]). [12] finds similar conclusions for an iterated local search on randomly generated NK landscapes (for a low number of perturbations) and UBQP landscapes. However on the contrary of [1] algorithms are compared by a fitness score or the number of problems for which the algorithm reach the optimum value. [13] observes that smooth landscapes lead to a better solution with a best improvement algorithm whereas more coarse landscapes lead to a better first improvement algorithm.

2.2 Papers finding different results of those of [1]

Among the papers that find different results, one of them tackles a vehicle routing problem: [14] compares the average final costs of the two improvement strategies on the capacited vehicle routing problem (CVRP), for a sequential search algorithm. An average improvement of 0.07% of BI over FI is found. The paper considers this improvement too small to consider that the property is still valid for the CVRP. For comparison, in [1] the average improvement for random points in a 100x100 square was between 0.07% and 3.54% (FI-CL and FI-NL compared to BI-NL).

On other problems than routing problems, 2 papers find different results from [1]. In [15], in average the Best Improvement strategy gives better result than the First Improvement strategy for the minimum dispersion problem with a random initialization. [16] finds no sigificant differences between the average final cost of the two strategies on a patient scheduling problem.

3 Papers mentionning the conclusion without using it for the core of the contribution

11 papers mainly only use the [1] as a reference for local search algorithms without using it in the core of their contribution ([17], [18], [19], [20], [21], [22], [23], [24], [25], [26], [27]). If the conclusion of [1] was not true, the impact on their contribution would be limited to the citation of this conclusion.

4 Excluded papers

31 papers has been excluded from the 119 of Google Scholar

- 1 of them lead to a deadlink

- 7 were inaccessible due to paywalls
- 23 were in other languages than french or english

5 Conclusion

To summarize, 45 papers would be affected if the conclusion of [1] is false. 22 of them would be especially impacted as they directly use the property of [1] to choose an improvement method. On the other hand 13 papers would be less impacted as they have done their experiments for their own problem and algorithms. However most of the time the comparison between the two improvement strategies is done with averages. Finally 11 papers would be impacted only regarding the citation of the property of [1] limiting the impact on the rest of the contribution. 31 papers were excluded. The other papers do not use [1] for this property but mainly as a reference for the First and Best Improvement comparison without focusing on the property investigated.

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