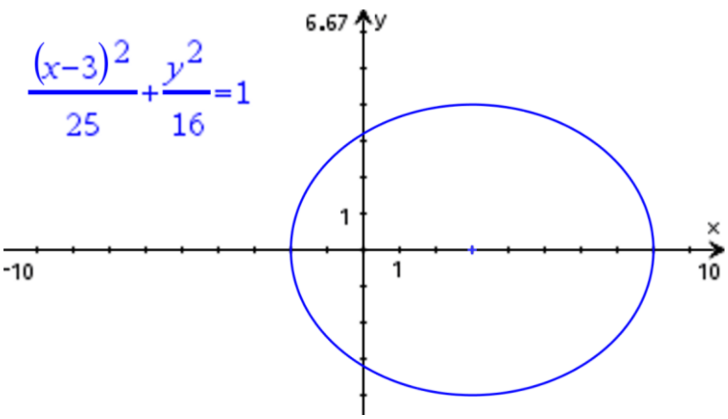
A satellite is in orbit around the moon. A coordinate plane containing the orbit is set up with the center of the moon at the origin. The equation of the satellite's orbit is:

$$\frac{(x-3)^2}{25}+\frac{y^2}{16}=1$$

(a). Use software to create a graph of this equation and determine when the satellite is the closest to and farthest from the center of the moon.

(b). There are two points in the orbit with y-coordinates = 2. Find the x-coordinate of these points and determine their distances to the center of the moon.

Answer (a):



The center of the moon is at the origin. The distance formula is:

$$d(A,\ B)=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$$

Since the center of the moon is (0, 0), we are simply looking to minimize and maximize this equation:

$$d(A,\ 0)=\sqrt{x_2{}^2+y_2{}^2}$$

At point (-2, 0), the distance is 2. At point (8, 0) the distance is 8. These are the min and max respectively for distance.

Answer (b):

To solve for x when y = 2, we substitute that into the question and solve for x.

$$\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$$

$$\frac{(x-3)^2}{25} + \frac{2^2}{16} = 1$$

$$\frac{(x-3)^2}{25} + \frac{1}{4} = 1$$

$$\frac{(x-3)^2}{25} = \frac{3}{4}$$

$$(x-3)^2 = \frac{75}{4}$$

$$x = \pm \frac{\sqrt{75}}{2} + 3$$

So, from this we know that the following are approximate coordinate when y = 2:

$(-1.33,\ 2)$ & $(7.33,\ 2)$

The distance from the origin is given by using the distance formula again with these new coordinates:

$$\sqrt{(-1.33)^2 + 2^2} \approx 2.401$$

$$\sqrt{(7.33)^2 + 2^2} \approx 7.597$$