

How to solve for the Sum of Powers:

$$\diamond S_n = \sum_{i=1}^n A^i = \frac{A^{n+1} - A}{A - 1} \quad (*)$$

Then the following pathway is used to come up with the formula:

$$\begin{aligned} S_n &= A + A^2 + A^3 + \dots + A^{n-1} + A^n \\ A * S_n &= A^2 + A^3 + A^4 + \dots + A^n + A^{n+1} \end{aligned}$$

Let's take these two and subtract the top from the bottom:

$$A * S_n - S_n = A^{n+1} - A$$

Factoring out the  $S_n$  and simplifying, gives the following:

$$\begin{aligned} S_n(A - 1) &= A^{n+1} - A \\ S_n &= \frac{A^{n+1} - A}{A - 1} \quad (\blacksquare) \end{aligned}$$