OPTIMIZATION FOR DATA SCIENCE

METHODS OF PORTFOLIO OPTIMIZATION

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CONTENT

PROJECT OBJECTIVES

TOPIC

Overview of the **Frank-Wolfe algorithm** and its variations within Markowitz portfolio theory.

OBJECTIVE

Analyze the **performance** of these algorithms on different **stock indices**.

MODERN PORTFOLIO THEORY

DEFINITION

Developed by Harry Markowitz in 1952, is an investment strategy that aims to **optimize portfolio returns** for a given level of risk.

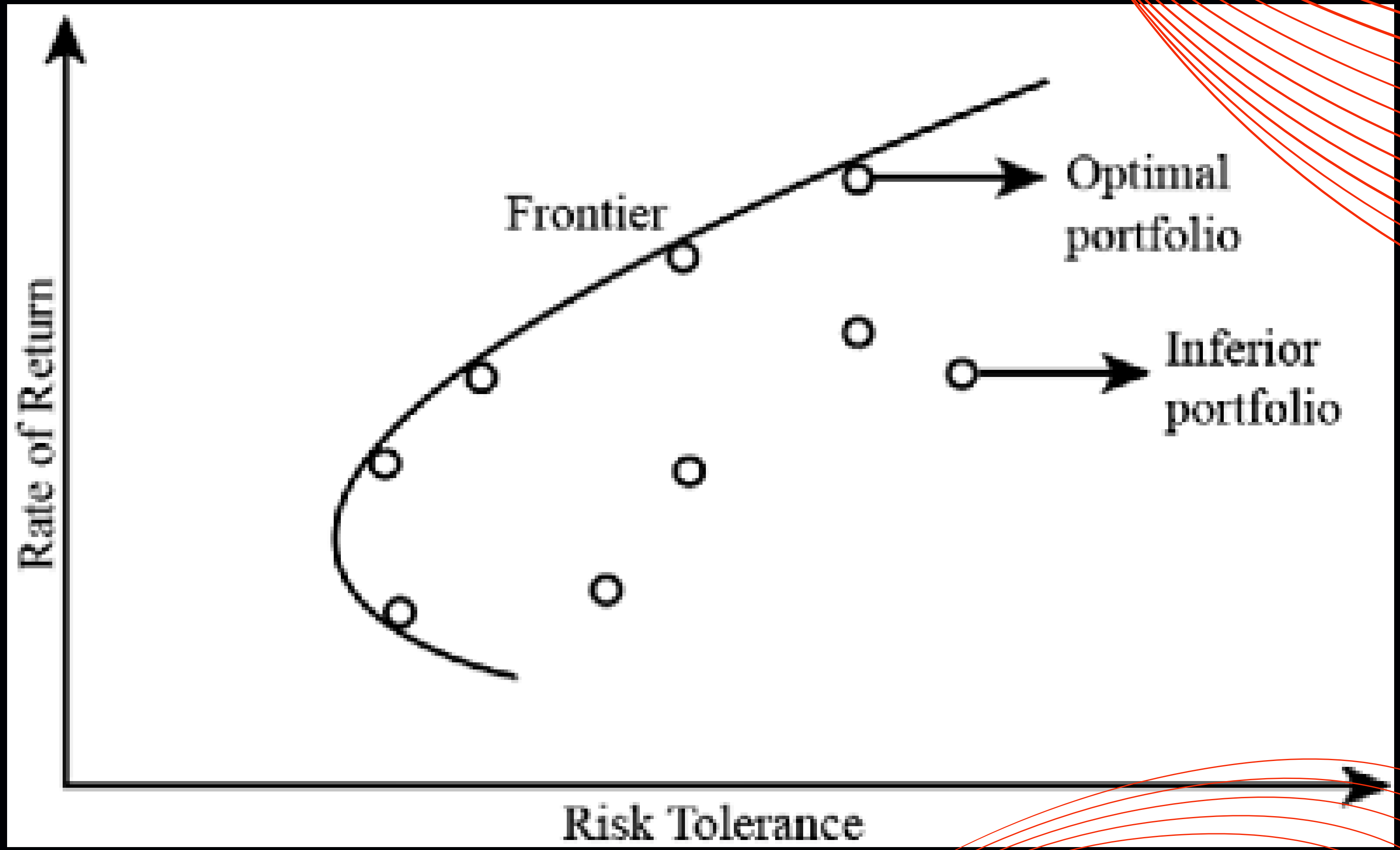
OBJECTIVE

- **Maximize** expected return
- **Minimize** risk through diversification

MODERN PORTFOLIO THEORY

EFFICIENT FRONTIER

The efficient frontier **represents optimal portfolios** that offer the highest expected return for a given level of risk.



RETURN

$\bar{r}^{\mathsf{T}} w$

RISK

$w^{\mathsf{T}} \Sigma w$

DEFINITION

- **w:** weights
- **Σ:** covariance matrix
- **r:** mean return of each asset

DATASET

Euro Stoxx 50

A stock index of the 50 largest companies in the **Eurozone**.

Ftse MIB

The main index of the **Italian Stock Exchange**, including the 40 largest Italian companies.

Ftse 100

An index of the 100 largest companies listed on the **London Stock Exchange**.

DATASET

Date
From 01-22-2007 to 05-06-2013

Index Returns
Weekly returns of the market index

Assets Returns
Weekly returns of each asset

Means Return
Compute from Assets Returns

Covariance Matrix
Compute from Assets Returns

CONSTRAINED PROBLEMS

LOSS FUNCTION

$$\min_{w \in R^n} \quad \gamma \, w^\intercal \, \Sigma \, w - \bar{r}^\intercal \, w$$

CONSTRAIN

$$\begin{aligned} s.t. \quad e^\intercal w &= 1 \\ w &> 0 \end{aligned}$$

DEFINITION

- **γ:** risk aversion = 6
- **w:** weights
- **Σ:** covariance matrix
- **r:** mean return of each asset

FIRST ALGORITHM

FRANK WOLFE

1- COMPUTE GRADIENT

$$\nabla f(w) = 2\gamma\Sigma w - \bar{r}^{\mathsf{T}}$$

2- DIRECTION

Find atom:

$$s_k = \arg\min_{s \in C} \nabla f(x_k)^T s$$

Compute direction:

$$d_k^{FW} = s_k - x_k$$

3- LEARNING RATE [γ]

Line Search to find **best step size** that minimize loss function

4- UPDATE WEIGHTS

w = w + γ * (d)

Direction

SECOND ALGORITHM

AWAY-STEP FRANK WOLFE

1- COMPUTE GRADIENT

$$\nabla f(w) = 2\gamma \Sigma w - \bar{r}^{\mathsf{T}}$$

2- Find vertex v:

$$v_k = \arg\max_{v \in A_k} \nabla f(x_k)^T v$$

3- Choose direction:

$$d_k = \begin{cases} d_{FW} & \text{se } -\nabla f(x_k)^T d_{FW} \geq -\nabla f(x_k)^T d_{AS} \\ d_{AS} & \end{cases}$$

4- Update:

Depends on the direction we take

$$x_{k+1} = x_k + \alpha_k d_{FW}$$

or

$$x_{k+1} = x_k + \alpha_k d_{AS}$$

THIRD ALGORITHM

PAIR-WISE FRANK WOLFE

1- COMPUTE GRADIENT

$$\nabla f(w) = 2\gamma \Sigma w - \bar{r}^{\top}$$

2- DIRECTION [d]

$$d_k = s_k - v_k$$

3- LEARNING RATE [γ]

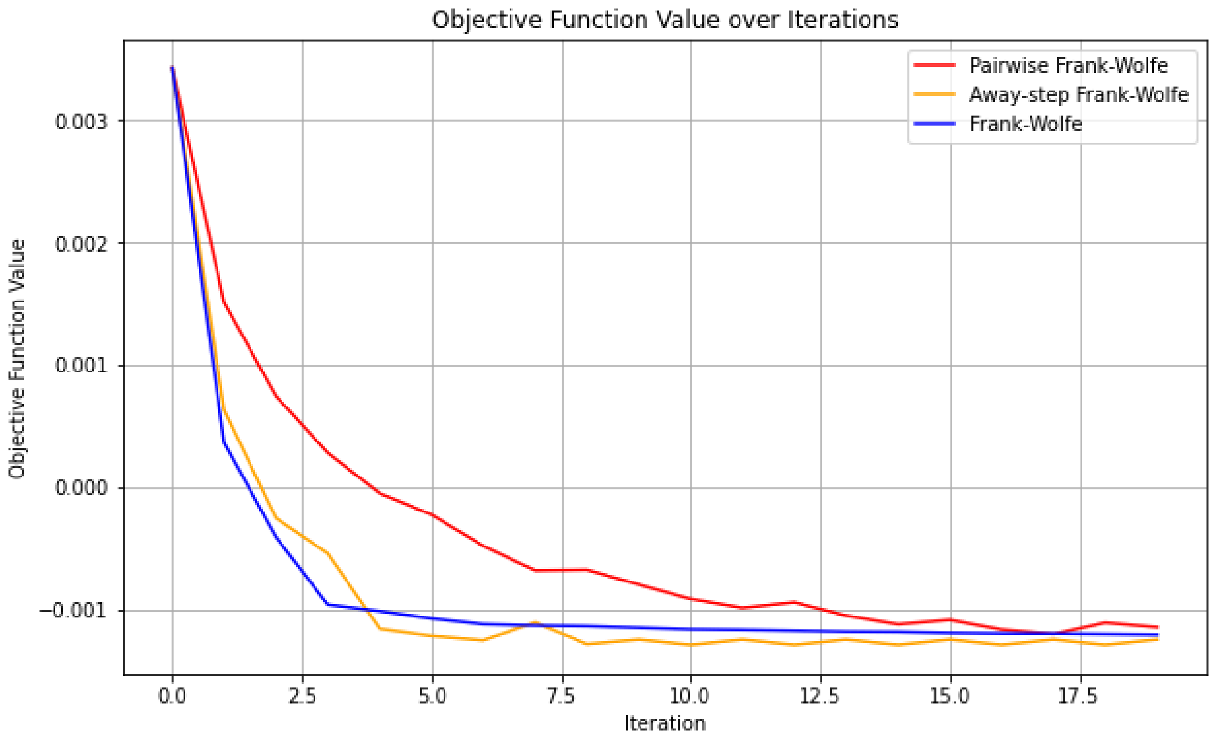
Line Search to find **best step size** that minimize loss function

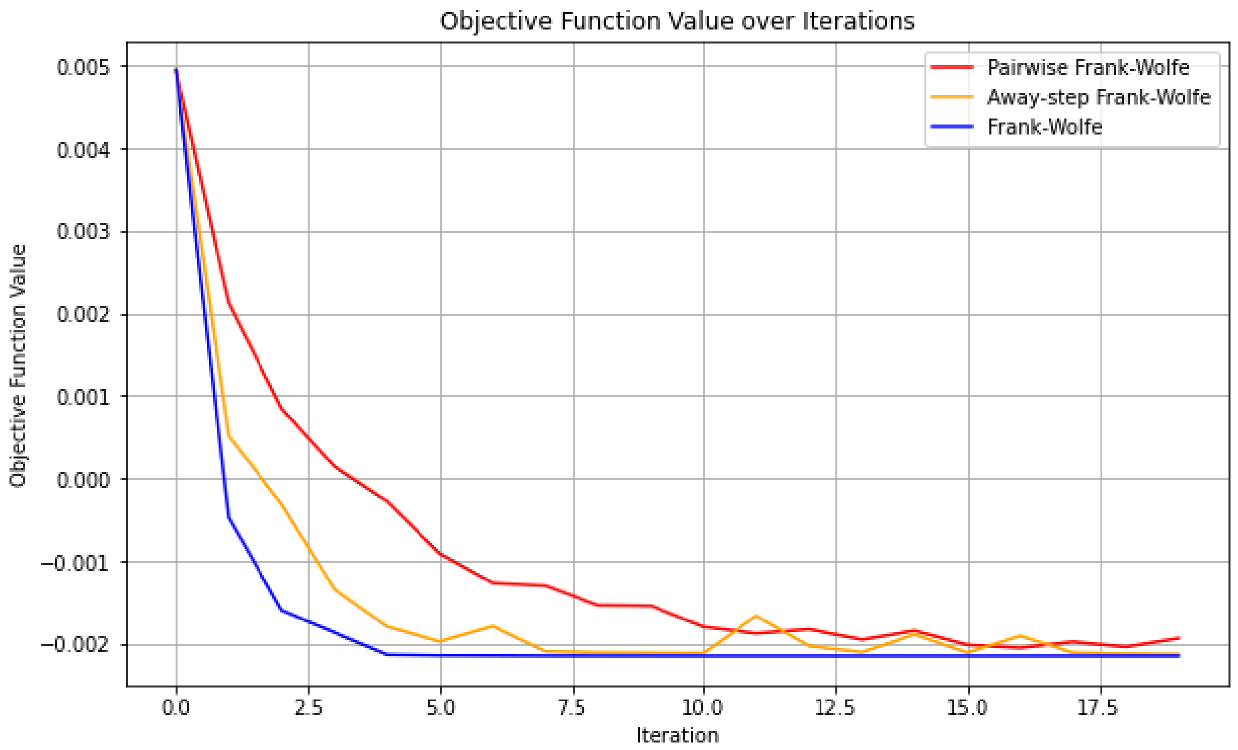
4- UPDATE WEIGHTS

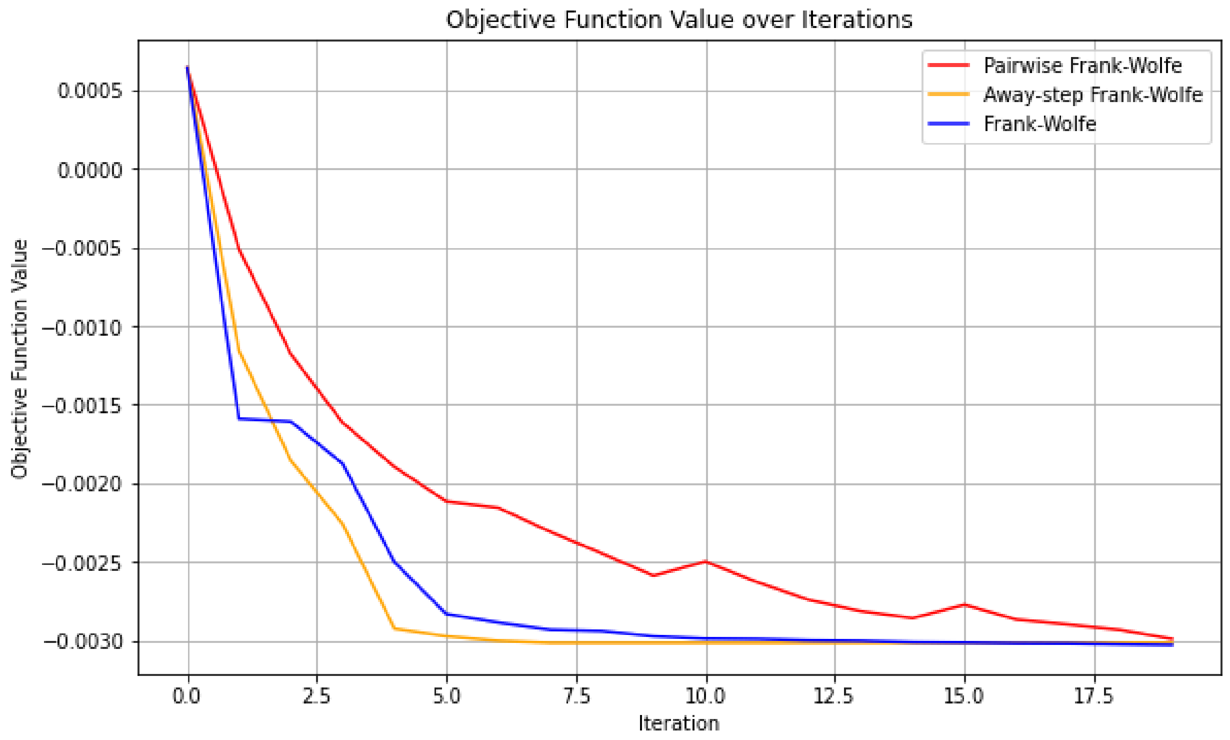
w = w + γ * d

LOSS FUNCTION COMPARISON

Euro Stoxx 50 | Ftse MIB | Ftse 100







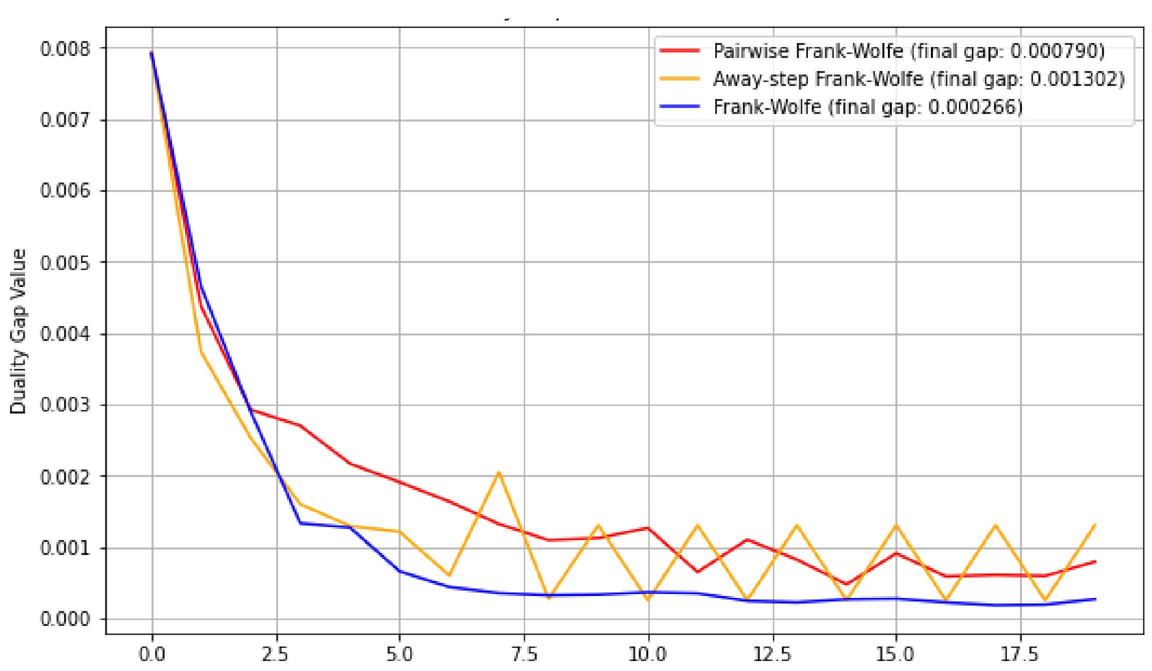
Frank Wolfe

Converges faster and reaches a lower loss value

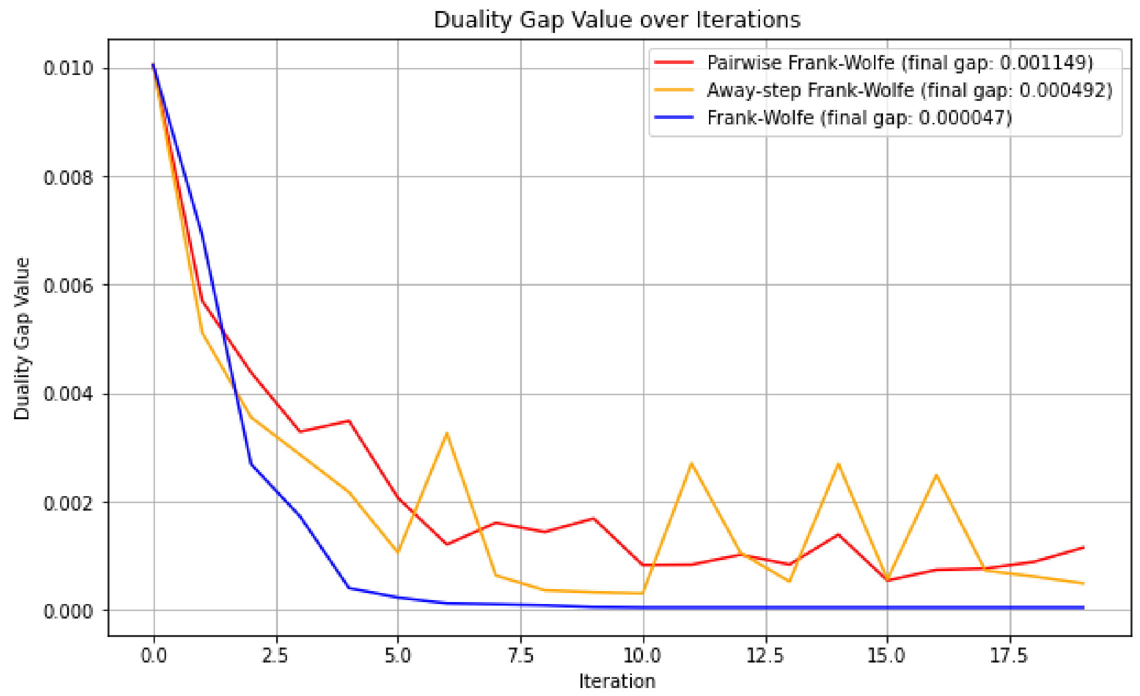
DUALITY GAP COMPARISON

Indicates how close the current solution is to the
optimal solution

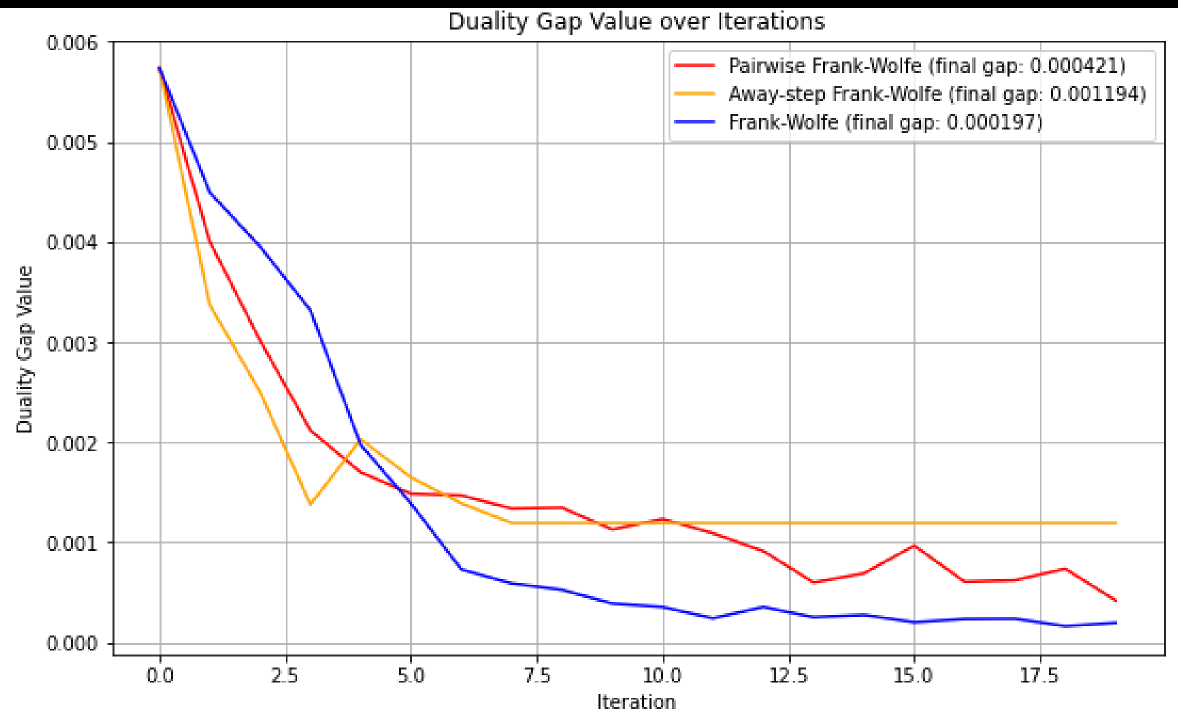
Euro Stoxx 50



Ftse MIB



Ftse 100



Frank Wolfe

Lower duality gap on all dataset

EXECUTION TIME COMPARISON

AWAY STEP FRANK WOLFE

Is the most efficient in terms of **computation time**

Dataset	FW	PFW	AFW
Euro Stoxx 50	*35 ms*	*33 ms*	*32 ms*
Ftse MIB	*40 ms*	*41 ms*	*35 ms*
Ftse 100	*30 ms*	*32 ms*	*28 ms*

RISK COMPARISON

PAIR WISE FRANK WOLFE

Is the worst in terms of **risk**

Dataset	FW	PFW	AFW
Euro Stoxx 50	*1.89%*	*1.98%*	*1.89%*
Ftse MIB	*1.73%*	*1.82%*	*1.68%*
Ftse 100	*1.68%*	*1.83%*	*1.79%*

RETURN COMPARISON

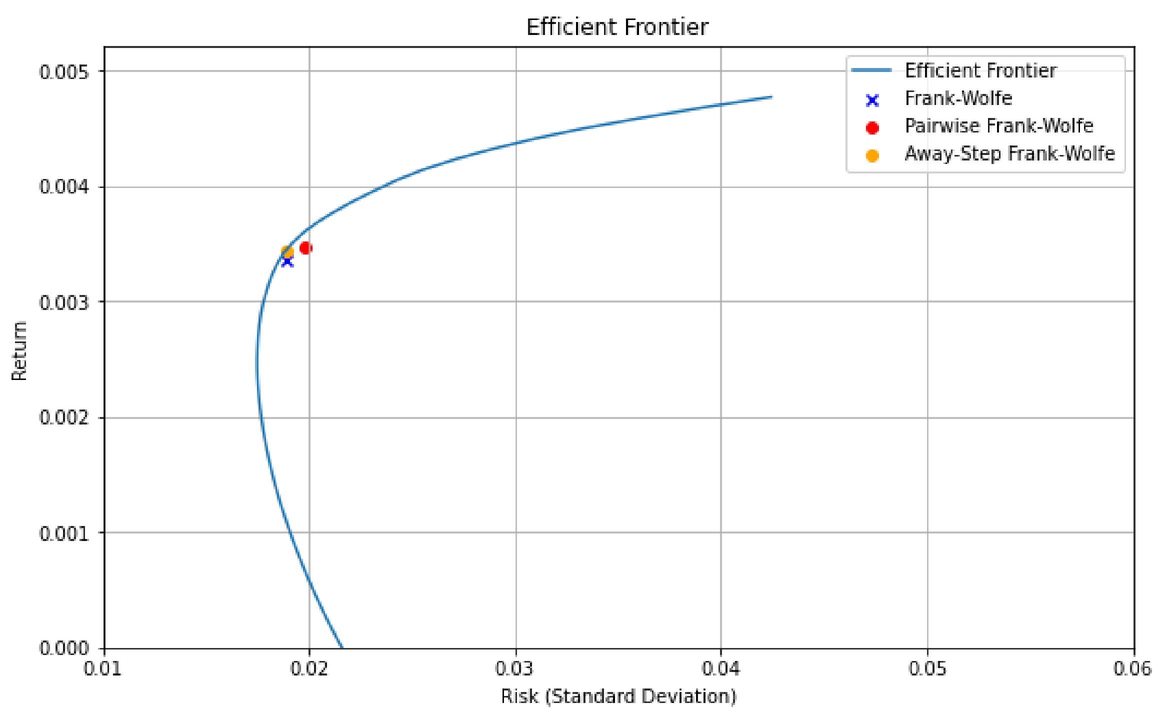
PAIR WISE FRANK WOLFE

Is the most efficient in terms of **weekly expected return**

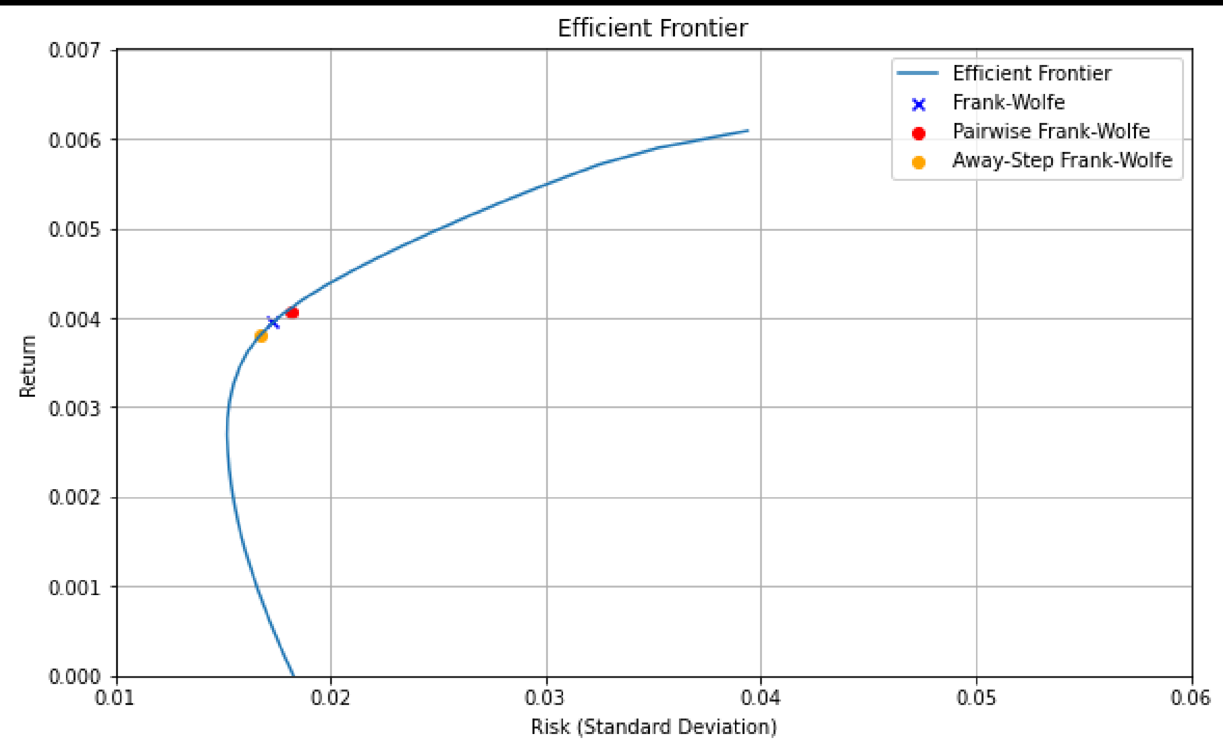
Dataset	FW	PFW	AFW
Euro Stoxx 50	*0.34%*	*0.35%*	*0.34%*
Ftse MIB	*0.40%*	*0.41%*	*0.38%*
Ftse 100	*0.47%*	*0.49%*	*0.49%*

EFFICIENT FRONTIER COMPARISON

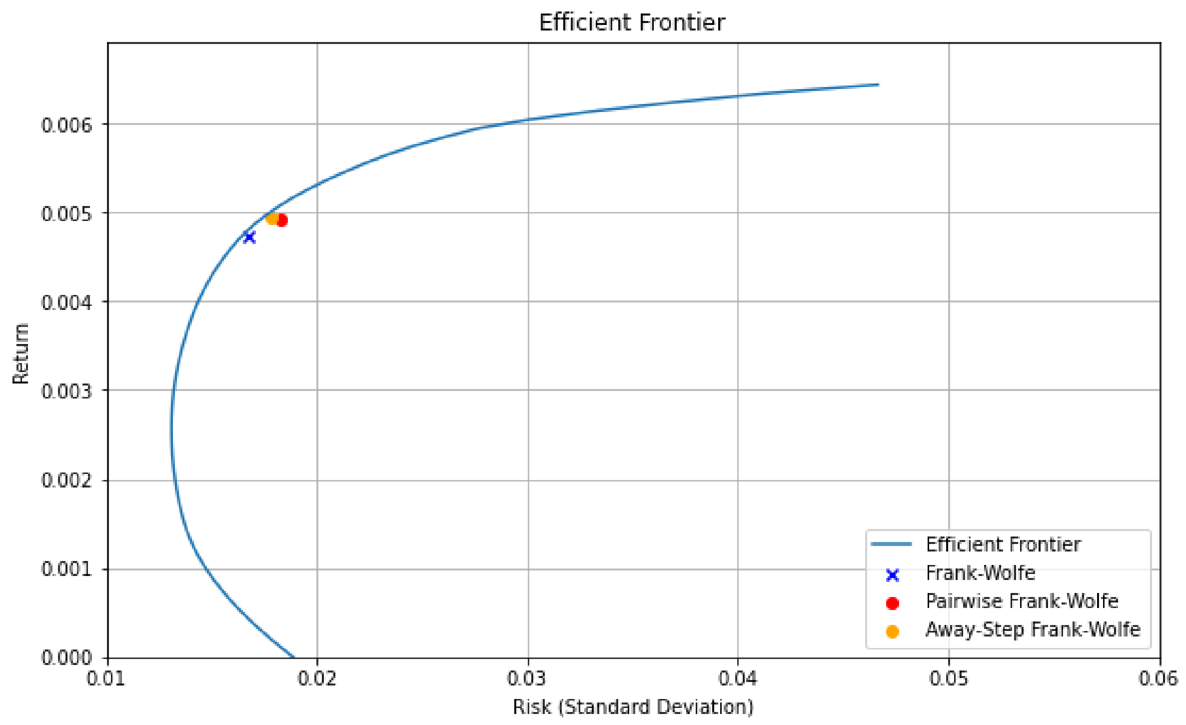
Euro Stoxx 50



Ftse MIB

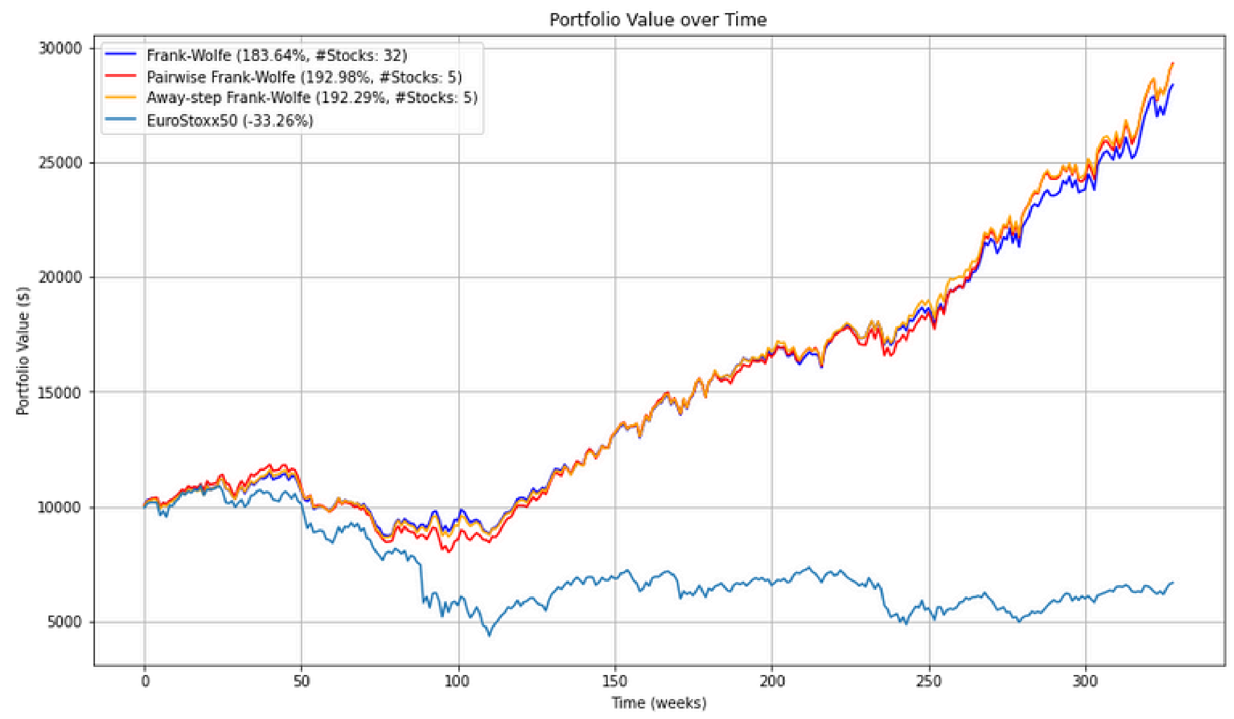


Ftse 100

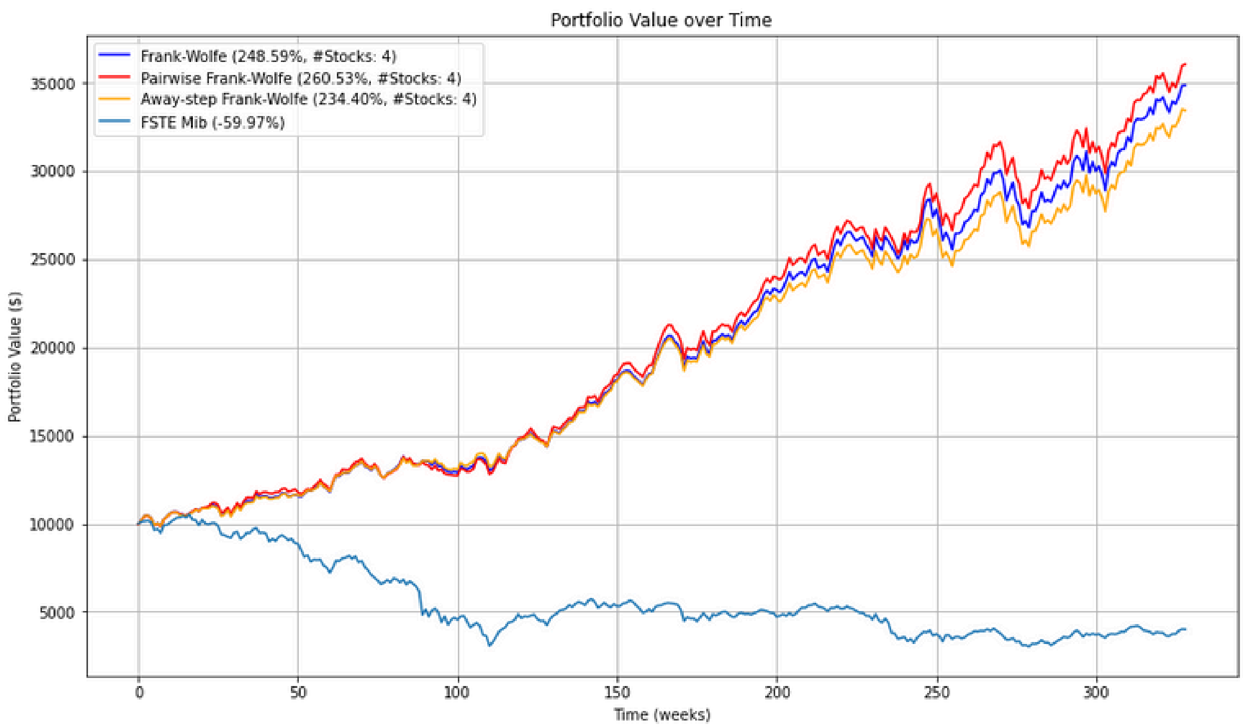


INVESTMENT SIMULATION

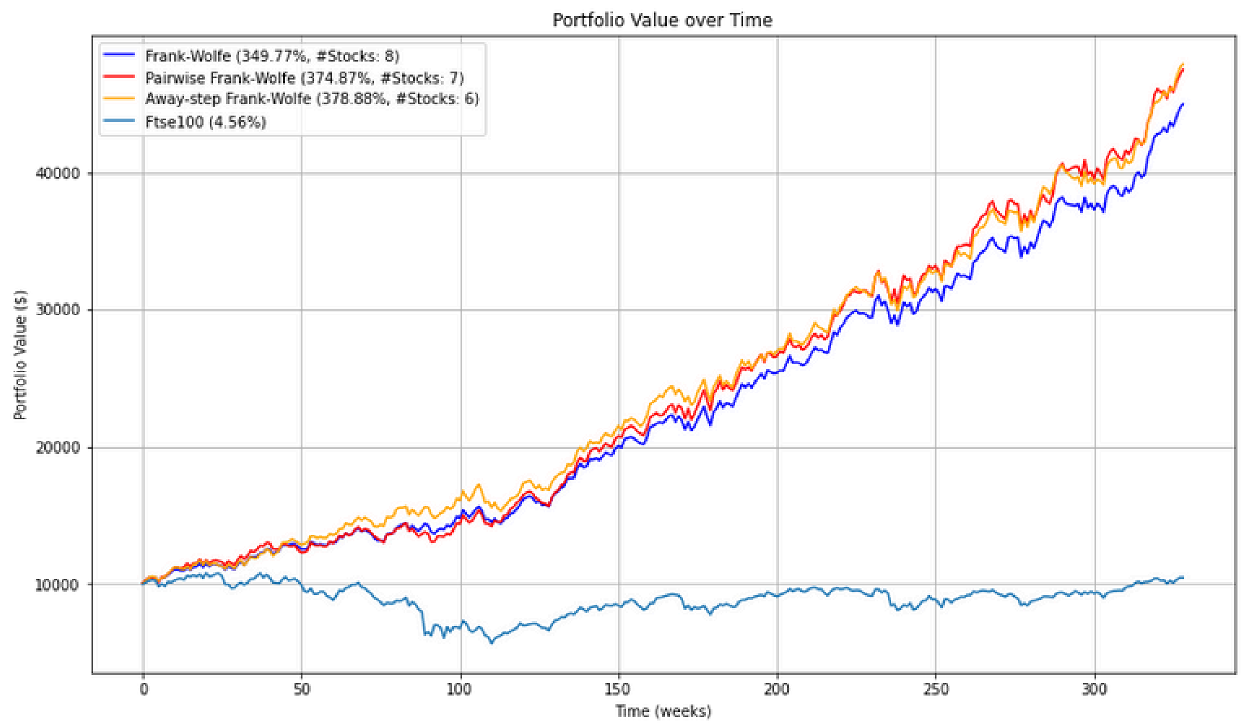
Euro Stoxx 50



Ftse MIB



Ftse 100



CONCLUSION

Frank-Wolfe Algorithm:

Best Performance:

- **Loss minimization**
- **Duality gap reduction.**

Away-Step Frank-Wolfe Algorithm:

- **Fastest execution times.**
- **Risk minimization**
- **Return optimization**

Pairwise Frank-Wolfe Algorithm:

- **Improvements over Away-Step FW in terms of loss and duality gap.**
- **Less efficient than the standard Frank-Wolfe.**

Future Work:

- **Adaptations for more complex financial models and larger datasets.**
- **Potential integration with machine learning techniques.**

THANK YOU