Solutions due by 10.30am Friday 26$^{\text{th}}$ February.

1. Show that if a square matrix is lower triangular or upper trianguar, then its determinant is the product of its diagonal entries.

   Recall lower triangular matrices have zeros below the main diagonal: $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}$.

2. Suppose A is diagonalizable. Show that $A^n = \underbrace{A \times A \times \ldots A}_{n}$ can be expressed more simply as $A^n = P^{-1}D^nP$.

3. Suppose A is diagonalizable. Show that

   a) $trace(A) = \lambda_1 + \cdots + \lambda_n$.
   b) $\det(A) = \lambda_1 \cdot \cdots \cdot \lambda_n$.

   Note that this is true in general for square matrices, whether they're diagonalizable or not, but the proof is harder. [Hint: you can use $\det(AB) = \det(A)\det(B)$.]

4. Consider an economy where agents can be in one of the states 'employed' or 'unemployed'. Suppose the probability that an employed person stays employed from one period to the next is 0.95. Suppose the probability that an unemployed person becomes employed in the next period is 0.7.

   a) Write this setup as a system of first order difference equations.
   b) Solve the system in terms of eigenvalues and eigenvectors.
   c) What is the long run rate of unemployment is this economy?

5. We defined $\text{Var}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2]$. Show that $\text{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

6. The pdf of the exponential distribution is

$$f(x, \lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

   its cdf is

$$F(x, \lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

a) Verify that f is a valid density function.

b) Verify that F is the distribution function corresponding to this density.

7. Show that $M_{aX+b}(t) = e^{bt}M_X(at)$.

8. Let $X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$. Show that $\sqrt{n}(\bar{X} - \mu) \sim N(0, \sigma^2)$.

9. An economic agent needs to make a decision based on an unknown random variable $\theta \sim N(0, \sigma_\theta^2)$. The agent knows the means and variances of these distributions. They would like to know the realization of θ exactly but are only given a noisy signal $y = \theta + \varepsilon$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is iid noise. Suppose the agent's strategy is to guess θ using a multiple of the signal: $\hat{\theta} = cy$. What constant c should the agent pick in order to minimize expected squared error: $\mathbb{E}[(\hat{\theta} - \theta)^2]$?