No solutions due in Week 1.

1. Use truth tables to determine whether the following propositions are tautologies, contradictions, or contingencies.

   a) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ b) $(p \Rightarrow q) \Rightarrow (\neg p \Rightarrow \neg q)$ c) $(p \Leftrightarrow q) \Leftrightarrow (\neg p \Leftrightarrow \neg q)$

2. Prove De Morgan's laws for sets.

   a) $(A \cup B)^c = A^c \cap B^c$ b) $(A \cap B)^c = A^c \cup B^c$

3. Find the power set for each of the following sets.

   a) $\{R, G, B\}$ b) $\{\emptyset\}$ c) $\mathcal{P}(\{\emptyset\})$

4. Prove by contradiction that the sum of a rational number and an irrational number is irrational.

5. Prove by induction that $1 + x + x^2 + \cdots + x^n = \frac{1-x^{n+1}}{1-x}$ for $|x| < 1$.

6. Prove by induction that $n! \geq n^2 \quad \forall n \geq 4$.

7. Let R be a binary relation from a set X to itself. Prove the following properties of R:

   a) If R is asymmetric then it is anti-symmetric.
   b) If R is asymmetric then it is irreflexive.
   c) If R is irreflexive and transitive then it is asymmetric.

8. Let $X = \{a, b, c, d\}$, and $(X, \succsim)$ be a rational weak preference relation. Under these preferences we have $a \succ b \sim c \succ d$. What is the graph of the relation?

9. Find an example of a function $f : \mathbb{N} \to \mathbb{N}$ that is:

   a) surjective but not injective.
   b) injective but not surjective.
   c) neither injective or surjective.
   d) bijective.

10. Let $f : X \to Y$ be a function. Let U_1 and U_2 be subsets of X, and let V_1 and V_2 be subsets of Y. Show that

a) if $U_1 \subseteq U_2$ then $f(U_1) \subseteq f(U_2)$.

b) if $V_1 \subseteq V_2$ then $f^{-1}(V_1) \subseteq f^{-1}(V_2)$.

c) $\forall U$, $U \subseteq f^{-1}(f(U))$.

d) $\forall V$, $f(f^{-1}(V)) \subseteq V$.

For c) and d), produce an example where the left hand side is a *strict subset* of the right hand side.

11. Show that a preference relation $(X, X, \succsim)$ can be represented by a utility function *only if* preferences are complete and transitive.

12. Let (X, d_1) and (X, d_2) be metric spaces. Show that $d_3(x, y) = \max\{d_1(x, y), d_2(x, y)\}$ is a valid metric.

13. Prove that the union of any number of open sets is open.

14. Show that the intersection of finitely many open sets is open.