1. There is an infinitely lived agent with time progressing in discrete steps, $t = 0, 1, 2, \ldots$. The agent receives utility from consumption in each period, $U(c_t) = \log(c_t)$. They begin at time $t = 0$ with a stock of savings $s_0 = 100$. Savings do not earn interest and the agent must choose each period how much of their savings to spend on consumption. The agent aims to maximize the total discounted utility across their life, with discount factor $\beta$. That is, the agent solves

$$\max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t)$$
$$\text{s.t. } c_t + s_{t+1} = s_t$$
$$\lim_{t\to\infty} s_t = 0$$

The first order conditions for this problem are

$$\beta \frac{1}{c_{t+1}} = \frac{1}{c_t} \tag{1}$$
$$c_{t+1} + s_{t+1} = s_t \tag{2}$$

a) Write the first order conditions as a system of linear difference equations.

b) Find the eigenvalues and eigenvectors of the system.

c) Find the solution to the system.

d) How many periods until the agent has used up half of their original wealth?