Introduction to Regression

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Outline

1. Modelling
2. Ordinary Least Squares
3. OLS Assumptions
4. Identifying and dealing with problems
5. Further Reading

Model for securities

Model security return. You have thought about this already as it is an important component of

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- CAPM
  - Beta is the relationship between securities returns and the market
- Diversification
  - Distinguish market risk and ideosyncratic risk
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  - What is the ideosyncratic or individual performance of the security?

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- ε_t is the error that covers omitted variables, measurement error and other stochastic or random elements

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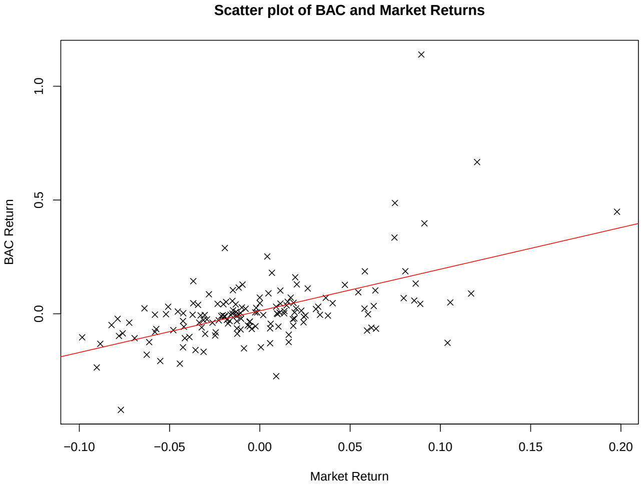
- x_t is the return of the market (S&P 500)
- β is the relationship between BAC returns and the market returns
- ε_t is all the other factors that affect BAC returns

Caution!

> *"Essentially all models are wrong, but some are useful"*
>
> (Box, 1987, p. 424)

S&P 500 and BAC



Solution 1

$y_t = a + bx_t + u_t$

Minimise the residuals

$$Min \sum_{t=1}^{t=T} u_t^2$$

$$Min \sum_{t=1}^{t=T} (y_t - a - bx_t)^2$$

Take, partial derivative to get the condictions.

Solution 2

$$\frac{\delta u}{\delta a} = \sum_{t=1}^{t=T} 2(y_t - a - bX_t) = 0$$

$$\frac{\delta u}{\delta b} = \sum_{t=1}^{t=T} 2X_t(y_t - a - bx_t) = 0$$

Re-arrange

$$\hat{b} = \frac{\sum x \sum y - \frac{\sum x \sum y}{T}}{\sum x^2 - \frac{(\sum x)^2}{T}}$$

$$\hat{a} = \bar{Y} - b\bar{X}$$

Solution: matrix form

In matrix form

$$\begin{aligned}\mathbf{y} =& \mathbf{X}\beta + \mathbf{u} \\ \mathbf{u} =& \mathbf{y} - \mathbf{X}\beta \\ \mathbf{u}`\mathbf{u} =& (\mathbf{X}\beta + \mathbf{u})`(\mathbf{X}\beta + \mathbf{u})\end{aligned}$$

Taking derivative and re-arranging (see textbook for proof)

$$\beta = (\mathbf{X}`\mathbf{X})^{-1}\mathbf{X}`\mathbf{y}$$

Regression Table

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.0130	0.0105	1.23	0.2203
SPY.R	1.8303	0.2240	8.17	0.0000

The Adjusted R^2 is 0.29, therefore nearly 30% of the BAC returns are explained by the returns of the market. 95% confidence intervals for the β are 1.39 to 2.27.

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Given a number of assumptions OLS is the BLUE **B**est, **L**inear, **U**nbiased, **E**stimator.

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- Explanatory variables are not related to the error
- Additionally, assume *normal errors* if we want to use normal assumption to compute *t-tests* of coefficients

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Solution: Add missing variable or a proxy

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Solution: Remove superfluous variable. Be careful of *the dummy variable problem*

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    - Use dummy variables

Eviews

- Eviews Website

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- Tutorials

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- User Guide 1 Chapter 11 (p. 315 to 321)

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- User Guide 2 Chapter 18 (p. 1 to 22)

Textbooks

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- D Gujarati, "Basic Econometrics", McGraw-Hill

Bibliography

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