Math Information

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1 Introduction

2 Eigenvalues

This comes from the Khan academy. The selection of videos begins Eigene Everything. it may even be good to go back further to the earlier documents on linear algebra. That would come Alternative coordinate systems: basis.

The key idea is that eigenvectors and eigenvalues can be used as useful basis vectors. A three element vector can be an arrow in three-dimensional space starting at the origin, an eigenvector $\mathbf{v}$ is an arrow whose direction si preserved or reversed when multiplied by $\mathbf{A}$.

If a square matrix $\mathbf{A}$ is multiplied by a vector $\mathbf{v}$, the result will be another vector, $\mathbf{w} = \mathbf{Av}$.

V is mapped to W by AV. v and w will not usually by parallel (meaning that they are on the same three dimensional line running through the origin). When they are parallel, v is an eigenvector or A and λ is the eigenvalue.

In that case,

$$w_i = A_{i,1}v_1 + A_{i,2}v_2, + \cdots + A_{i,n}v_n = sum^n_{j-1} A_{i,j}v_j \tag{1}$$

Take a look at Wikipedia.

The eigenvalue equation for a matrix A is

$$Av = \lambda v = 0 \tag{2}$$

This is equivalent to

$$(A - \lambda I)v = 0 \tag{3}$$

Linear Algebra says that an equation $Mv = 0$ has a non-zero solution *if and only if* $det(M)$ is zero. Therefore,

$$det(A - \lambda I) = 0 \tag{4}$$

To find the eigenvalues use the fact that

$T(x) = A\overrightarrow{x}$ can be represented as $T(\overrightarrow{v}) = A\overrightarrow{v} = \lambda\overrightarrow{v}$ where $\overrightarrow{v}$ is the *eigenvector* and λ is the *eigenvalue*.