Introduction to Time Series Analysis

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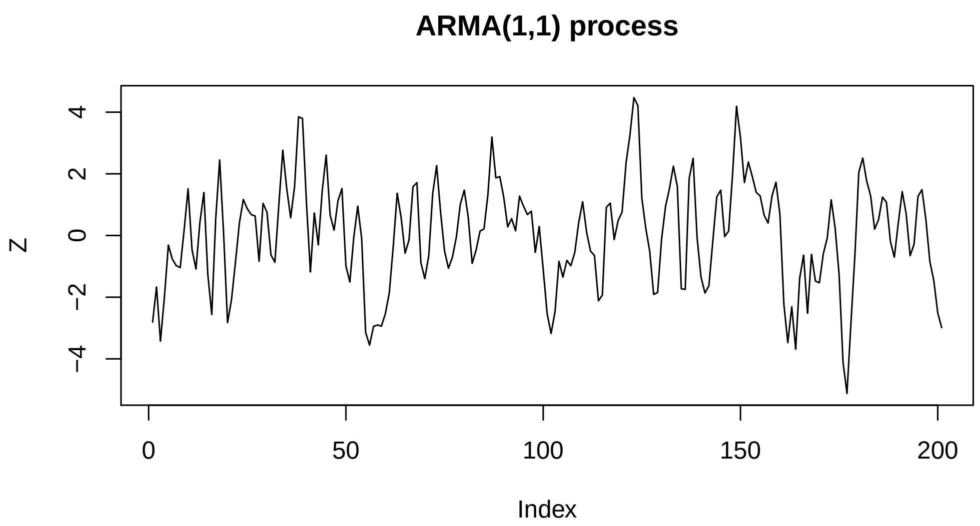
1 ARMA(p,q) time series

This comes from Arthur Charptentier. Given the ARMA(1,1) model

$$X_t = \phi X_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \tag{1}$$

```
theta = 0.7
phi = 0.5
n = 1000
Z = rep(0, n)
set.seed(1)
e = rnorm(n)
for (t in 2:n) Z[t] = phi * Z[t - 1] + e[t] + theta * e[t - 1]
Z = Z[800:1000]
plot(Z, type = "l", main = "ARMA(1,1) process")
```



If the MA element is not identified and purely AR model is estimated by least squres the estimate of ϕ is not consistent.

$$X_t = \phi X_{t-1} + \varepsilon_t \tag{2}$$

```
base = data.frame(Y = Z[2:n], X = Z[1:(n - 1)])
regression = lm(Y ~ 0 + X, data = base)
summary(regression)

##
## Call:
## lm(formula = Y ~ 0 + X, data = base)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -3.245 -0.791  0.063  0.971  3.069
##
## Coefficients:
##   Estimate Std. Error t value Pr(>|t|)
## X    0.696      0.051    13.6   <2e-16 ***
## ---
## Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
##
## Residual standard error: 1.22 on 199 degrees of freedom
##   (799 observations deleted due to missingness)
## Multiple R-squared: 0.483,Adjusted R-squared: 0.481
## F-statistic:  186 on 1 and 199 DF,  p-value: <2e-16
```

Compute the autocorreltion of the noise.

```
n = 200
cor(residuals(regression)[2:n], residuals(regression)[1:(n - 1)])

## [1] 0.2663
```

More formlly, with the Durbin-Watson sttistic

```
require(car)

## Loading required package:  car
```

```r
durbinWatsonTest(regression)

##  lag Autocorrelation D-W Statistic p-value
##    1          0.2657         1.463   0.002
##  Alternative hypothesis: rho != 0
```

It should be assumed that

$$u_t = \varepsilon_t + \theta_{t-1} \tag{3}$$

and

$$\rho(1) = \frac{\theta}{1 + \theta^2} \tag{4}$$

```r
polyroot(c(1, -1/cor(residuals(regression)[2:n], residuals(regression)[1:(n -
    1)]), 1))

## [1] 0.2885-0i 3.4666+0i
```